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Multisensor Data Integration in the NASA/Stanford Gravity Probe B Relativity Mission

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Abstract *Gravity Probe B (GP-B) is a gravitational experiment designed to measure two predicted by General Theory of Relativity precessions of a free-falling gyroscope placed in a polar orbit around the Earth. The frame-dragging effect (drift perpendicular to the orbital plane) has never been directly measured before, while the geodetic effect (drift in the orbital plane) will be measured with an unprecedented accuracy. GP-B Relativity Mission is an example of a system with multisensor data integration: it requires the optimal processing of information coming from at least nine physical sources (four science gyroscopes, the science telescope, the attitude control system of the spacecraft, on-board GPS receivers, NASA/JPL Earth ephemerides, and the astronomical data on the proper motion of the reference star). This paper presents the structure of the GP-B data integration and the core filtering approach to the state-vector estimation. Two-step nonlinear filter (that may be applied for different problems with nonlinear measurements) is discussed and the specifics of its implementation for the GP-B data analysis is presented.*

Keywords: Data analysis, Multi-sensor signal processing, Nonlinear filtering.

1 Overview of the Experiment

The NASA/Stanford Relativity Mission, also known as the Gravity Probe B (GP-B) experiment, will test two predictions of Albert Einstein's General Theory of Relativity based on observations of free-falling electrically sus-

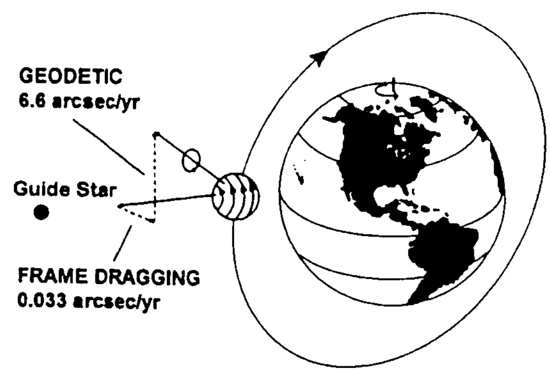

Figure 1: GP-B experimental concept

pended cryogenic gyroscopes in an 650 km circular orbit around the Earth [1].

General Relativity predicts that an ideal free-falling gyroscope (that, according to Newtonian Mechanics, is supposed to keep the direction of its spin axis) in reality would exhibit two relativistic precessions: geodetic drift, due to the curvature of the space-time, and frame-dragging drift, caused by the Earth, that as a massive rotating body drags space-time around it. The magnitudes of the two effects in a 650 km polar orbit are 6.6 arcsec/year for the geodetic effect and between 33 and 42 milliarcsec/year for the frame-dragging precession, depending upon the choice of a reference Guide Star. The measurements will be done by comparing orientations of the gyroscope's spin axis to the reference direction to the distant fixed star. Figure 1 shows a schematic representation of the GP-B experimental concept, and also demonstrates that the polar orbit will re-

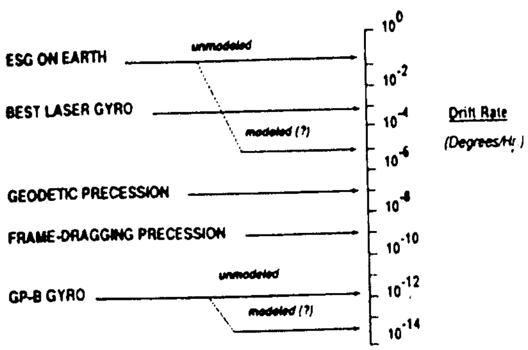

Figure 2: Geodetic and frame-dragging precessions, and performance of gyroscopes

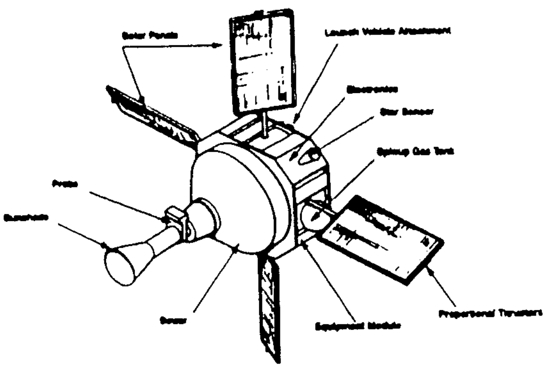

Figure 3: GP-B Spacecraft (schematic view)

sult in orthogonal geodetic and frame-dragging precessions, thus significantly simplifying the corresponding data analysis.

The fundamental objective of the Relativity Mission is to measure the angular rate between the local frame, free-falling about the Earth, and a distant inertial frame (defined by the 'fixed' stars) to an accuracy better than 0.5 marcsec/year.(To get the impression how small the relativistic drifts are and how high the required measurement accuracy is we can emphasize that 1 milliarcsecond is "the width of a human hair seen from the 15 kilometers"!). Figure 2 shows the relative performance of existing gyroscopes on Earth, GP-B gyroscopes, and the geodetic and frame-dragging precessions (0.3 marcsec/yr = 10^{-11} deg/hr).

2 GP-B Multisensor Structure and Data Integration

Gravity Probe B Relativity Mission represents an example of a complex multisensor integrated system that is supposed to function in space continuously for about two years. In order to measure the described above relativistic drifts with required accuracy it will be necessary to use at least nine sources of information: four science gyroscopes, the science telescope, the attitude control system of the spacecraft, on-board GPS receivers, NASA/JPL Earth ephemerides, and the astronomical data on the proper motion of the Guide Star.

Changes in the direction of local inertial space are detected by measuring the science gyroscope's spin axis direction relative to the optical direction to the chosen guide star. The gyroscope's readout system is based on the effect of magnetic field generated by a spinning superconductor(London moment). Precession of the gyroscope angular momentum results in the variations of the London moment, that is aligned with the instantaneous gyroscope's spin axis. The science signal measured by the readout system represents the London moment magnetic flux through the gyroscope pick-up loop converted to the output voltage by the SQUID magnetometer [2]. There will be four electrically suspended cryogenic science gyroscopes with the corresponding readout systems and their signals will be used for the redundancy and mutual calibration purposes.

The GP-B satellite is referenced to the distant inertial space by the special on-board telescope with its specific eight photo-detectors. The spacecraft's attitude control system is designed to point the telescope continuously towards the Guide Star (IM Pegasus)and to minimize the body-fixed pointing error. The 'proper' motion of the Guide Star (20 milliarcsec) will be taken into account based on the multi-year astronomical observations provided by the Harvard-Smithsonian Center for Astrophysics).

The GP-B spacecraft rotates at a constant

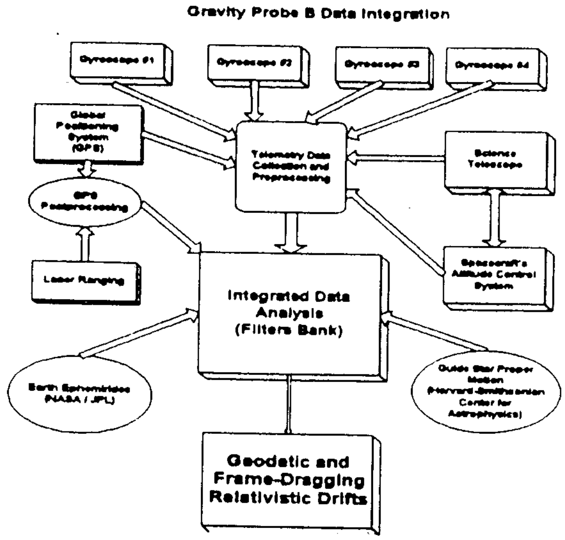

Figure 4: GP-B Multisensor Data Integration

roll rate about the line of sight to the Guide Star. Rolling the satellite allows to move the science signal to the roll frequency, where the readout measurement noise (which has 1/f spectrum) is lower. Rolling also allows a single gyroscope to measure both the geodetic and frame-dragging precessions. A schematic representation of the GP-B satellite is shown in Figure 3.

The observability of the relativistic drifts from the gyroscopes' readout signals is based on the effects of optical aberration caused by orbital motion of the GP-B satellite and annual motion of the Earth around the Sun. Orbital information will be provided by a GPS system (with the specially designed on-board GPS receivers). It is also planned to use a periodic laser ranging of the GP-B spacecraft. The best up-to-date information about the annual motion of the Earth will be obtained from NASA/JPL Ephemerides database.

The structure of the data integration that is being implemented in the GP-B Data Analysis is shown in Figure 4.

3 GP-B Data Analysis

To clarify the essence of the estimation problems to be solved in the GP-B data analysis we present below only one of the several existing simplified structures of the measurement configuration: one gyroscope, single-axis telescope signal, perfectly circular orbit, and the ideal GPS data.

The state of the system (parameters to be estimated) in this case is defined by the 8-dimensional vector

$$\mathrm{X} = \left[R_g,\ R_f,\ C_g,\ \delta\phi,\ NS_0,\ EW_0,\ b,\ C_T\right]^T, \tag{1}$$

where R_g and R_f are the relativistic drift rates, C_g is the gyroscope's readout system scale factor, C_T is the lelescope scale factor, $\delta\phi$ is the roll phase offset (misalignment between the roll axis of the spacecraft and the sensitivity axis of the readout system, NS_0 and EW_0 are North-South (in the orbital plane) and East-West (perpendicular to the orbital plane) initial misalignments of the gyroscope relative to the direction to the guide star, and b is the readout system bias. We also will assume that the state-vector components are either remained constant or slightly evolve during the 1-year experiment (particularly C_g, C_T, $\delta\phi$, and b).

The measurement equation (science model) can be reduced to the following structure:

$$\begin{aligned} z_G(t) = x_3\Big[&[x_5 + x_1 t + A_o\cos(\omega_o t) + L_1\cos(\omega_a t) \\ &+ L_2\sin(\omega_a t)]\cos(\omega_r t + x_4) \\ &+ [x_6 + x_2 t + L_3\cos(\omega_a t) + L_4\sin(\omega_a t)]\times \\ &\sin(\omega_r t + x_4)\Big] + x_7 + \frac{x_3}{x_8} z_T(t) + \nu(t), \end{aligned} \tag{2}$$

where $z_G(t)$ and $z_T(t)$ are the main science signals coming from the gyroscope and the telescope, ω_o, ω_a, and ω_r are the orbital, annual, and roll frequencies; A_o, L_1, L_2, L_3, and L_4 are parameters that are known (measured) based on the information from the GPS, laser ranging and the Earth ephemerides. Measurement noise $\nu(t)$, that includes both the gyroscope and the telescope readout noises, is assumed to be a white noise with zero mean and known standard deviation σ.

Nonlinear structure of the measurement equation (2) shows that the estimation of the state-vector (1) has to be formulated as the *nonlinear filtering problem*: for the linear state vector model

$$x_{k+1} = \Phi_k x_k + \Gamma_k w_k, \; w \sim N(0, Q_k), \quad (3)$$

and nonlinear measurement model

$$z_k = F(x_k, t_k, \omega_r, \omega_o, \omega_a) + \nu_k, \; \nu \sim N(0, R_k) \quad (4)$$

find the estimate $\hat{x}$ that would minimize the least-square cost function

$$J = \frac{1}{2}\Big[\sum_{k=1}^{N}(z_k - F(x_k, t_k))^T {R_k}^{-1} \times$$

$$(z_k - F(x_k, t_k)) + \sum_{k=1}^{N-1} w_k^T Q_k^{-1} w_k\Big] \quad (5)$$

For the GP-B science signal structure (2), as it has been shown by numerous simulations, the traditional nonlinear estimators, such as the extended Kalman filter (EKF) and the iterated extended Kalman filter (IEKF) [3] give a *biased* estimates of relativistic drifts R_g and R_f. The reason, as usual, is that both EKF and IEKF linearize the measurement equation (4) and fail to minimize the cost function (5). To overcome that difficulty and achieve the required extremely high accuracy of the estimation, we use the new two–step nonlinear recursive estimator [4].

4 Two–Step Estimator: optimal static solution

In this section we assume that the state–vector $\mathbf{x}$ consists of the constant parameters, so the estimation problem (4)–(5) can be rewritten in the batch form:

$$\mathbf{Z} = \mathbf{F}(x, t) + \mu, \quad (6)$$

$$J = \frac{1}{2}(\mathbf{Z} - \mathbf{F}(x, t))^T \mathbf{R}^{-1}(\mathbf{Z} - \mathbf{F}(x, t)) \quad (7)$$

where

$$\mathbf{Z} = [z_1, \ldots, z_N]^T, \mathbf{F} = [F_1, \ldots, F_N]^T,$$
$$\mu = [\nu_1, \ldots, \nu_N]^T.$$

The two-step nonlinear least-squares estimator divides the problem of minimizing the cost function into two steps. A new set of variables $y = f(x)$ is defined as the nonlinear combinations of the unknown original states, so that the measurement equation becomes linear with respect to the new states. The first–step linear estimation problem can be solved optimally by exploiting a linear Kalman filter. The second step is an iterative, nonlinear least squares fit of the first-step estimates.

The method for choosing the first-step variables is entirely problem dependent and for the GP-B problem is addressed in a later section. Here we assume only that there exists a transformation $y = f(x)$ such that the equation (4) becomes linear:

$$z_k = H_k y + \nu_k, \qquad k = 1, 2, \ldots, N \quad (8)$$

or in the batch form

$$\mathbf{Z} = \mathbf{H}y + \rho, \quad (9)$$

The cost function J_y for estimating the first-step states becomes

$$J_y = \frac{1}{2}(\mathbf{Z} - \mathbf{H}y)^T \mathbf{R}^{-1}(\mathbf{Z} - \mathbf{H}y) \quad (10)$$

and its minimization leads to the well-known linear weighted least square solution

$$\hat{y} = P_y \mathbf{H}^T \mathbf{R}^{-1} \mathbf{Z}, \qquad P_y = \left(\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\right)^{-1} \quad (11)$$

The estimates of the first-step states are treated as measurements in the second step. The second step measurement equation is

$$\hat{y} = f(x) + \mu \quad (12)$$

where the measurement noise μ has covariance matrix P_y. The second-step cost function is then defined as

$$J_x = \frac{1}{2}(\hat{y} - f(x))^T {P_y}^{-1}(\hat{y} - f(x)) \quad (13)$$

Taking the derivative of J_x yields

$$\frac{\partial J_x}{\partial x} = -(\hat{y} - f(x))^T P_y{}^{-1} \frac{\partial f(x)}{\partial x}$$

$$= -(\mathbf{Z} - \mathbf{H}f(x))^T \mathbf{H}^T \mathbf{R}^{-1} \frac{\partial \mathbf{H} f(x)}{\partial x}$$

$$= -(\mathbf{Z} - \mathbf{F}(x,t))^T \mathbf{R}^{-1} \frac{\partial \mathbf{F}(x,t)}{\partial x}$$

$$= \frac{\partial J}{\partial x}.$$

Thus minimizing the cost functions J_y and J_x in this manner is equivalent to minimizing the original cost function J, which proves the optimality of the two-step estimator in the static case.

The first-step optimization can be obtained with either a batch fit, or if recursive structure is desired, a Kalman filter represented in any of its well-known alternative forms [3]. Note that in the static case the first step is decoupled from the second step, i.e. all noisy measurements may be processed with a Kalman filter and the second step - conversion from the estimates $\hat{y}$ and P_y to $\hat{x}$ and P_x, needs to be done just once.

The second-step optimization for the nonlinear fit (12)-(13) is carried out by using the Newton-Raphson algorithm [3] and may be represented in the following iterative form:

$$\hat{x}_{i+1} = \hat{x}_i - \Gamma_i^{-1} \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}_i} P_y{}^{-1}(\hat{y} - f(\hat{x}_i)) \quad (14)$$

$$\Gamma_i = \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}_i}^T P_y{}^{-1} \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}_i} \quad (15)$$

$$P_x = \Gamma_i^{-1} \quad (16)$$

where the number of iterations i depends on a convergence criterion.

5 Two–Step Estimator: dynamic case

Two-step estimator can be generalized to incorporate the dynamics of the state-vector x. Without discussing details of the derivation we present below a summary of the two-step estimator in general case.

Given: the nonlinear measurement model (4), the linear state dynamics (3), the nonlinear state transformation $y_k = f_k(x_k)$

First–Step Optimization ($\hat{y}$ and P_y are the optimal first step estimate and covariance matrix):

Measurement Update:

$$\hat{y}_k = \bar{y}_k + P_{y,k} H_k^T R_k^{-1}(z_k - H_k \bar{y}_k),$$

$$P_{y,k} = \left(M_{y,k}^{-1} + H_k^T R_k^{-1} H_k\right)^{-1} \quad (17)$$

Time Update:

$$\bar{y}_{k+1} = \hat{y}_k + f_{k+1}(\bar{x}_{k+1}) - f_k(\hat{x}_k)$$

$$M_{y,k+1} = P_{y,k} + \left.\left[\left(\frac{\partial f}{\partial x}\right) M_{x,k+1} \left(\frac{\partial f}{\partial x}\right)^T\right]\right|_{x=\bar{x}_{k+1}} - \left.\left[\left(\frac{\partial f}{\partial x}\right) P_{x,k} \left(\frac{\partial f}{\partial x}\right)^T\right]\right|_{x=\hat{x}_k} \quad (18)$$

Second–Step Optimization ($\hat{x}$ and P_x are the optimal second step estimate and covariance matrix):

Iterative Measurement Update (i– iteration number):

$$\hat{x}_{k,i+1} = \hat{x}_{k,i} - P_{x,k,i}\, q_{k,i}^T; \qquad k = 1, 2, \ldots, N$$

$$P_{x,k,i+1}^{-1} = \left.\left[\left(\frac{\partial f}{\partial x}\right)^T P_{y,k}^{-1} \left(\frac{\partial f}{\partial x}\right)\right]\right|_{x=\hat{x}_{k,i}}$$

$$q_{k,i} = -(\hat{y}_k - f_k(\hat{x}_{k,i}))^T P_{y,k}^{-1} \left.\left(\frac{\partial f_k}{\partial x_k}\right)\right|_{x_k=\hat{x}_{k,i}} \quad (19)$$

Time Update:

$$\bar{x}_{k+1} = \Phi_k \hat{x}_k; \qquad k = 1, 2, \ldots, N-1$$

$$M_{x,k} = \Phi_k P_{x,k} \Phi_k{}^T + \Gamma_k Q_k \Gamma_k{}^T \quad (20)$$

It is interesting to note that for the cases where the number of the first-step states is the same as the number of the second-step states, it is sometimes possible to invert functions f to obtain $\hat{x}$ as a function of $\hat{y}$, making iterations unnecessary.

6 Two–Step Estimator for the GP-B problem

The structure of the GP-B gyroscope's readout signal (1)-(2) allows the following nonlinear transformation of variables:

$$y = f(x) = \begin{bmatrix} x_3\cos(x_4) \\ x_4\sin(x_4) \\ x_3(x_1\cos(x_4)+x_2\sin(x_4)) \\ x_3(-x_1\sin(x_4)+x_2\cos(x_4)) \\ x_3(x_5\cos(x_4)+x_6\sin(x_4)) \\ x_3(-x_5\sin(x_4)+x_6\cos(x_4)) \\ x_7 \\ x_3/x_8 \end{bmatrix} \quad (21)$$

that converts the nonlinear measurement equation (2) into a linear one:

$$z = H(t)y + \nu, \quad (22)$$

where

$$H = \Big[\varepsilon_1\cos\omega_r t+\varepsilon_2\sin\omega_r t,\ \varepsilon_1\sin\omega_r t-\varepsilon_2\cos\omega_r t,$$

$$t\cos\omega_r t,\ t\sin\omega_r t, \cos\omega_r t,\ \sin\omega_r t,\ 1,\ z_T\Big], \quad (23)$$

and

$$\varepsilon_1 = L_1\cos(\omega_a t) + L_2\sin(\omega_a t),$$

$$\varepsilon_2 = L_3\cos(\omega_a t) + L_4\sin(\omega_a t),$$

The first-step estimation is carried out by the linear batch estimator or Kalman filter. As for the second step we use the fact, that the transformation (21) can be explicitly inverted:

$$\hat{C}_g = \hat{x}_3 = \sqrt{\hat{y}_1^2+\hat{y}_2^2},$$

$$\hat{\delta\phi} = \hat{x}_4 = arctan(\frac{\hat{y}_2}{\hat{y}_1}),$$

$$\hat{R}_g = \hat{x}_1 = \frac{1}{\hat{C}_g^2}(\hat{y}_3\hat{y}_1 - \hat{y}_4\hat{y}_2), \quad (24)$$

$$\hat{R}_f = \hat{x}_2 = \frac{1}{\hat{C}_g^2}(\hat{y}_3\hat{y}_2 + \hat{y}_4\hat{y}_1),$$

$$\hat{NS}_0 = \hat{x}_5 = \frac{1}{\hat{C}_g^2}(\hat{y}_5\hat{y}_1 - \hat{y}_6\hat{y}_2),$$

$$\hat{EW}_0 = \hat{x}_6 = \frac{1}{\hat{C}_g^2}(\hat{y}_5\hat{y}_2 + \hat{y}_6\hat{y}_1),$$

$$\hat{C}_T = \hat{x}_8 = \frac{\hat{C}_g}{\hat{y}_8},$$

It is clear from the equations (24) that the accuracy of determination of the relativistic drifts R_g and R_f depends greatly on the accuracy of the estimation of the scale-factor C_g. As it has been pointed out by Y.Bar-Shalom, the transformation from the variables (y_1, y_2) to (x_3, x_4) is the Cartesian-Polar transformation and the straightforward use of the first equation (24) should lead to the biased estimate of the scale-factor C_g, as it is known in the tracking theory for the Polar-Cartesian transformation [5]. The corresponding debiased technique has been developed for this case and results in the following correction

$$\hat{C}_g = \sqrt{\hat{y}_1^2+\hat{y}_2^2} + \frac{\sigma^2}{2\sqrt{\hat{y}_1^2+\hat{y}_2^2}} \quad (25)$$

where σ is a standard deviation of the measurement noise ν.

The two-step nonlinear estimator has been used intensively for the general error analysis of the GP-B experiment. Figure 5 shows the dynamics of the estimation process and the potentially achievable accuracy of estimation. Qualitatively, the combination of the orbital (100-min period) and annual (1-year period) aberrations allows to determine the readout system scale factor C_g and roll phase offset $\delta\phi$ (science instrument dynamic calibration), which in turn allows to get the best estimate of the geodetic (R_g) and frame-dragging (R_f) relativistic drifts.

The two-step filtering approach, described above, is also being used for various GP-B Data Analysis problems of the multi-sensor signal processing, where in order to achieve the required accuracy of the relativistic drift measurements, it is necessary to combine and optimally process data from the mentioned above

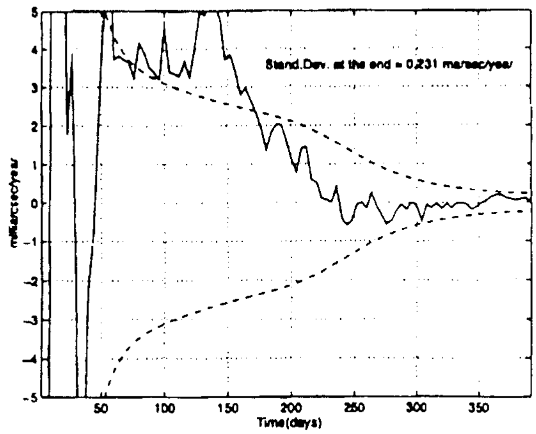

Figure 5: Relativistic drift estimation

nine main physical sources together with the auxiliary information about the on-board environmental temperature and magnetic fields variations during the science mission. The 'bank' of filters with the different content of the state vectors, based on the above described methodology, will be be used in the data reduction that will start soon after the GP-B satellite's launch scheduled for May of 2002 year.

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