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DATA REDUCTION, ERROR ANALYSIS AND IDENTIFICATION OF SYSTEMATIC ERRORS IN THE GRAVITY PROBE B EXPERIMENT

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Abstract

In addition to the basic data analysis to determine the geodetic and frame-dragging relativistic effects, an essential part of the Gravity Probe B Relativity Mission is the in-flight verification and calibration of possible sources of systematic errors. In preparation for the analysis of the telemetry information, a) nonlinear optimal filters that estimate the system state vector and calculate the statistical error due to the SQUID and telescope readouts have been designed and tested; b) quantitative *a priori* estimates of the magnitude and directions of more than 100 classical torques on the GP–B gyroscope have been made; c) procedures for calibrating the magnitude or verifying the absence of potential systematic experimental effects have been developed.

Detailed simulations and torque analysis demonstrate that the expected standard deviation of estimation error for both components of the relativistic drift is 0.2 *milli-arcsec/year* for a single gyroscope in a 12 month measurement time, and the additional error due to classical torques on the gyroscope is less than 0.1 *milli-arcsec/year*. In-flight and ground-based calibration and verification methodology is discussed. These procedures will allow reduction and even elimination of possible sources of systematic experimental errors.

1 Introduction

Along with the technological development of experimental hardware and appropriate techniques, it is extremely important for the Gravity Probe B experiment to carry out a thorough error analysis and to design and to implement an adequate data reduction algorithm. It is the unprecedented accuracy of the experiment, in the first place, that requires serious consideration of many different factors that could affect the overall result. In plain words, it is not easy to extract the values of relativistic drifts from the readout signal and to ascertain its required accuracy. Moreover, GP–B science mission has been set up as an uncalibrated experiment, which means that the non-relativistic drift of gyroscopes is not included in the measurement model, being expected to be surprisingly small (less than 0.1 *marcsec/yr*) under the unique experimental conditions.

In this paper we give a brief survey of efforts to carry out the necessary error analysis and create the means of reliable and precise data reduction. We assume the reader to be familiar with the general concept of the GP–B experiment, which is described, in particular, in paper [1] (see also the references therein).

2 Classical Torques and Error Tree

Classical torques on the GP–B gyro had been primarily considered as the most important source of experimental errors, therefore they have been extensively investigated for a long time; a survey of earlier studies of them is found in [2]. There are two basically different types of torques: support dependent (SD) torques, which are caused by operation of the electrostatic gyro suspension system, and support independent ones (SI), due to various physical factors, such as the London moment coupling with the local shield, or residual gas pressure, etc. The results on torques of either group are given in [3] and [4], respectively.

In particular, in [3] it is shown that each SD torque is proportional to the sum or difference of squares of voltages applied to a pair of suspension system electrodes, and there are only 15 major different proportionality coefficients called *torque coefficients* which depend entirely on the rotor asphericity and are expressed via spherical harmonics coefficients of the rotor shape by the formulas from fundamental paper [5] containing the first version of SD torque theory. The torque coefficients of GP–B gyros are studied in detail in [6].

A *position factor* is also present in the SD torque equation which equals unity when the rotor spin axis and the spacecraft roll axis are perfectly aligned (*nominal* position), equals the appropriate misalignment angle for the *misaligned* position, and equals the appropriate miscentering of the rotor for the *miscentered* position. Finally, a sum of squared electrode voltages is related to a suspension system *preload*, while the difference is always proportional to some *acceleration* acting on the gyro; all known non-relativistic accelerations are analyzed in [7] and listed in [3]. This allows one to relate SD torque to the physical sources causing them.

All in all, the results of [3] and [4] provide a consistent classification of over 100 currently known torques, and a formula for the gyro drift rate due to each of them; those are used in the GP–B Error Tree, which is implemented in a convenient tool, an Excel spreadsheet book called New Error Tree (NET).

The NET Excel workbook consists of two main parts describing non-relativistic drifts and measurement (readout) errors, respectively. The second part contains the results of simulations of GP–B data reduction (see the next section). The simulations, of course, cannot be included in the spreadsheet, so all the numbers in this part are fixed and do not automatically change with the change of the values in the 'input' NET sheets (*"Constants", "Parameters", "Coefficients", "Preloads", and "Accelerations"*).

On the contrary, the first part of the NET is a 'live' one, i. e., one only has to specify (type in) numerical values of all necessary parameters in the 'input' sheets, and the values of all classical drift rates are immediately computed by the spreadsheet, giving also the total experimental error after combining those with the measurement error and the proper motion of the Guide Star. Here is how it works.

The input sheets include, first of all, the table *"Constants"* where the quantities are collected which either are fundamental constants, like speed of light, or have a well specified value in the GP–B project, such as the rotor radius. The numbers here can, in principle, be changed, causing automatically respective changes in the NET as a whole, but such changes typically are not needed. Still, any variation of any constant affects the value given in every dependent cell, that is, in all the cells of the NET spreadsheet where this constant is used.

All the quantities listed in the 'input' sheets have two numerical values corresponding to what is called 'worst' and 'plausible' cases; these two cases go also through all the 'output' sheets described below, so that two values, worst and plausible, are given for every classical drift rate, and thus ultimately the total experimental error in sheet *"Main"* is calculated for the worst and plausible case as well, since the measurement error is also estimated for the worst and plausible case.

The worst case values are either taken directly from the requirements, or derived from them, or else are the result of our estimate of what the worst case value may be for a particular parameter in reality; these estimates are based on the collective experience of the GP–B team. The plausible case values are either those already achieved experimentally, or are expected to be achieved "with the 99%" certainty. So, the plausible case shows what we strongly hope to have in the real experiment, while the worst case represents the upper bound of errors: they definitely are not going to be worse than that if all the hardware works as required. Naturally, the things in the NET book are arranged in such way that a change of a worst case value automatically implies variations of dependent *worst case* classical drift values *only*, not affecting the plausible case at all, and vice versa.

The table *"Parameters"* contains three groups: 1) rotor and housing, mechanical, 2) rotor and housing, electrical and magnetic, and 3)satellite, orbit, guide star; there are a little below 100 changeable parameters listed. A 'standard set' of torque coefficients is given in the sheet *"Coefficients"*; their worst case values are calculated from requirements under the assumption that the rotor asphericity is represented by two spherical harmonics only, the mass unbalance (l=1) and the oblateness (l=2), including the centrifugal one. The plausible values are obtained from the worst case ones by a one quarter reduction expected due to spin and polhode averaging. Precise measured data on the shape of three anodized GP-B rotors show that even the 'standard' plausible value of torque coefficients are typically larger than realistic ones [5], so that our estimates of classical drift rate has an essential margin with respect to these parameters.

The values of preloads at different dangerous frequences (D.C., roll, twice roll, transient) are listed in the table *"Preloads"*; predominantly, the required preload values are used for both cases. Accelerations at the same frequences due to all major physical sources (Earth and satellite gravity gradients, centrifugal force due to spacecraft roll, roll frequency variation, etc.) are found in sheets *"Accelerations"* and *"Effective accelerations squared"*; with just few exceptions, they are not numerically specified, but calculated from other input data by appropriate formulas from [7].

The total number of input changeable quantities used in the NET is around 120.

The first 'output' sheet of the NET book is *"Main"*; as mentioned, it shows the top portion of the Error Tree with the boxes containing contributions from classical torques, measurement error and the proper motion of the Guide Star. There are four values of the drift rate, in *marcsec/yr*, given in every box throughout all the sheets starting with *"Main"*, namely, their worst and plausible case values in North–South (NS, geodetic effect) and East–West (EW, frame-dragging effect) directions.

The NET continues in the *"Sheet 0"* where our classification of all classical torques is depicted by means of boxes with contributions of each group of torques. The two major groups are support dependent and support independent torques.

Inside SD torque group we recognize ***acceleration dependent*** (sheets 1–3) and *preload dependent* (sheets 4–6) torques, which are subsequently subdivided according to the rotor's position in which they are produced - ***nominal*** (sheets 1,4), ***misaligned*** (sheets 2,5) and ***miscentered*** (sheets 3,6). Therefore there are exactly $6 = 2 \times 3$ sheets with SD torques from *"Sheet 1"* (acceleration dependent, nominal position torques) to *"Sheet 6"* (preload dependent, miscentered position torques). In each of the sheets, torques are again subdivided into large groups, such as those caused by either parallel or perpendicular to the roll axis accelerations, or by preloads or preload differences, etc. Finally, the torques are sorted out by their physical source, and each torque is represented by its own box. The boxes are connected by lines in an ascending way, so that the structure of every sheet with torques is that of a tree graph: each box of a lower level is connected to a single upper level box.

SI torques are primarily classified in sheet 0 as housing fixed (sheets 7–9) and inertially fixed (sheet 10); the former are again sorted relative to the nominal, misaligned and miscentered rotor's position. Particular SI torques are dealt with in each of sheets 1–10 in a similar manner as SD torques in sheets 1–6.

All the ten sheets with the torques are filled in in the following way. In each of them there are the lowest level boxes showing drift rates due to a torque belonging to some particular group which is caused by a particular physical source. The four values of drift rates given in the box (NS worst, NS plausible, EW worst, EW plausible) are calculated by the proper formula from [2] or [3] in which either worst or plausible values of the necessary parameters from the 'input' sheets are used. As soon as the values of all the input quantities used specified, the numbers appear in the lowest level boxes, as well as all upper level ones, all the way up to the top in sheet *"Main"*. The numbers in the upper level boxes are automatically computed by means of a built-in procedure operating by the rule based on the assumption that different torques act independently. So, say, the squares of EW, worst case, drift rates from all the boxes of a group at any given level are summed up, and the square root of this sum is inserted as a value of EW, worst case, drift rate into the next level box to which the group is attached. This operation is repeated level after level from the bottom to the top of a given sheet with torques, which results in the numbers given in the uppermost box of the sheet representing the combined contribution of all the torques included in it (for instance, SD, acceleration dependent, nominal position torques from *"Sheet 1"*). Those uppermost boxes of sheets 1–10 serve as the lowest level boxes of *"Sheet 0"*, whose uppermost box, containing the total drift rate from all classical torques, serves, in its turn, as one of the three lowest level boxes of sheet *"Main"*, along with the box containing measurement error and the other one with the contribution of proper motion of the Guide Star.The resulting total non-relativistic drift rate values from the current version of the NET, which is being kept as GP–B document S0292, are

Worst case:
$EW = 0.262\, marcsec/yr \qquad NS = 0.205\, marcsec/yr$
Plausible case:
$EW = 0.084\, marcsec/yr \qquad NS = 0.083\, marcsec/yr$

The *measurement* error estimated by simulations described in the next section currently is
Worst case:
$EW = 0.270\, marcsec/yr \qquad NS = 0.270\, marcsec/yr$
Plausible case:
$EW = 0.210\, marcsec/yr \qquad NS = 0.190\, marcsec/yr$

Finally, the *proper motion* of the Guide Star in either direction is estimated as $0.15\, marcsec/yr$ and $0.12\, marcsec/yr$ for the worst and plausible cases, respectively. All the three results combined present the following *estimate of the total error in relativistic drift rate:*
Worst case:
$EW = 0.404\, marcsec/yr \qquad NS = 0.256\, marcsec/yr$
Plausible case
$EW = 0.371\, marcsec/yr \qquad NS = 0.240\, marcsec/yr$

These numbers are dominated by the measurement error, especially in the plausible case.

The New Error Tree proves to be a useful instrument of analysis of systematic experimental errors, their sensitivity to various parameters, as well as of the GP–B project requirement validation.

3 Data Reduction: Nonlinear Optimal Filtering

The GP–B readout system is based on the effect of magnetic field generated by a spinning superconductor. This field, the London moment dipole, is aligned with the instanteneous spin axis of the gyroscope [1]. Therefore the direction of the spin axis and thus its drift rate can be inferred from magnetic measurements.

The model of the readout system signal (the measurement equation) can be written as

$$z = C_g\Big[(NS_0+R_gt-\varepsilon_1)\cos(\omega_rt+\delta\phi)-(EW_0+R_ft-\varepsilon_2)\sin(\omega_rt+\delta\phi)\Big]+b+\nu, \quad (1)$$

where C_g is the scale factor, NS_0 and EW_0 are the initial misalignments, i. e., the angles between the gyro spin axis and the optical direction to the Guide Star in the North–South and East–West inertial directions, R_g and R_f are the geodetic and frame-dragging relativistic precession rates, ω_r is the spacecraft roll rate, $\delta\phi$ is the roll phase offset, ε_1 and ε_2 are the optical aberration components, b is the SQUID bias, and ν is the measurement noise.

The expression for the aberration signals are given in [8]:

$$\varepsilon_1 = \Lambda_o\sin(\omega_ot+\phi_o)+\Lambda_{a1}^{NS}\sin(\omega_at+\phi_a)+\Lambda_{a2}^{NS}\cos(\omega_at+\phi_a)$$

$$\varepsilon_2 = \Lambda_{a1}^{EW}\sin(\omega_at+\phi_a)+\Lambda_{a2}^{EW}\cos(\omega_at+\phi_a) \qquad (2)$$

Here $\Lambda_o = v_s/c$, v_s is the spacecraft orbital velocity, ω_o is the orbital angu velocity, ϕ_o is the initial orbital phase, ω_a and ϕ_a are the angular velocity a initial phase of Earth's motion around the Sun, and

$$\Lambda_{a1}^{NS} = \frac{v_E}{c}\sin\delta\cos\lambda, \quad \Lambda_{a2}^{NS} = \frac{v_E}{c}\left(-\sin\delta\sin\lambda\cos\alpha + \cos\delta\sin\alpha\right)$$

$$\Lambda_{a1}^{EW} = -\frac{v_E}{c}\sin\lambda, \qquad \Lambda_{a2}^{EW} = -\frac{v_E}{c}\cos\lambda\cos\alpha, \qquad ($$

with v_E being the orbital velocity of Earth, δ and λ being the declination a right ascension of the Guide Star, and α being the angle between the ecliptic a equatorial planes. Natural variation of aberrations is used to make the scale fact and roll phase offset observable.

Introducing the state vector of parameters to be estimated,

$$X = \left[C_g,\ \delta\phi,\ R_g,\ R_f,\ NS_0,\ EW_0,\ b,\right]^T, \qquad (\cdot$$

one is able to formulate the GP–B data reduction problem as follows.

Based on the measurements $\{z(t_1), z(t_2), \ldots\ldots, z(t_N),\}$, the known (measure or calculated) signals $\varepsilon_1(t)$, $\varepsilon_2(t)$, and model (1)—(3), find 'the best' estimate $\hat{x}$ (the state vector x.

Under a reasonable assumption that some components of the state vector x ma vary with the time during the experiment, the data analysis problem is recognize as the *nonlinear* filtering problem whose general form is: for

$$x_{k+1} = \Phi_k x_k + w_k, \quad z_k = F(x_k, t_k) + \nu_k, \qquad k = 1, 2, \ldots, N \qquad (5$$

find the estimate $\hat{x}$ of the state vector x that minimizes the least–square cost func tion

$$J = \frac{1}{2}\left[\sum_{k=1}^{N}\left(z_k - F(x_k, t_k)\right)^T R_k^{-1}\left(z_k - F(x_k, t_k)\right) + \sum_{k=1}^{N-1} w_k^T Q_k^{-1} w_k\right] \qquad (6$$

The dynamic model, i. e., the first of equations (5), describes how the unknow state vector is propagated through time by the transition matrix Φ, with the whit process noise w, which is Gaussian with the zero mean and covariance matrix Q The measurement noise ν is also assumed to be white, $E[\nu] = 0$, $E[\nu\nu^T]$.

As was shown by simulations, the standard nonlinear estimators, such as th extended Kalman filter (EKF) and the iterated extended Kalman filter (IEKF), give for the GP–B nonlinear measurement equation (1), a *biased* estimates of relativisti drifts R_g and R_g [8]. The reason is that both EKF and IEKF linearize cost functio (6), see [9]. To overcome this difficulty, a special nonlinear recursive two–ste estimator has been developed [10]. Instead of linearizing the cost function, it break the minimization procedure into two steps. A new set of states is defined for th first step using nonlinear combinations of the unknowns, so that the measuremen equation becomes linear with respect to the new ones. This first step linear proble can be solved optimally by applying a standard Kalman filter [11]. The second ste states are then calculated by treating the first step state estimates as measurements and using an iterative Gauss–Newton algorithm.

The two–step estimation approach was applied to the GP-B measurement model (1)—(4). By choosing the first step states as

$$y = f(x) = \Big[C_g\big(NS_0\cos\delta\phi - EW_0\sin\delta\phi\big),\ -C_g\big(NS_0\sin\delta\phi + EW_0\cos\delta\phi\big),$$
$$C_g\big(R_g\cos\delta\phi - R_f\sin\delta\phi\big),\ -C_g\big(R_g\sin\delta\phi + R_f\cos\delta\phi\big), \tag{7}$$
$$-C_g\cos\delta\phi,\ C_g\sin\delta\phi,\ b\Big]^T$$

we convert measurement equation (1) into a linear equation

$$z = H(t)y + \nu, \tag{8}$$

where

$$H = \Big[\cos\omega_r t,\ \sin\omega_r t,\ t\cos\omega_r t,\ t\sin\omega_r t,$$
$$\varepsilon_1\cos\omega_r t + \varepsilon_2\sin\omega_r t,\ \varepsilon_1\sin\omega_r t - \varepsilon_2\cos\omega_r t,\ 1\Big]^T \tag{9}$$

Applying now the two–step estimator [10] we obtain the following recursive estimation procedure:

First–Step Optimization

Measurement Update:

$$\hat{y}_k = \bar{y}_k + P_{y,k}H_k^T R_k^{-1}(z_k - H_k\bar{y}_k)$$
$$P_{y,k} = \left(M_{y,k}^{-1} + H_k^T R_k^{-1} H_k\right)^{-1} \qquad k = 1,2,\ldots,N$$

Time Update:

$$\bar{y}_{k+1} = \hat{y}_k + f_{k+1}(\bar{x}_{k+1}) - f_k(\hat{x}_k)$$
$$M_{y,k+1} = P_{y,k} + \left(\frac{\partial f_{k+1}}{\partial x_{k+1}}\right) M_{x,k+1}\left(\frac{\partial f_{k+1}}{\partial x_{k+1}}\right)^T\Bigg|_{x_{k+1}=\bar{x}_{k+1}} -$$
$$\left(\frac{\partial f_k}{\partial x_k}\right) P_{x,k}\left(\frac{\partial f_k}{\partial x_k}\right)^T\Bigg|_{x_k=\hat{x}_k}$$

Second–Step Optimization

Measurement Update: (i is the iteration index)

$$\hat{x}_{k,i+1} = \hat{x}_{k,i} - P_{x,k,i}\, q_{k,i}^T; \quad P_{k,i+1} = \left(\frac{\partial f_k}{\partial x_k}\right)^T P_{y,k}^{-1}\left(\frac{\partial f_k}{\partial x_k}\right)\Bigg|_{x_k=\hat{x}_{k,i}}$$
$$q_{k,i} = -\big(\hat{y}_k - f_k(\hat{x}_{k,i})\big)^T P_{y,k}^{-1}\left(\frac{\partial f_k}{\partial x_k}\right)\Bigg|_{x_k=\hat{x}_{k,i}}$$

Time Update:

$$\bar{x}_{k+1} = \Phi_k \hat{x}_k; \qquad \dot{M}_{x,k+1} = \Phi_k P_{x,k} \Phi_k^T + Q_k$$

Matrices Φ, H, Q and R, as well as nonlinear transformation $y = f(x)$, are already defined.

On the grounds of the two–step estimation approach we have developed *a family* of filters for processing signals from separate SQUIDS, combined 4–SQUID signals the SQUID–telescope system signal, and for express 12–hour analysis. We have also designed filters with augmented state vectors, for instance, to include optical effects, such as deflection of Guide Star light by the Sun, and stellar parallax. All these filters serve as a baseline for the GP–B data reduction algorithms, and, as mentioned in the previous section, they are being used for error analysis. They are also valuable for working out requirements to the proper elements and subsystems of GP–B hardware.

To make the measurement error analysis transparent and to outline its basic trends, a simplified analytic solution of the GP–B data reduction problem has been recently derived [12]. The major simplification is that there is no process noise in it ($Q = 0$). Despite a serious difference in the procedures, the analytical results demonstrate a remarkable agreement with those of full–scale simulation for $Q = 0$.

4 Verification and Calibration Procedures

The last of 12 fundamental scientific requirements of GP–B project [13] calls for various validation and verification procedures, to enhance the confidence of the experimental results. Those include sufficient redundancy in data acquisition (4 gyroscopes, etc., see below), extensive analysis of possible errors, and on-ground and in-flight calibration and verification tests. Special work has been done to pursue these goals which can be briefly described in the following way.

4.1 Internal Consistency of Experimental Data

a) Four independent gyroscopes made of different materials, with naturally different surface properties, asphericity and mass unbalance. The spin vectors of one pair of gyros will be in the direction to the Guide Star, the other pair will have oppositely directed spins. Each gyro will work under a slightly different gravity gradient, it will have its own suspension and electronics.

b) Two independent telescope readouts will provide the pointing in the direction of the Guide Star.

c) Two independent star trackers will be used for the spacecraft orientation.

d) At least two methods of spacecraft orbit determination will be exploited (GPS, laser ranging, TDRSS transponder)

4.2 External Consistency of Experimental Data

Several effects below have been previously measured, namely,

a) Geodetic effect

b) Gravitational deflection of starlight

c) Stellar parallax

d) Orbital motion of binary components of Guide Star

These effects will be available from GP–B experimental data and will be compared to the known results.

Gravity Probe B has been designed to make all classical torques and readout errors sufficiently small that they can be neglected in the data reduction. However, to demonstrate that these classical torques and readout errors have the expected magnitude an extensive ground-based and in-flight calibration procedure has been designed. This calibration procedure allows for full recovery of the experimental accuracy in the event of higher than expected torques or readout errors. It may also allow for a modest improvement in the overall experiment accuracy. Both ground-based and in-flight verification and calibration tests are being done and are planned; a flight phase after the end of the observational period is especially devoted to the in-flight verification. Several examples of such measures are given below.

4.3 *Ground Based Calibration*

a) The shape of three anodized GP–B rotors have been precisely measured, and the range of torque coefficients (see sec. 2) has been determined, to demonstrate the validity of classical torque error analysis carried out in the NET. This also allows one not to include even major classical torques as states in the data reduction algorithms.

b) SQUID bias temperature sensitivity is measured.

c) Temperature sensitivity of telescope readout bias and scale factor is measured.

4.4 *In-flight Calibration*

After the data acquisition period calibration of many sources of systematic errors will be carried out, mostly by a deliberate enhancement of error causing factors, for instance:

a) To calibrate classical torques, preload voltages, acceleration environment, misalignments and rotor miscentering will be deliberately enhanced, and the resulting drift rate measured.

b) To search for mismodeled errors, the satellite roll rate will be deliberately changed.

c) The temperature of the telescope solid state detectors will be changed to check optical readout errors.

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