CRYOGENIC EQUIVALENCE PRINCIPLE EXPERIMENT
Discussion and Present Status

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The Weak Equivalence Principle is the hypothesis that the ratio of passive gravitational mass to inertial mass is the same for all bodies regardless of their composition. This principle has a fundamental place in physics as the experimental basis for Einstein's Strong Equivalence Principle, which is a postulate of General Relativity. Since any violation of the principle would have very profound consequences for gravitational theory, it is important to test the principle to the greatest possible accuracy.

The immediate consequence of the Weak Equivalence Principle is the uniqueness of free fall, that is, all test bodies fall with the same acceleration. Thus the natural test of the principle consists in dropping two bodies of different composition and seeing that they have the same acceleration within the measurement error. Any violation of the equivalence principle may be measured by the quantity η , defined as

$$\eta = \frac{2(M_p/M_i)_A - (M_p/M_i)_B}{(M_p/M_i)_A + (M_p/M_i)_B} \qquad (1)$$

where M_p/M_i is the ratio of passive gravitational mass to inertial mass of body A or B . Equation (1) is equal to the difference in acceleration of the two bodies divided by their average acceleration. The present upper limit on η is about 3×10^{-11} . [1]

Unfortunately the simple "free-fall" experiment of dropping two masses from a tower is seriously limited by the time of fall and the accuracy with which the masses can be released; optimistically it could reach only about $\eta = 10^{-10}$. [2] Most modern measurements have been made with torsion balances using the sun's gravity as the accelerating field; a torsion balance has the advantages of simplicity, noise immunity, and indefinite period of measurement. These experiments are limited by the small size of the sun's gravity at the earth's orbit (about 0.6 cm/sec^2) and the noisy environment of the earth. A free-fall equivalence principle experiment in earth orbit eliminates the chief disadvantage of free-fall experiments on earth, and avoids a serious limitation on orbiting torsion balance experiments due to gravity gradients. [2] We expect that such an experiment might do much better than present measurements because it uses the earth's gravity (in orbit, about 800 cm/sec^2) instead of the sun's, and because a satellite environment may be much quieter than the surface of the earth (Fig. 1). The present experiment is an earth-based version of a proposed orbital experiment, which uses the sun's gravity as a source modulated by the earth's rotation. Its projected sensitivity compares very favorably with torsion balance experiments and should approach $\eta = 10^{-13}$.

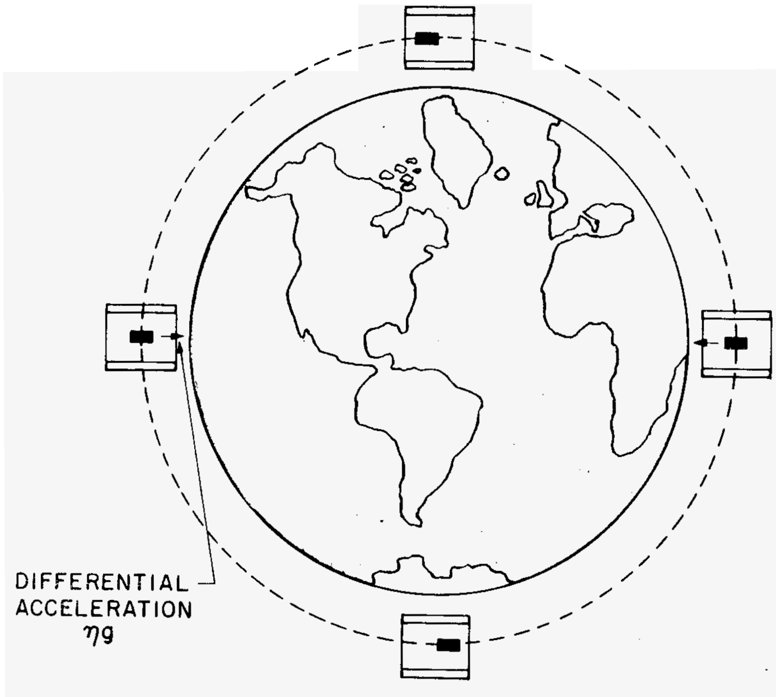

Figure 1: Concept of an Orbital Equivalence Principle Experiment

The experiment consists in directly comparing the accelerations of two test bodies floating in superconducting magnetic bearings. [3] There are three reasons for doing the experiment this way. First, it is always desirable to perform as many variations of a crucial experiment as possible. Second, analysis shows that even on the earth, this type of experiment can improve on existing measurements. Finally, the present version of the experiment is a stepping stone to a much more sensitive orbital experiment; the apparatus and techniques used for the ground experiment are almost directly applicable to the orbital version.

There are essentially five reasons for using cryogenics and superconductivity in this experiment: these are mechanical and electrical stability, extremely small loss factors, perfect magnetic shielding, reduction of gas pressure disturbances, and an excellent position readout by SQUID magnetometers. Thermal noise, which is commonly cited as a reason for operating at low temperature, is not a significant factor here because of low losses, the large masses used, and the very much larger disturbances which determine the practical limits of the experiment. The improvement in gas disturbances deserves brief discussion, because some gas pressure disturbances--for example viscous coupling and temperature gradient effects--seem to get worse as temperature decreases. The trick is that it is much easier to get very low pressures at low temperature [4], and the disturbances decrease faster with pressure than they increase with cooling.

The sensitivity of this ground experiment will be limited by the same cause that limits torsion balance experiments: seismic noise. In fact, because magnetic bearings have rather different responses to seismic noise than torsion fibers (in particular a simpler vibrational mode structure, and less intermode coupling), there is hope for improvement here. Furthermore the free-fall type of experiment can be made relatively more immune to gravity gradient noise. This disturbance, due to randomly moving masses (people, traffic, weather fronts, etc.) is expected to become a significant limitation at sensitivities somewhere below $\eta = 10^{-12}$. It is of critical importance to an orbiting experiment. A torsion balance cannot be used in earth orbit because of the earth's gravity gradient: small mass imbalances in the rotor cause it to tend to point in the direction of the gradient and limit the experiment to not much better than can be done on earth. In orbit, with two independent test bodies, it is possible to use the measured differential acceleration to estimate the center of mass offset between the two masses, and then control their positions so that the first-order gravity gradient force vanishes. By properly choosing the shapes of the masses, higher-order forces can also be made to vanish. It is not feasible to perform the equivalent balancing operations on the rigid rotor of a torsion balance.

Description

The concept of the apparatus is illustrated in Figure 2. Two cylindrical test masses, one of niobium and the other of lead-plated aluminum, are suspended in superconducting magnetic bearings which orient and position them. The masses are essentially free to move along their common axis and are rigidly constrained radially. The axial positions of the test masses are measured by superconducting coils near the ends

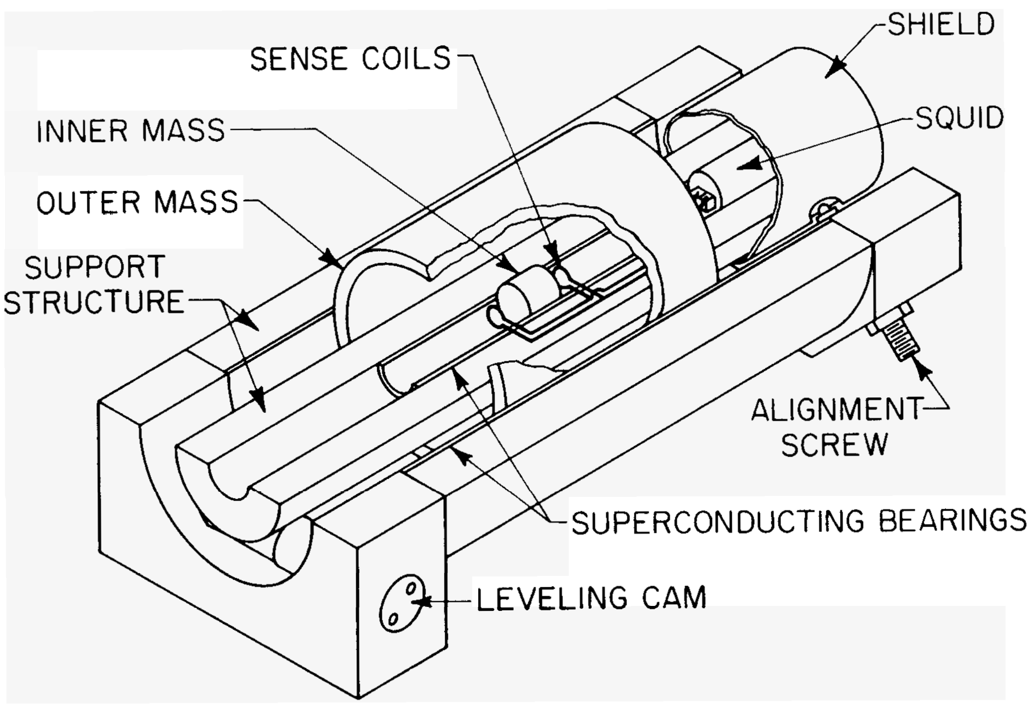

Figure 2: Equivalence Principle Accelerometer

of the test masses (see below). [5] Motion of the superconducting mass modifies these inductances and changes the flux coupled to a SQUID magnetometer. Control of the axial positions of the test masses is presently by a pair of coils under either end of each mass; in effect, currents in these coils tilt the magnetic support, without coupling much to the position detector coils. The position measurements are used to calculate a control signal for each mass which centers them on each other and in the bearings, and gives them identical periods and a small amount of coupling. In this condition the differential normal mode of the masses is not excited by seismic noise and their motions may be compared directly. The comparison is made by subtracting the control currents in a precision resistor at room temperature. Because of the large loop gain of the controller, drifts in the room temperature portions of the apparatus can be eliminated. The experiment is controlled by a microcomputer which keeps track of the masses, calculates the control signals, and records data.

In operation, the apparatus is mounted in a helium dewar on a specially designed antivibration platform. This platform incorporates a spherical air bearing to simulate the motion of a 200 meter long pendulum; since gravitational force is used to maintain the position of the platform, the apparatus is in free fall horizontally and provides excellent isolation from seismic noise at all frequencies. The practical limits to the isolation are determined by gas turbulence in the bearing, the necessity of making electrical and mechanical connections to the apparatus, and the sphericity of the bearing surface.

The magnetic bearings and their support structures are a critical component of the system [3,6], and have undergone substantial development. The requirement is for bearings having small dissipation, excellent stability, and large radial stiffness while exerting the least possible force along the cylinder axis. These requirements are met by a superconducting magnet of the design shown in Figure 3. The

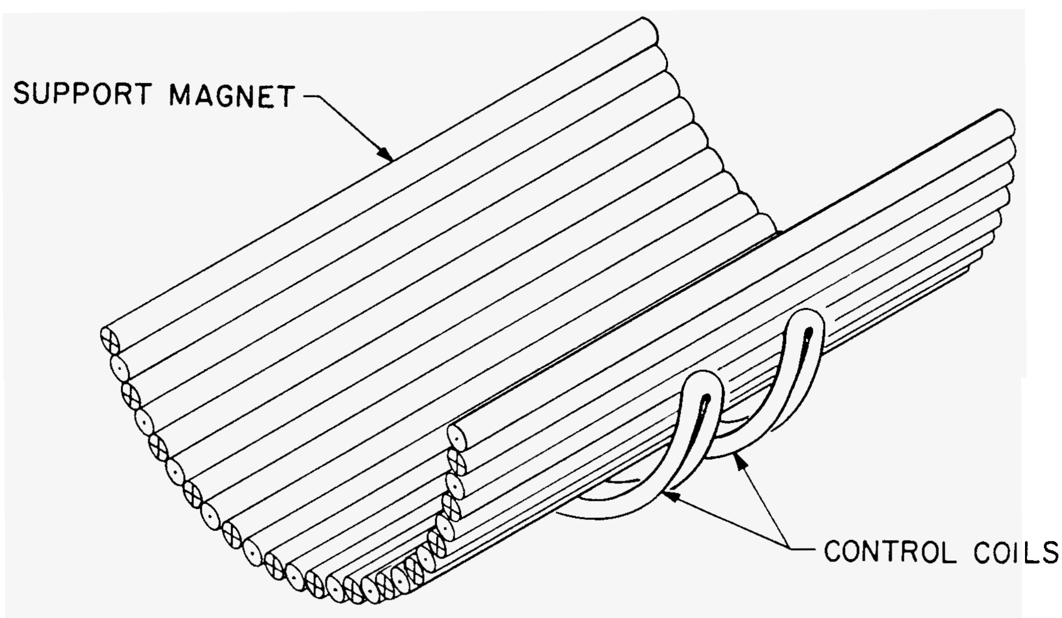

Figure 3: Magnetic Bearing Concept

dissipation is determined mostly by the residual gas in the system and the amount of normal metal present. Similarly, the stability of a persistent current (well below the critical current) is probably limited by the mechanical stability of the magnet, which can be very high at low temperature because the thermal expansion coefficient of most materials approaches zero. [7] The stiffest possible bearing with minimum fringing fields is made by arranging that current flows in opposite directions in adjacent wires. The large difference in stiffness between the radial and axial directions reduces coupling of noise into the sensitive axis, and small fringing fields reduce stray forces on the test mass and interference with the SQUID position detectors.

A measurement of $\eta = 10^{-13}$ corresponds to an acceleration difference of about 10^{-13} dynes/gram between the two masses. In order to measure this at all, the axial forces from the bearing must be minimized. The measurement is made by a SQUID position detector which can detect a change in position δx less than 10^{-10} cm in 0.01 second. The magnetic bearing exerts a residual force $F(x)$ along the measurement axis, which is a possibly known function of position, but which, because of the uncertainty in the position measurement, causes an uncertainty $(dF/dx)\delta x$ in the force on the test mass. We may treat the residual force of the bearing as if it is all due to curvature, so that the mass has a different height $h(x)$ at each position x along the bearing: thus $F = mg(dh/dx)$. This determines a condition on the second derivative of the height which must be met to get the force sensitivity δF:

$$d^2h/dx^2 < (\delta F/\delta x)/mg = (\delta A/\delta x)/g \qquad (2)$$

where g is the acceleration of gravity and δA is the desired acceleration sensitivity. To achieve an acceleration sensitivity δA of 10^{-13} cm/sec^2, it is necessary that $d^2h/dx^2 < 10^{-6}$ cm^{-1}, which is equivalent to 0.005 cm of smooth variation per meter of length - somewhat better than a typical straightedge. This requirement is much relaxed in the orbital version of the experiment, essentially because it is in free fall.

The bearings for the equivalence principle experiment must have this straightness over a distance scale of about one millimeter, the range of motion of the masses. The force the bearings produce is more important than the physical straightness, so that it is necessary to use magnetically homogeneous materials throughout. Microscopic particles of iron contaminated the first version of these bearings and produced bumps much larger than desired. Ultimately, the limits to smoothness may be due to electrostatic patch effect and the finite size of magnetic flux quanta trapped in the superconducting wires. Bearings approaching the required smoothness have been manufactured by a "lost wax" process, and we are investigating several other methods of making them as well. [3] These smooth bearings have not yet been incorporated into the apparatus, which is still using an early pair, one of which is badly contaminated with iron particles and one which has cracked due to repeated thermal cycling. The quality of these bearings is a major limitation on the present sensitivity. The axial forces of the bearing can be determined by measuring the period of a test mass in the bearing, or by measurement of its acceleration as it slides along it.

The position detector circuit is shown in Figure 4. This type of detector provides an exceptionally stable and sensitive position measurement. In operation, a persistent current I_1 is trapped in the loop

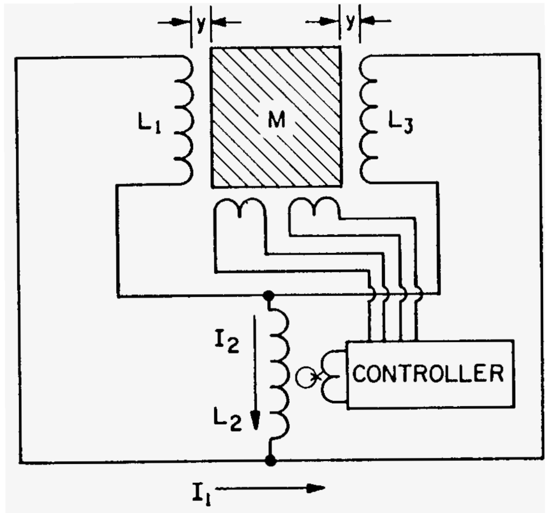

Figure 4: Basic Position Detector Circuit

L_1--L_3 ; motion of the superconducting masses changes L_1 and L_3 , and flux conservation forces a current I_2 to flow through L_2 which is proportional to the motion and to I_1 . The sensitivity is

$$\frac{dI_2}{dx} = \frac{2I_1}{(2L_2 + L_1)} \left(\frac{dL_1}{dx}\right) \tag{3}$$

where $L_1 \cong L_3$ and $dL_1/dx \cong dL_3/dx$. I_2 is detected by a SQUID magnetometer. The position sensitivity is continuously adjustable and limited by the critical current of the wire and the resolution of the SQUID. The present system can resolve less than 10^{-10} cm with $I_1 = 1$ ampere.

We are investigating several simple changes to the position detector circuit to improve the overall performance of the experiment. If the inductances L_1 and L_3 and current I_1 are large enough, appreciable restoring forces can be exerted on the test mass; this could simplify the control scheme by changing the total spring constant. [8] Two circuits similar to that in Figure 3 can be connected with L_2 as a common inductance to perform a direct subtraction of the mass positions [9], while a circuit analogous to an inductance bridge may allow independent adjustment of the equilibrium position, sensitivity and frequency. These modifications may simplify the control and subtraction scheme and also improve the measurement by using the inherent stability of supercurrents.

After seismic noise, the next largest disturbances to the system are gas pressure disturbances. If one end of a test mass has area A , and is outgassing at an equivalent pressure P , while the other end is not, the condition that the mass be disturbed less than the signal $g\eta$ from a violation of equivalence η is

$$P < \frac{Mg\eta}{A} \tag{4}$$

where g is the driving acceleration and M is the mass. For a 500 gram lead mass of area $7\ cm^2$ and length $5\ cm$, P must be less than 4×10^{-9} dynes/cm^2 or 3×10^{-12} torr [4] in an experiment designed to measure $\eta = 10^{-13}$. This condition is much too conservative, because a steady disturbing force has no component at the frequency of the signal. Other gas disturbances may, however, vary at or near this frequency. One such effect is the radiometer effect, due to differences in temperature in the chamber surrounding the test masses. In a well designed dewar these may be much less than 10^{-4} K, and we assume that all of this occurs as a gradient dT/dx changing at the signal frequency. The pressure limit is given by multiplying equation (4) by $T/(L dT/dx)$ where L is the length of the mass. For the same mass at $4°$ K, a pressure less than 10^{-8} torr seems to be more than adequate. Other gas disturbances are even smaller.

Present Status and Conclusion

We began taking preliminary equivalence principle data in May 1982. The data confirms the operation of the system and suggests that some simple improvements will make an enormous improvement in sensitivity.

Figure 5 is an example of the data. Trace 1 is the horizontal position of the outer test mass as measured during the night of July 5-6, 1982. Trace 2 is the difference in position between the inner and outer test masses. Each point in Figure 5 is a weighted average of points

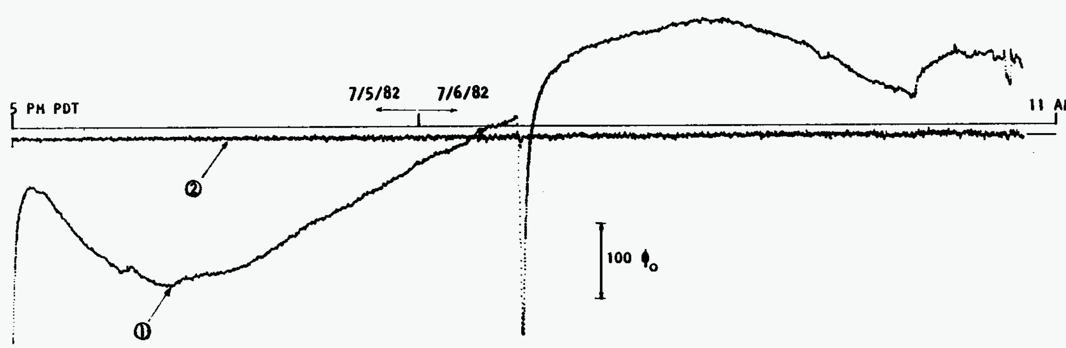

Figure 5: Preliminary Equivalence Principle Data

taken every 0.015 seconds for 7.5 seconds. This represents a compromise between anti-aliasing, control requirements, and memory capacity of the microcomputer.

Trace 2 is a straight line to within the noise amplitude, showing that the masses track each other accurately at low frequency. The overall displacement in Trace 1 (about one micron) is due to the daily variation in both the seismic tilt and the temperature of the laboratory. The large disturbance at 2 a.m. was caused by an automatic transfer of liquid nitrogen. The cold boil-off gas poured over one side of the apparatus and cooled it by about $1°$ K.

In Trace 2 of Figure 5, the noise amplitude at one cycle per day is about $0.25\ \phi_0$, which with the test mass period of about 4 seconds corresponds to $\eta < 8 \times 10^{-7}$, or an acceleration sensitivity of 5×10^{-7} cm/sec^2. This is the acceleration sensitivity of one test mass, not the differential acceleration sensitivity. It is presently impossible to make a good differential measurement because the controllers for the test masses are overwhelmed by the imperfections of the bearings and are not at this stage useful for matching the masses' periods. They could only be used to control the mass positions for brief times before saturation or integer overflow occurred, or, in the case of Figure 5, to add damping to reduce ringing and overall noise response.

There are four simple changes already in progress which should extend the sensitivity of the experiment from $\eta = 10^{-7}$ to 10^{-12} or better.

First, as mentioned above, the poor quality of the original magnetic bearings forces us to work with short periods of 3--4 seconds and prevents matching the periods of the test masses. We have constructed bearings with horizontal periods of 15 seconds and new bearings under construction will have periods of 30 seconds. Since the sensitivity depends on the square of the period while the high frequency noise remains essentially constant, the longer period bearings should immediately increase the sensitivity from 10^{-7} to about 10^{-9}.

Second, the data illustrated by Figure 5 was taken without the antivibration system in operation. Although the antivibration system reduces high frequency noise by at least 30 db from less than 0.1 to 50 Hz, it requires a leveling controller for long term operation. After 15 minutes to an hour, the ground tilts enough to cause the platform to bump against its safety stops. When a leveling system has been put into operation, the background noise will be reduced by at least a factor of 10.

Third, we discovered that the structure of the dewar probe is rather compliant and has resonances at about 8, 12, and 25 Hz. Motions of the probe at these frequencies are excited by ground noise and liquid helium boiling, and are about a factor of ten larger than the seismic background. No attempt was made to suppress these resonances while gathering the data in Figure 5, but they may be reduced by bracing the bottom of the probe rigidly against the inner wall of the helium well. The dewar structure itself is very stiff.

Fourth and finally, for these measurements the masses were operated as essentially two unmatched and uncoupled oscillators. Because they have different frequencies and initial conditions, these oscillators respond differently to seismic noise and the sensitivity to differential acceleration is essentially the same (or somewhat worse) than the sensitivity of either oscillator alone. Once we can use the control servo (or modified position detector circuit) to adjust the periods to the oscillators to be the same, the situation is quite different. The oscillators then respond identically to seismic noise except for possibly different initial conditions, the effect of which is removed by providing a small amount of coupling between the test masses. In effect this procedure changes the original uncoupled system into a coupled system with two normal modes, a common mode and a differential mode. When the periods are matched, the differential mode is not excited by seismic noise. We have several options for achieving this matching, including the existing controllers and the modifications to the position detectors discussed above. A noise rejection ratio of 10^5 is reasonable [9] which would give a sensitivity of $\eta = 10^{-12}$ even without the previous three changes.

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