DATA ANALYSIS IN THE GRAVITY PROBE B RELATIVITY EXPERIMENT

M.I.Heifetz
Gravity Probe B, W.W.Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA, U.S.A.

G.M.Keiser
Gravity Probe B, W.W.Hansen Experimental Physics Laboratory, Stanford University, Stanford, CA, U.S.A.

Abstract *Gravity Probe B (GP-B) is a gravitational experiment designed to measure two predicted by General Theory of Relativity precessions of a free-falling gyroscope placed in a polar orbit about the Earth. The frame-dragging effect (drift perpendicular to the orbital plane) has never been directly measured before, while the geodetic effect (drift in the orbital plane) will be measured with an unprecedented accuracy. GP-B Data Analysis includes processing telemetry data from several physical sources placed on the GP-B spacecraft: the science gyroscope's readout system, telescope optical system, Global Positioning System (GPS), and the spacecraft's attitude control system. We discuss here only one of the numerous problems that need to be resolved through Data reduction: precise estimation of the gyroscope's relativistic drift rates. The two-step nonlinear filtering approach is presented and estimation recursive algorithms that will be used in the GP-B Data Analysis are discussed.*

Keywords: Data analysis, Multi-sensor signal processing, Nonlinear filtering.

1 Introduction

The Gravity Probe-B Relativity experiment [1] makes use of gyroscopes in Earth polar orbit to measure two effects of Einstein's General Theory of Relativity with previously unachieved accuracy - the precessions of the local inertial frame free falling about the Earth with respect

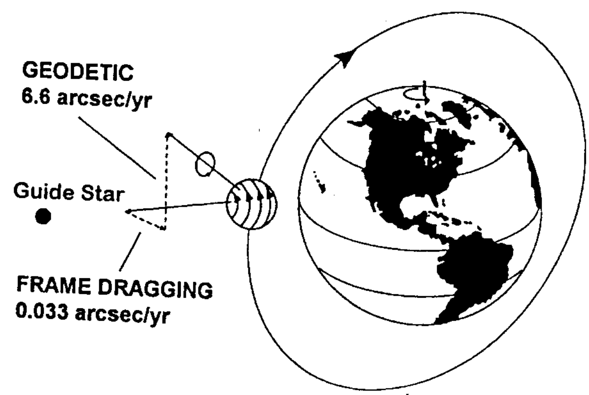

Figure 1: GP-B experimental concept

to the inertial frame of the distant universe. In a polar-orbiting spacecraft, with the gyroscope spin axes lying in the plane of the orbit and perpendicular to the Earth's rotation axis, the two effects to be measured - the geodetic effect and the never before measured frame-dragging effect - are at right angles with respect to each other. The magnitudes of the two effects in a 650 km polar orbit are 6.6 arcsec/year for the geodetic effect and between 33 and 42 marcsec/year for the frame-dragging precession, depending upon the choice of guide star. Figure 1 is a schematic representation of the GP-B experimental concept.

The experiment will measure these precessions with respect to the line-of-sight to a reference star whose position and proper motion with respect to the inertial frame of the distant universe (the "fixed" stars) is determined in separate astrometric measurements. The goal is to measure the geodetic precession to better than 1 part in 10^4 and the frame-dragging precession to better than 1 percent. The fundamental objective of the relativity mission is to measure the angular rate between the local frame (free-falling about the Earth) and distant inertial space (defined by the "fixed" stars) to an accuracy better than 0.5 marcsec/year, independently in each direction, for a one year experiment.

Changes in the direction of local inertial space are detected by measuring the Science Gyroscopes (SG) spin axis direction relative to the spacecraft (S/C) in which the SG are contained with a low noise, non-interfering readout system (four gyroscopes are used for redundancy with certain systematic effects removed by spinning them in opposite directions). The spacecraft is referenced to distant inertial space, as calibrated by a guide star, by the Science Telescope (ST) fixed to the spacecraft. The SG and ST data are subtracted from each other and corrected for known effects (such as aberration of starlight and others) in a data reduction process whose output is the measured drift between the local and distant inertial spaces; i.e., general relativistic drift. Extremely low levels of acceleration on the SG's are required to keep the Newtonian drifts of the gyroscopes from overly corrupting the experiment data. The spacecraft is therefore operated drag free to minimize the effects of disturbances on the science gyroscopes and guarantee that they remain in a purely gravitational orbit (geodesic). One science gyroscope will be used as the drag free proof mass to virtually eliminate the disturbing forces on that SG. The GP-B spacecraft and its attitude control system are designed to point the telescope continuously towards the guide star (or towards its apparent position) and to minimize the body-fixed pointing error.

The GP-B spacecraft also rotates at a constant roll rate about the line of sight to the guide star. Rolling the satellite allows to move the science signal to the roll frequency, where the readout measurement noise (which has spectrum) is lower. Rolling also allows a s gle gyroscope pick-up loop and its readout measure both the geodetic and frame-draggi precessions. Good orbital information of bo the Earth and the S/C is required for the e periment calibration against known fundame tal processes. The readout system scale fact is precisely calibrated during the experime using the optical aberration of starlight due the spacecraft motion around the Earth an the Sun.

The GP-B gyroscope has been designed a near perfect inertial instrument: Newtonia precession due to the classical torques is su posed to be less than 0.3 milliarcsec/year. Th means that the gyroscope's measured prece sion angle is assumed to be caused mainly b the relativistic effects. A detailed analysis c classical torques acting on the GP-B gyroscop is presented in [2].

In this paper we describe our approach t the GP-B Data Analysis as the set of fil ters that estimate the model-dependent systen state vectors and calculate covariance matrice that represent statistical errors of the relativis tic drift measurements due to the gyroscop and/or telescope readout noise and unmodelec disturbances.

2 GP-B Science Signal Model

The accuracy required in the GP-B experiment demands resolving numerous problems of the 'optimal' data processing of the GP-B science signals. Here we discuss only one of them: estimation of the relativistic geodetic and frame-dragging drift rates of the GP-B gyroscope from the data provided by the gyroscope's readout system. The GP-B readout system is based on the effect of magnetic field generated by a spinning superconductor(London moment) [3]. Precession of the gyroscope angular momentum results in the variations of the London moment, that is aligned with the instantaneous gyroscope spin axis. The science signal measured by the readout system represents the London moment magnetic flux through the gyroscope pick-up loop converted to the output voltage by the SQUID magnetometer.

The simplified model of the gyroscope science signal can be represented as

$$z(t) = C_g\Big[(NS_0 + R_g t - \varepsilon_1(t))\cos(\omega_r t + \delta\phi) - (EW_0 + R_f t - \varepsilon_2(t))\sin(\omega_r t + \delta\phi)\Big] + b(t) + \nu(t), \quad (1)$$

where C_g is the readout system scale factor, NS_0 and EW_0 are North-South (in the orbital plane) and East-West (perpendicular to the orbital plane) initial misalignments, R_g and R_f are the average drift rates, ω_r is the spacecraft roll rate, $\delta\phi$ is the roll phase offset, ε_1 and ε_2 are optical aberration components, b is the readout system bias. Measurement noise $\nu(t)$ is assumed to be a white noise with zero mean and known covariance matrix R.

Optical aberration is a shift in the apparent direction towards the guide star due to the velocity of the spacecraft to the line of sight to the star. There are two categories of aberration for the GP-B spacecraft: orbital aberration caused by the satellite's orbital motion around the Earth, and annual aberration, due to the motion of the Earth around the Sun. Optical aberration signals are continuously calculated based on the information from the on-board GPS and NASA/JPL Earth ephemerides.

3 Relativistic drift rate estimation

Introducing the state vector of parameters to be estimated,

$$\mathbf{X} = \Big[R_g,\ R_f,\ C_g,\ \delta\phi,\ NS_0,\ EW_0,\ b\Big]^T, \quad (2)$$

and under reasonable assumption that some components of the state vector (2) may vary with time during the experiment, the data analysis problem is recognized as the *nonlinear* filtering problem: for the linear state vector model

$$x_{k+1} = \Phi_k x_k + \Gamma_k w_k, \quad w \sim N(0, Q_k), \quad (3)$$

and nonlinear measurement model

$$z_k = F(x_k, t_k, \varepsilon_1(t_k), \varepsilon_2(t_k), \omega_r) + \nu_k, \quad k = 1, 2, \ldots, N \quad (4)$$

find the estimate $\hat{x}$ that minimizes the least-square cost function

$$J = \frac{1}{2}\left[\sum_{k=1}^{N}(z_k - F(x_k, t_k))^T R^{-1}(z_k - F(x_k, t_k)) + \sum_{k=1}^{N-1} w_k^T Q_k^{-1} w_k\right] \quad (5)$$

For the GP-B science signal structure (1), as it has been shown by numerous simulations, the standard nonlinear estimators, such as the extended Kalman filter (EKF) and the iterated extended Kalman filter (IEKF) [4] give, , a *biased* estimates of relativistic drifts R_g and R_f. The reason is that both EKF and IEKF linearize the measurement equation (4) and the cost function (5). To overcome that difficulty, a new nonlinear recursive two–step estimator has been developed [5]).

Instead of linearizing the cost function, it breaks the minimization procedure into two steps. A new set of states is defined for the first–step filter using nonlinear combinations of the unknowns, so that the measurement equation becomes linear with respect to the new ones. The choice of first step states is dependent on the particular problem being addressed. The first–step linear problem can be solved optimally by exploiting a linear Kalman filter. The second-step states are then calculated by treating the first step state estimates as 'new' measurements and by using an iterative Newton–Raphson searching algorithm.

By choosing the first step states as

$$y = f(x) = \begin{bmatrix} C_g(NS_0\cos\delta\phi - EW_0\sin\delta\phi) \\ -C_g(NS_0\sin\delta\phi + EW_0\cos\delta\phi) \\ C_g(R_g\cos\delta\phi - R_f\sin\delta\phi) \\ -C_g(R_g\sin\delta\phi + R_f\cos\delta\phi) \\ -C_g\cos\delta\phi \\ C_g\sin\delta\phi \\ b \end{bmatrix} \quad (6)$$

we convert the nonlinear measurement equation (1) into a linear one:

$$z = H(t)y + \nu, \tag{7}$$

where

$$H = \Big[\cos\omega_r t,\ \sin\omega_r t,\ t\cos\omega_r t,\ t\sin\omega_r t,$$

$$\varepsilon_1\cos\omega_r t + \varepsilon_2\sin\omega_r t,\ \varepsilon_1\sin\omega_r t - \varepsilon_2\cos\omega_r t,\ 1\Big] \tag{8}$$

Applying now the two–step estimator [5] we obtain the following recursive estimation procedure.

First–Step Optimization: ($\hat{y}$ and P_y are the optimal first step estimate and covariance matrix).

Measurement Update:

$$\hat{y}_k = \bar{y}_k + P_{y,k}H_k^T R_k^{-1}(z_k - H_k\bar{y}_k),$$

$$P_{y,k} = \left(M_{y,k}^{-1} + H_k^T R_k^{-1} H_k\right)^{-1} \tag{9}$$

Time Update:

$$\bar{y}_{k+1} = \hat{y}_k + f_{k+1}(\bar{x}_{k+1}) - f_k(\hat{x}_{k+1})$$

$$M_{y,k+1} = P_{y,k} + \left[\left(\frac{\partial f}{\partial x}\right) M_{x,k+1} \left(\frac{\partial f}{\partial x}\right)^T\right]\Bigg|_{x=\bar{x}_{k+1}};$$

$$-\left[\left(\frac{\partial f}{\partial x}\right) P_{x,k} \left(\frac{\partial f}{\partial x}\right)^T\right]\Bigg|_{x=\hat{x}_k} \tag{10}$$

Second–Step Optimization: ($\hat{x}$ and P_x are the optimal second step estimate and covariance matrix).

Iterative Measurement Update (i– iteration number):

$$\hat{x}_{k,i+1} = \hat{x}_{k,i} - P_{x,k,i}\, q_{k,i}^T; \qquad k = 1, 2, \ldots, N$$

$$P_{x,k,i+1}^{-1} = \left[\left(\frac{\partial f}{\partial x}\right)^T P_{y,k}^{-1} \left(\frac{\partial f}{\partial x}\right)\right]\Bigg|_{x=\hat{x}_{k,i}}$$

$$q_{k,i} = -(\hat{y}_k - f_k(\hat{x}_{k,i}))^T P_{y,k}^{-1} \left(\frac{\partial f_k}{\partial x_k}\right)\Bigg|_{x_k=\hat{x}_{k,i}} \tag{11}$$

Time Update:

$$\bar{x}_{k+1} = \Phi_k \hat{x}_k; \qquad k = 1, 2, \ldots, N-1$$

$$M_{x,k} = \Phi_k P_{x,k} {\Phi_k}^T + \Gamma_k Q_k {\Gamma_k}^T \tag{12}$$

Matrices H, Φ, Γ, Q and R, as well as nonlinear transformation $y = f(x)$, are defined above.

The two-step nonlinear estimator (2)–(12) has been used intensively for the general error analysis of the GP-B experiment. Figure 2 shows the dynamics of the estimation process and the potentially achievable accuracy of estimation. Qualitatively, the combination of the orbital (100-min period) and annual (1-year period) aberrations allows to determine the readout system scale factor C_g and roll phase offset $\delta\phi$ (science instrument dynamic calibration), which in turn allows to get the best estimate of the geodetic (R_g) and frame-dragging (R_f) relativistic drifts.

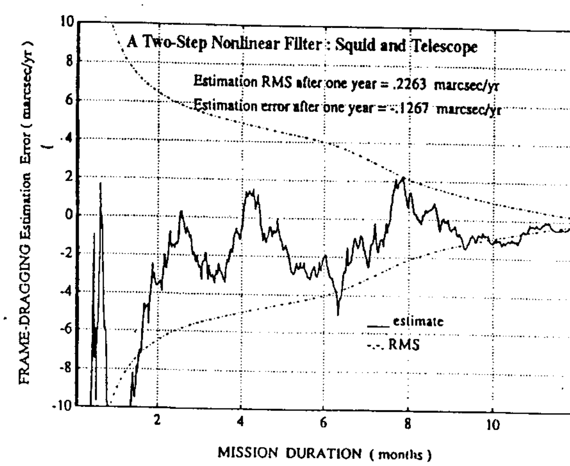

Figure 2: Relativistic drift estimation

The two-step filtering approach, described above, is also being used for various GP-B Data Analysis problems of the multi-sensor signal processing, where in order to achieve the required accuracy of the relativistic drift measurements, it is necessary to combine and optimally process data from the four GP-B science gyroscopes, science telescope's photo-detectors, spacecraft attitude control system, on-board GPS, together with the auxiliary information about the on-board environmental temperature and magnetic fields variations during the science mission. The 'bank' of filters with the different content of the state vectors, based on the above described methodology, is planned to be used in the data reduction that will start soon after the GP-B satellite's launch scheduled for October of the year 2000.

Acknowledgements

This work was supported by NASA grant NAS 8-39225 to Gravity Probe B.

References

[1] C.W.F.Everitt. *The Stanford Relativity Gyroscope Experiment: History and Overview.*, in *Near Zero,pp.587-639, W.H.Freeman and Co., New York, NY, 1980.*

[2] A.S.Silbergleit,M.I.Heifetz,G.M.Keiser. *Classical Torque Errors in Gravity Probe B Experiment*, in *Proceedings of the Third William Fairbank Meeting on the Lense-Thirring Effect, Rome-Pescara, Italy 1998.*

[3] F.London. *Superfluids Vol 1: Macroscopic Theory of Superconductivity.* Wiley, New York, 1953.

[4] T.F.Elbert, *Estimation and Control of Systems.* Van Nostrand Reinhold, New York, 1984.

[5] G.Haupt,N.Kasdin,G.Keiser,B.Parkinson. *Optimal Recursive Iterative Algorithm for Discrete Nonlinear Least-Squares Estimation.* Journal of Guidance, Control and Dynamics, **19**, 3, 643–649, 1996.