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Gravity Probe B Relativity Mission

MONITORING THE STABILITY OF AND COMPENSATING FOR CHANGES IN THE GYROSCOPE-READOUT-SYSTEM SCALE FACTOR

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Monitoring the Stability of and Compensating for Changes in the Gyroscope-Readout-System Scale Factor

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Introduction

The gyroscope readout system is used to determine the orientation of the gyroscope spin axis with respect to the satellite roll axis. The directly measured quantity, after application of the appropriate scale factor, is the angular change between the gyroscope spin axis and the plane of the gyroscope pick-up loop. For the GP-B experiment, the scale factor of this readout system may be determined to better than one part in 10^5 using the known magnitude and direction of the orbital and annual aberration of starlight as calibrating signals [1]. However, if the scale factor is not stable to this accuracy over periods as long as a year, then there will be some degradation of the overall experiment error [2]. This report describes methods of quantitatively measuring the readout-system scale-factor stability. Additional information on the flux slipping operations (by J. Lockhart) and the flux slipping procedure (by B. Clarke) are attached as Appendices A and B. An alternative procedure for directly measuring the scale factor at a lower gain setting is described in Appendix C.

A. Gyroscope Readout System

Figure 1 is a schematic of the gyroscope readout system. The gyroscope readout system converts a magnetic flux change through the gyroscope pick-up loop to an output voltage, V_R. The gyroscope pickup-loop is part of the superconducting pick-up loop circuit, which includes the gyroscope pick-up loop, the feedback transformer with mutual inductance M_F, and the SQUID coupling transformer with mutual inductance, M_I. The current flowing in the pick-up loop circuit depends on the magnetic flux in the gyroscope pick-up loop, the magnetic flux from the feedback transformer due to the current I_T, and the current in the SQUID's inductive ring. (We ignore the induced current in the SQUID's ring since it has no effect when the SQUID is operated as a flux locked loop.) A change in the current, I_T, flowing in the pick-up loop circuit induces a magnetic flux change in the SQUID's inductive ring. The flux-locked-loop (FLL) electronics acts as an error amplifier that by integration of its input generates the output voltage, V_R, which in turn generates a current that is fed back to the transformer with mutual inductance, M_F. The FLL electronics changes its output voltage, V_R, until the flux and current in the pick-up loop circuit returns to almost exactly its original value and the input to the FLL electronics approaches zero. Since the FLL electronics is an error amplifier, the scale factor is very insensitive to its gain. The output voltage, V_R, is post amplified and filtered before being read out by a 16 bit ADC. In summary, the gyroscope readout system acts to produce an output voltage change proportional to a magnetic flux change in the gyroscope pick-up loop by keeping the total magnetic flux in the superconducting pick-up loop circuit constant.

Special consideration has been given in the design of the readout system to methods of checking its long-term stability. Over time scales up to 15 days, the gyroscope-readout-system scale factor has been designed to be stable to better than 1 part in 10^5. Over intermediate time scales up to 90 days, the stability of the scale factor may be checked by measuring the amplitude of the injected calibration signal. This calibration current is generated by a D/A converter (using the voltage reference, V_{RC}) with the calibration resistor, R_C. Over longer time scales, the stability of the voltage reference

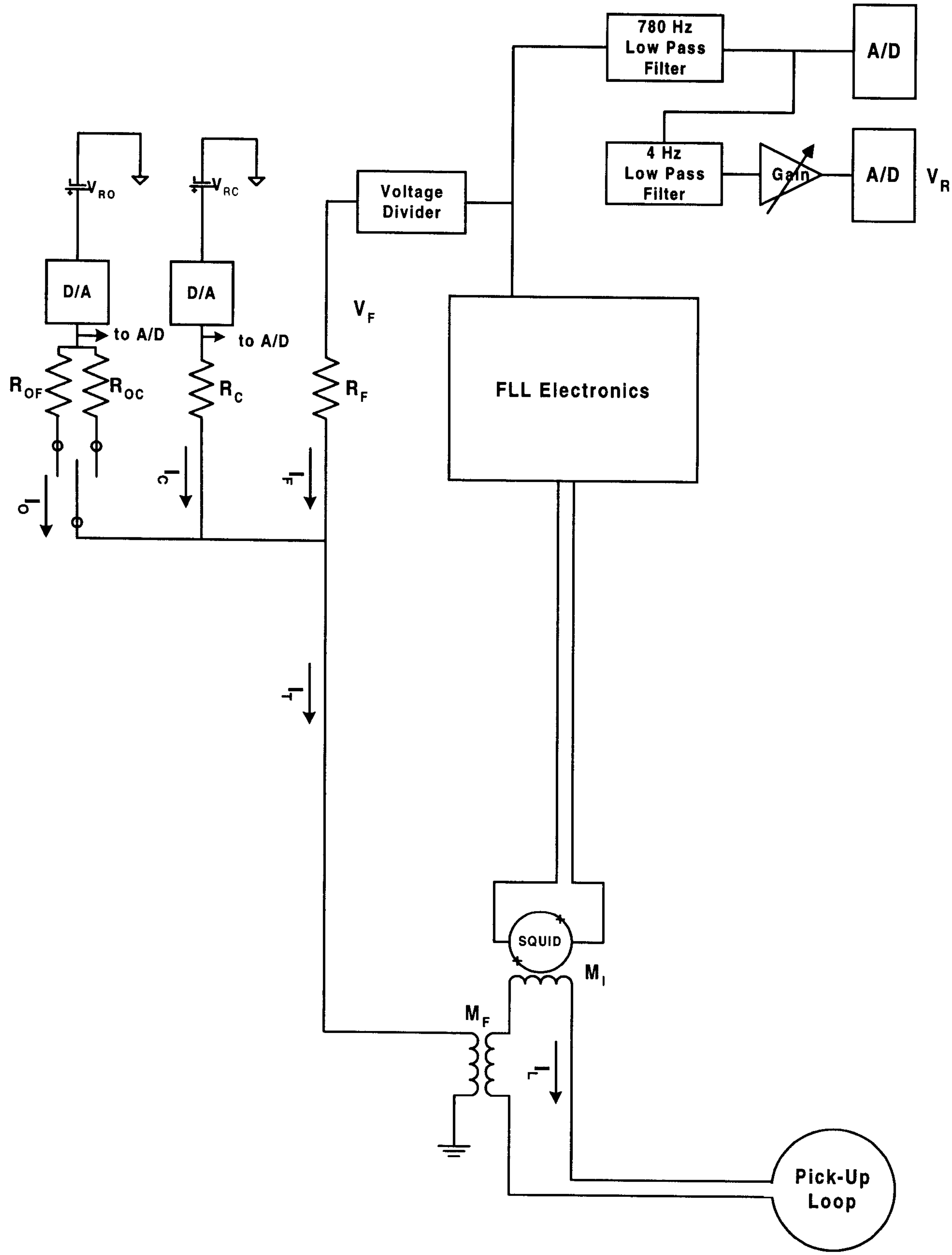

Figure 1. Diagram of Gyroscope Readout System

cannot be guaranteed, but it may be measured using the flux slipping technique described in section E. In this technique, changes in the readout voltage, V_R, corresponding to flux changes by an integral number of flux quanta in the SQUID are used to check the stability of the offset reference voltage, V_{RO}. Finally, the stability of the calibration signal voltage reference may be compared with the offset voltage reference by measuring the change in the voltage produced by commands to the offset and calibration D/A converters.

The relationship between the feedback voltage, the number of flux quanta in the SQUID, and the calibration and offset voltages is derived below. The equations in this document are in SI units unless otherwise noted.

C. Operation of the Flux Locked Loop

Since the pick-up loop circuit is superconducting, the current flowing in it, I_L, is related to the applied magnetic flux through the pick-up loop circuit, ϕ, by the equation

$$L\,I_L = \phi_C + \phi \tag{1}$$

where L is the inductance of the pick-up loop circuit and ϕ_C is the value of the magnetic flux through the pickup loop circuit when the magnetic flux ϕ is zero. The applied magnetic flux through the pick-up loop circuit is the sum of the magnetic flux applied by the gyroscope, ϕ_G, and the magnetic flux applied by the feedback coil, ϕ_F,

$$\phi = \phi_G + \phi_F\,. \tag{2}$$

The magnetic flux applied by the feedback transformer is

$$\phi_F = M_F\,I_T = M_F\left(I_F + I_C + I_O\right) \tag{3}$$

where M_F is the mutual inductance between the primary and the secondary of the feedback transformer, and I_T is the total current through the primary of the feedback coil. This current is the sum of the feedback current, I_F, the calibration current, I_C, and the offset current, I_O. Each of these currents is determined by the voltage applied to a resistor, which is carefully chosen to have a low temperature coefficient:

$$\begin{aligned} I_F &= V_F / R_F \\ I_C &= V_C / R_C \\ I_O &= V_O / R_O \end{aligned} \tag{4}$$

In these equations, R is the resistance of each of the resistors, V_F is the output of the voltage divider shown in Figure 1, and V_C and V_O are the calibration and offset voltages.

Combining the above equations, the current in the pick-up loop circuit is given by

$$I_L = \frac{1}{L}\left\{\phi_C + \phi_G + M_F\left(\frac{V_F}{R_F} + \frac{V_C}{R_C} + \frac{V_O}{R_O}\right)\right\} \tag{5}$$

Then the magnetic flux applied to the SQUID is the product of the mutual inductance between the SQUID input coil and the SQUID ring, M_I, and the current in the pick-up loop circuit,

$$\phi_S = \frac{M_I}{L}\left\{\phi_C + \phi_G + M_F\left(\frac{V_F}{R_F} + \frac{V_C}{R_C} + \frac{V_O}{R_O}\right)\right\} \tag{6}$$

The flux locked loop maintains the flux in the SQUID at a constant value. When the flux locked loop is closed, the servo loop adjusts the feedback voltage to keep the flux through the SQUID at a constant value, ϕ_{S0}, within the limitations of the noise in the electronics:

$$\phi_S = \phi_{S0} + \phi_N \qquad \textit{FLL closed} \tag{7}$$

where ϕ_N is the noise in the flux-locked-loop electronics referred to the flux in the SQUID. If the flux locked loop is unlocked and then relocked, it will relock at its original value plus or minus an integral number of flux quanta:

$$\phi_S = \phi_{S0} + n\,\phi_0 + \phi_N \qquad \textit{FLL unlocked and relocked} \tag{8}$$

If the magnetic flux applied to the SQUID by the input circuit while it is unlocked has changed so the net applied flux is within ±½ flux quantum of the original value of n ϕ_0, the SQUID will usually relock with the applied flux equal to its original value of ϕ_{S0}, *i.e.*, n = 0.

From equations (6) and (8), the general expression for the feedback voltage whenever the FLL is closed may be found. Setting the magnetic flux applied to the SQUID by the input circuit to a constant value, ϕ_{S0}, plus an integral number of flux quanta, ϕ_0, and then solving equation (6) for V_F gives

$$V_F = \frac{R_F}{M_F}\left\{\frac{L}{M_I}(\phi_{S0} + n\phi_0 + \phi_N) - \phi_C - \phi_G\right\} - \frac{R_F}{R_C}V_C - \frac{R_F}{R_O}V_O, \tag{9}$$

The readout voltage, which is the voltage measured by the A/D converter in either the high or low frequency signal chains, is then

$$\begin{aligned} V_R &= k_{AD}k_{LP}k_{VD}V_F \\ &= k_{AD}k_{LP}k_{VD}\left[\frac{R_F}{M_F}\left\{\frac{L}{M_I}(\phi_{S0} + n\phi_0 + \phi_N) - \phi_C - \phi_G\right\} - \frac{R_F}{R_C}V_C - \frac{R_F}{R_O}V_O\right], \end{aligned} \tag{10}$$

where k_{LP} is the attenuation (or gain) of the low pass filters and the post-gain amplifier, k_{AD} is the attenuation (or gain) of the A/D converter, and k_{VD} is the attenuation of the voltage divider shown in Figure 1. The readout scale factor depends both on the voltage divider and the post FLL gain in front of the ADC as well as the values of R_F and M_F.

The calibration and offset voltages are produced by a digital-to-analog converter, and the stability of these applied voltages depend on the stability of the 10 volt reference used for each D/A converter. The applied voltages are determined by the commanded voltages and the voltage references:

$$V_C = W_C\frac{V_{RC}}{10} \tag{11a}$$

$$V_O = W_O\frac{V_{RO}}{10} \tag{11b}$$

Here, V_C and V_O are the applied calibration and offset voltages, W_C and W_O are the commanded calibration and offset voltages, and V_{RC} and V_{RO} are the D/A reference voltages for the calibration voltage D/A and the offset voltage D/A. Since drift in the reference voltages are one possible source of the error in the following calibration, the differences between the applied and commanded voltages must be taken into account.

D. Gyroscope Readout System Scale Factor

The SQUID scale factor may be defined as the change in the readout voltage for a given change in the flux through the pickup loop applied by the gyroscope. From equation (11), it is

$$C_S = \frac{\delta V_R}{\delta \phi_G} = -k_{AD} k_{LP} k_{VD} \frac{R_F}{M_F} = -k \frac{R_F}{M_F} \tag{12}$$

where the electronic scale factor, *k,* is the product of the scale factors of the voltage divider, the low pass filters and gain, and the A/D converter. The overall gyroscope scale factor is the change in the readout voltage for a given change in the angle between the gyroscope spin axis and the plane of the pick-up loop:

$$C_G = \frac{\delta V_R}{\delta \theta} = \frac{\delta V_R}{\delta \phi_G} \frac{\delta \phi_G}{\delta \theta} = -k_{AD} k_{LP} k_{VD} \frac{R_F}{M_F} \alpha = -k \frac{R_F}{M_F} \alpha . \tag{13}$$

Here α is the change in the magnetic flux through the gyroscope pick-up loop for a given change in the angle between the gyroscope spin axis and the plane of the readout loop. This factor depends on the gyroscope spin speed and the component of the trapped magnetic flux along the gyroscope spin axis [3].

E. Measurement of the Scale-Factor Stability

From equation (12), it can be seen that the SQUID scale factor depends on the scale factor of the A/D converter, k_{AD}, the scale factor of the low pass filters and post-gain amplifier, k_{LP}, the attenuation of the voltage divider, k_{VD}, and the ratio of the feedback resistor, R_F, to the mutual inductance of the feedback coil, M_F. Methods for checking on the stability of these parameters are described below.

1. Monitoring the Scale Factor Using the Calibration Signal

A low frequency calibration signal will be injected into the pickup loop circuit at a frequency of either 1/62 or 1/124 Hz and a London moment equivalent amplitude of up to 30 arc seconds. This calibration voltage is produced by the calibration signal D/A converter, which uses the calibration signal voltage reference, V_{RC}. From equation (10), the change in the measured readout voltage, V_R, due to a change in the commanded calibration voltages, W_C, is

$$V_{R,Cal} = -k \frac{R_F}{R_C} \frac{V_{RC}}{10} W_C \tag{14}$$

As long as the resistors and the voltage references for the D/A converters are stable, the calibration or offset D/A converters may be used to check on the stability of the attenuation by the low pass filters, post-gain amplifier, A/D converter, and voltage divider. In practice, the change in the commanded calibration voltage is not a step but a sinusoidally varying input at the calibration signal frequency of 1/62 or 1/124 Hz.

Over intervals of less than 90 days the resistors and voltage references will be stable to better than 1 part in 10^5. The combined effects of a low temperature coefficient and a thermally controlled environment produce fractional changes in the resistance of less than 1 part in 10^7 at orbital and annual periods [4]. Long term drift in the resistance due to aging may produce larger changes in the resistance. In reference [4], these long term drifts due to aging were estimated to be less than 6 parts in 10^6 over a period of 90 days. Since this document was written, additional information was supplied by the manufacturer [5], which indicates that the long term drift of the metal foil resistors in the GP-B SQUID Readout System will be significantly smaller over a 90 day interval as well as longer intervals. The long term drift may be reduced by the 500 hours of burn-in, by operating the resistors at temperatures less than 25^0C, and by operating the resistors with a power dissipation of less than 0.1 W. The feedback, offset, and calibration resistors satisfy all of these conditions. In this case, the long term drift after the 500 hour burn-in period will be less than 3 parts in 10^6 over a year. In addition, test data show that the resistance values tend to drift in the same direction with time, so that the change in the ratio of two resistors will be less than the change in either resistor alone.

Radiation effects on the voltage references in the SQUID Readout Electronics will cause their voltage output to drift slowly with time [6]. The thickness of the SRE covers were doubled to decrease the radiation exposure to these critical parts. Table 1 below shows the expected output shift in each of the three voltage references over intervals of one month and three months. Data for this table is taken from reference [6] and is based on a total estimated dosage of 700 rad(Si) over the 18 month lifetime of the mission. Over a three month interval, the expected drift rate of the calibration signal voltage reference is 5.0 ppm. Combining this drift with the long term drift of the calibration and feedback resistors, the overall drift in the product of the voltage reference and the ratio of the feedback to calibration resistors over a 90 day interval is expected to be less than 8 ppm. Over a 30 day interval the drift rate is expected to be less than 2.7 ppm.

Table 1. Expected Shift in Output of SRE Voltage References due to Radiation

Voltage Reference Part Number	Purpose	Output Shift	
		1 month	3 months
AD780	A/D Converter	0.8 ppm	2.3 ppm
REF02	Calibration Signal D/A Converter	1.7 ppm	5.0 ppm
AD588	Offset Signal D/A Converter	0.45 ppm	1.3 ppm

The amplitude and phase of the SRE Readout Voltage, V_R, at the calibration signal frequency will be continuously monitored. With a SQUID noise of 190 mas/√Hz (required value, T002, Req. 5), the amplitude of this signal may be determined to a fractional accuracy of 10 ppm (or 0.3 mas) in 4.64 days or 5.6 ppm in 15 days. Note that these measurements may also be made during guide star invalid.

2. Flux Slipping to Check Stability of Offset Reference Voltage

The method of flux slipping may be used to check on the stability of the D/A voltage references using the following procedure, which is described in detail in Appendices A and B. The SQUID readout electronics is set in its nominal operating range where the peak-to-peak output is equivalent to a 200 arc sec London Moment signal (0.05 ϕ_0). The SQUID is locked, the offset current is set to zero, and the readout voltage is measured. Then, the SQUID is unlocked, and an offset current is applied which will shift the number of flux quanta in the SQUID by approximately 100 ϕ_0. With the required value of the readout noise, this d.c. level may be measured to an accuracy of 50 $\mu\phi_0$ in 1 sec. To apply this large offset current, the coarse offset current resistor, R_{OC}, which is shown in Figure 1, must be used. Then the FLL is relocked, the readout voltage, V_R, is re-measured. From equation (10), neglecting the noise, the change in the readout voltage, V_{FS}, due to the change in the commanded offset voltage, W_O, and the flux slipping will be

$$V_{R,FS} = k\left[\frac{R_F}{M_F}\left\{\frac{L}{M_I}(100\phi_0)\right\} + \frac{R_F}{R_{OC}}\frac{V_{RO}}{10}W_O\right] \tag{15}$$

If this measurement is repeated at regular intervals, the difference between the measured change in the readout voltage will be primarily sensitive to the change in the offset voltage reference since the quantity in the square brackets is the difference between two large quantities. However, as J. Turneaure pointed out and as shown below in section 4, repeated sets of measurements will give an exact solution for the change of all the relevant quantities.

For a flux shift of 100 ϕ_0 and a readout noise of 50 $\mu\phi_0$, changes in the offset voltage reference may be resolved to a limit set by the stability of the feedback and coarse offset resistors by repeating these measurements at regular intervals over the course of the mission. As noted in section 1 above the stability of these resistors is expected to be better than 3×10^{-6}, this flux slipping directly measures the stability of the offset voltage reference to this same accuracy.

3. Comparison of Calibration and Offset Voltage References

The calibration and offset voltages are also directly measured by the A/D converter. The change in the directly measured calibration and offset voltages, V_{CM} and V_{OM}, for a given change in the commanded calibration and offset voltages, W_C and W_O, are

$$V_{CM} = k_{AD}\frac{V_{RC}}{10}W_C \tag{16a}$$

$$V_{OM} = k_{AD}\frac{V_{RO}}{10}W_O \tag{16b}$$

Since these measurements are both made by the same A/D converter, the ratios of these two quantities are a direct measure of the ratio of the voltage references. With these measurements, stability of the calibration voltage reference may be determined relative to the offset voltage reference. These measurements are sensitive to both gain of the A/D converter and the changes in the offset and calibration voltage references. By combining both these measurements, it would be possible to determine any change in the ratio of the voltage references. Since the noise in these measurements is limited only by the quantization of the 16-bit A/D converter, these measurements could be made quickly. However, since the 16-bit A/D converter has a resolution of 1.5×10^{-5} at best, multiple measurements or some form of dithering is necessary to reduce this number to the several parts in 10^6.

4. Exact Solution for Change in Scale Factors and Voltage References

J. Turneaure has pointed out that an exact solutions for the change in the voltage references and the relevant scale factors may be found from repeated sets of four measurements described by equations (14), (15), and (16). From equation (14) the fractional change in the readout voltage for given change in the commanded calibration voltage is:

$$\frac{\delta V_{R,Cal}}{V_{R,Cal}} = \frac{(V_{R,Cal})_1 - (V_{R,Cal})_2}{V_{R,Cal}} = \frac{\delta k}{k} + \frac{\delta V_{RC}}{V_{RC}} \tag{17}$$

Similarly, from the flux slipping the difference in the readout voltage after two flux slipping calibrations will be

$$\frac{\delta V_{R,FS}}{V_{R,FS}} = \frac{(V_{R,FS})_1 - (V_{R,FS})_2}{V_{R,FS}} = \frac{\delta k}{k} + C\frac{\delta V_{RO}}{V_{RO}} \tag{18a}$$

where

$$C = \frac{\frac{R_F}{R_O}\frac{V_{RO}}{10}W_O}{\frac{R_F}{M_F}\frac{L}{M_I}100\phi_0 + \frac{R_F}{R_O}\frac{V_{RO}}{10}W_O} \tag{18b}$$

The quantity C will much larger than one because the denominator is the difference between two large quantities. The change in the readout voltage after the flux slipping calibration will be primarily sensitive to the change in the offset voltage reference. Finally, the fractional change in the directly measured offset and calibration voltages will be given by

$$\frac{\delta V_{CM}}{V_{CM}} = \frac{(V_{CM})_1 - (V_{CM})_2}{V_{CM}} = \frac{\delta k_{AD}}{k_{AD}} + \frac{\delta V_{RC}}{V_{RC}} \tag{19a}$$

$$\frac{\delta V_{OM}}{V_{OM}} = \frac{(V_{OM})_1 - (V_{OM})_2}{V_{OM}} = \frac{\delta k_{AD}}{k_{AD}} + \frac{\delta V_{RO}}{V_{RO}} \tag{19b}$$

Equations (17), (18), and (19) are a set of four equations which may be solved for the four variables δk, δk_{AD}, δV_{RC}, δV_{RO}. These changes in the scale factors k and k_{AD} as well as the changes in the calibration and offset reference voltages, V_{RC} and V_{RO}, may be determined from these sets of measurements. The solution for the change in the scale factor k is

$$\frac{\delta k}{k} = \frac{C}{C-1}\left(\frac{\delta V_{R,Cal}}{V_{R,Cal}} - \frac{\delta V_{CM}}{V_{CM}} + \frac{\delta V_{CM}}{V_{CM}}\right) - \frac{1}{C-1}\frac{\delta V_{R,FS}}{V_{R,FS}} \quad (20)$$

With this knowledge, the change in the overall SQUID readout scale factor, given by equation (12) may be determined.

F. Capability for Monitoring and Compensating for Long Term Drifts in the Scale Factor

From equation (12), it can be seen that the changes in the SQUID scale factor are due to changes in the product of the scale factors, k, and the any changes in the feedback resistor, R_F, or the mutual inductance of the feedback transformer, M_F.

$$\frac{\delta C_S}{C_S} = \frac{\delta k}{k} + \frac{\delta R_F}{R_F} - \frac{\delta M_F}{M_F} \quad (21)$$

The fractional error in the monitored value of k is given by equation (20). The first term on the right hand side of this equation is the accuracy with which the change in the calibration signal may be measured in the output of the SRE. With a SQUID noise of 190 mas/√Hz over an interval of one month the calibration signal may be measured to a relative accuracy of 4×10^{-6} by monitoring the readout voltage at the calibration signal frequency. Over the next month the measurement may be repeated, and the difference between the two measurements may be use to determine the fractional change in $V_{R,Cal}$ to an accuracy of 5.6×10^{-6}. At the same time the change in amplitude of the calibration signal measured directly by the A/D converter, V_{CM}, to an accuracy significantly better than the quantization limit because the sinusoidal signal sweeps through many bits. The fractional error in this measurement should be less than 10^{-6}. Similarly, the error in the directly measured offset voltage, with repeated measurements or some dithering, should also be better than the quantization limit, but perhaps not as accurate as the directly measured calibration signal. This error is estimated to be less than 3×10^{-6}. The contribution of the measured readout voltage to the fractional error in k is not significant since it is multiplied by the small factor 1/C in equation (20).

These contributions to the overall accuracy of the electronic scale factor, k, are summarized in Table 2 below. The flux slipping operations are assumed to occur at one month intervals. Continuous measurements of the calibration signal in the SRE readout (and also directly measured by the A/D converter) are made over a one month interval centered around each of the flux slipping operations. The fractional change in the electronic scale factor may be measured to an accuracy of $< 6.5 \times 10^{-6}$ at one month intervals.

Table 2. Contributions to Fractional Error in Electronic Scale Factor Measured at One Month Intervals

Contribution	Fractional Error
Fractional Error in Measured Amplitude of Calibration Signal Over One month	5.6×10^{-6}
Error in Directly Measured Calibration Signal	$< 1 \times 10^{-6}$
Error in Directly Measured Offset Voltage	$< 3 \times 10^{-6}$
Contribution from Measured Readout Voltage After Flux Slipping	$< 1 \times 10^{-6}$
Root Sum Square	$< 6.5 \times 10^{-6}$

If the change in the electronic scale factor is measured to this accuracy every month, then the variation in the scale factor at the sine of the annual frequency, the cosine of the annual frequency, and the linear drift over a 1 year period may be determined. Appendix D is a short Matlab program to calculate the covariance matrix for these measurements for a given number of measurements and a given standard error in each measurement. For measurements made at intervals of once per month with a standard error of 7×10^{-6} for one year, these components may be measured to the following accuracy:

Table 3. Accuracy of Long Term Drift of Electronic Scale Factor Based on Flux Slipping Measurements at One Month Intervals

Component	Accuracy
Sine of Annual Frequency	3.1×10^{-6}
Cosine of Annual Frequency	2.9×10^{-6}
Linear with Time Over One Year	3.8×10^{-6}

The two other contributions to the error in the SQUID scale factor are the long-term drift in the feedback resistor and the long-term drift in the mutual inductance of the feedback transformer. In section E.1 above, the long-term stability of the feedback resistor was estimated to be 3×10^{-6}. In reference [4], the long-term stability of the mutual inductance of the feedback transformer was estimated to be less than 3×10^{-13} over a one year period. All of the contributions to the uncertainty in the long-term drift in the SQUID scale factor are summarized in Table 4 below:

Table 4. Contributions to Uncertainty in Long Term Drift of SQUID Scale Factor

Component	Electronic Scale Factor, k From Table 3	Long Term Drift in Feedback Resistor	Long Term Drift in Mutual Inductance of Feedback Transformer	Root Sum Square
Sine of Annual Frequency	3.1×10^{-6}	3×10^{-6}	$< 3 \times 10^{-13}$	4.3×10^{-6}
Cosine of Annual Frequency	2.9×10^{-6}	3×10^{-6}	$< 3 \times 10^{-13}$	4.2×10^{-6}
Linear with Time Over One Year	3.8×10^{-6}	3×10^{-6}	$< 3 \times 10^{-13}$	4.8×10^{-6}

G. Summary

The gyroscope readout scale factor is the product of the pickup loop scale factor and the SQUID scale factor. The SQUID scale factor, in turn, depends on the electronic scale factor, the resistance of the feedback resistor, and the mutual inductance of the feedback transformer. By monitoring the calibration signal injected into the pickup loop and performing a flux slipping calibration at intervals of once per month, it is possible to continuously monitor the electronic scale factor. Using any pair of flux slipping calibrations, it is possible to monitor the change in the scale factor to an accuracy better than 7×10^{-6}. If these measurements are made at one month intervals, the long term drift and the annual components of the electronic scale factor may be determined to an accuracy better than 3.8×10^{-6}. With the contribution of the long term drift in the feedback resistor, the SQUID scale factor may be monitored to an accuracy of 4.8×10^{-6}.

Appendix A – Operation Sequences

Flux Slip Calibration **- Plan and *Command Sequence***
(J.M. Lockhart, Dec. 2001)
Flux slip calibration is designed to calibrate the response of the SQUID readout system; i.e., the change in Analog-to-Digital Converter (ADC) counts produced by a unit change in the flux in the SQUID loop. It is important to have a means to correct variations in readout system response caused by drift of the ADC voltage reference (principally due to energetic particle radiation and, to a much lesser extent, to temperature changes).

It is important to note that all flux changes applied to the SQUID by the SRE (feedback, offset, and calibration signal) are produced by changes in the current in the primary of the SQUID feedback transformer. The SQUID feedback transformer has extremely high stability, so both the offset and calibration signal sources can be calibrated against the flux quantum in the SQUID by finding the amount of feedback transformer current which produces a change of one flux quantum in the SQUID.

The sequence of operations for calibrating the offset current starts with making the SQUID environment as free of stray flux as possible by turning off calibration sources and avoiding operation during periods in the SAA. The offset source is set to high range, and the SQUID is "zeroed" (an offset current which is sufficient to zero the FLL output is applied). The SQUID is unlocked (taken out of flux lock), and an offset current corresponding as nearly as can be estimated to one hundred flux quanta at the SQUID loop is applied, using the high range offset mode. The SQUID is re-locked, and the FLL output A/D reading is recorded. This process is repeated several times to average out changing flux from the gyro.

This calibration of the offset current can now be transferred to the internal calibration source by using the "DC Cal" mode. The SQUID is flux locked and zeroed, the maximum value of dc calibration source current is applied (a value of 14 nA or 0.007 flux quanta), and the SQUID is re-zeroed using the offset current in low-range mode. This allows the calibration source level to be cross-compared with the offset current levels which were just related to the size of the flux quantum in the SQUID loop. The resolution of the low range offset current is 1.2 nA (600 $\mu\Phi_0$), so nulling can be performed to a precision of 0.6 nA (300 $\mu\Phi_0$). A 10 PPM drift in the calibration source would amount to a 0.14 fA or 0.07 $\mu\Phi_0$ flux change at the SQUID, requiring 5 x 10^5 seconds or 140 hours to resolve. Inter-calibration to 100 PPM would need only 1.4 hours.

II. Resolution, Stability, and Noise Effects

The offset current for the SQUID has a low (standard) range of +/- 2.5 uA (+/- 1.25 flux quanta) and a high range of +/- 250 uA.

The on-board calibration signal generator provides a dc calibration mode with four levels and a maximum current of 14 nA (0.007 flux quanta, or 28 arc seconds for 130 Hz gyro spin).

The SQUID noise level PSD is (hopefully) white at 7 $\mu\Phi_0/\sqrt{Hz}$ above 0.1 Hz or so and 1/f below 0.1 Hz, with a value of < 50 $\mu\Phi_0/\sqrt{Hz}$ at roll frequency. Measurements of a dc feedback effort will have an rms noise corresponding to the integral of the PSD from a low frequency corresponding to the inverse of twice the total observation time to a high frequency corresponding to the inverse of the total measurement time; thus typically a noise of 75 $\mu\Phi_0$ for measurements over one second. It is desired to calibrate the offset current to 10 PPM, or the equivalent of 12.5 $\mu\Phi_0$ on high-range offset, which would require only about 20 such measurements to average out the SQUID noise. A larger problem is that of gyro motion, with accompanying trapped flux variations of up to 1.7 Φ_0 at the SQUID; these are brought down to 50 $\mu\Phi_0$ variations by on-board analog and digital filtering (assuming a post-gain of X20). Roll frequency variations on the order of 50 arc seconds (about 0.13 flux quanta) may be more of a problem, but can be averaged out.

Flux Slip Calibration Operations

Notes: The flux slip operation shall not take place while the spacecraft in exposed to the SAA

These operations shall be repeated sequentially for all of the SQUIDs.

SQUID would be unavailable to ATC during this test.

Appendix B – Flux Slip Procedure

(B. Clarke)

Usage:

The first template to run will be LargeOffset_Flux_Slip_Cal. This template measures the offset current in high mode which is equivalent to ~ 100 Phi_0 for SQUIDs 1, 2 and 3 and 60 Phi_0 for SQUID 4. It is anticipated these values will be the same as those found during Payload Test II. For reference, the PL-II values are:

SQUID	100 Phi_0 offset equivalent
1	x57E8
2	x571E
3	x57DE
4	x645A (60 Phi_0)

If the 100 (60) Phi_0 offset equivalents are found to be different after running LargeOffset_Flux_Slip_Cal on orbit, LargeOffset_Flux_Slip_SQUIDS_1_4 and LargeOffset_Flux_Slip_SQUIDS_2_3 will need to be modified to reflect the new offset equivalents.

LargeOffset_Flux_Slip_SQUIDS_1_4 and LargeOffset_Flux_Slip_SQUIDS_2_3 are each broken into three separate templates, an initialization template, a slip template, and a termination template.

The initialization template puts the offset in high mode and zeroes the science low pass filter signal in gain 1.

The slip template performs a measurement of the science low pass filter output for 180 seconds (roughly one roll period) at the endpoints of the 100 (60) Phi_0 flux slip. This template should be scheduled repeatedly in order to record the number of iterations required by the science team. At the time of this documents release 10 iterations are required.

The termination template restores the offset to normal mode, puts the SQUID in gain 1, locks the FLL and zeroes the science low pass filter output.

LargeOffset_Flux_Slip_Cal

Entry conditions

- Calibration signal off.
- Science Mode Offset Adjust disabled.
- Science Low Pass Filter enabled.
- All gyros caged.

Setting Initial Offset

Unlock all FLL's.
Set all four OFFSET commands to x8000.
Set all four OFFSET modes to High.
Set all four SQUIDs to range 1.
Set all four SQUID Post Gain settings to 1.
Lock all four FLL's.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 128,
Avg = 4,
Niter = 1000,
Wait = 10;

Wait 30 Seconds.
Increase the Post Gain settings for all four SQUIDs to 2.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 2,
Avg = 6,
Niter = 600,
Wait = 10;

Wait 30 Seconds.
Increase the Post Gain settings for all four SQUIDs to 3.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 2,
Avg = 6,
Niter = 600,
Wait = 10;

Wait 30 Seconds.
Increase the Post Gain settings for all four SQUIDs to 4.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 2,
Avg = 6,
Niter = 600,
Wait = 10;

Wait 30 Seconds.
Increase the Post Gain settings for all four SQUIDs to 5.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 2,
Avg = 6,
Niter = 600,
Wait = 10;

Wait 30 Seconds.

Apply 100 Phi_0 Slip (60 Phi_0 for SQUID 4)
Unlock FLL on all four SQUIDs.
Change the Post Gain settings for all four SQUID channels to 1.
Use the SQUID Init Ramp_Offset_Current command with the following values to get SQUIDs 1 to 3 close to 100 $\Phi 0$ out:

Increment = 1024,
Niter = 21,
Wait = 5;

Use the SQUID Init Ramp_Offset_Current command with the following values to get SQUID 4 close to 60 $\Phi 0$ out:

Increment = 1024,
Niter = 24,
Wait = 5;

Use the SQUID Init Ramp_Offset_Current command with the following values for the final adjustment:

	SQUID 1	SQUID 2	SQUID 3	SQUID 4
Increment	1000	798	990	1114
Niter	1	1	1	1
Wait	5	5	5	5

Lock all four FLL's.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 128,
Avg = 4,
Niter = 1000,
Wait = 10;

Wait 30 seconds.
Increase the Post Gain settings for all four SQUIDs to 2.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 2,
Avg = 6,
Niter = 600,
Wait = 10;

Wait 30 seconds.
Increase the Post Gain settings for all four SQUIDs to 3.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 2,
Avg = 6,
Niter = 600,
Wait = 10;

Wait 30 seconds.
Increase the Post Gain settings for all four SQUIDs to 4.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 2,

Avg = 6,
Niter = 600,
Wait = 10;

Wait 30 seconds.
Increase the Post Gain settings for all four SQUIDs to 5.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 2,
Avg = 6,
Niter = 600,
Wait = 10;

Wait 30 seconds
Calibration Termination
Unlock FLL on all four SQUIDs.
Change the Post Gain settings for all four SQUID channels to 1.
Ramp OFFSET on all four SQUIDs to x8000 in steps of x1000 waiting 0.2 seconds at each step.
Set all four OFFSET modes to Normal.
Set all four OFFSET commands to x7D28.
Lock FLL on all four SQUIDs.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 128,
Avg = 4,
Niter = 1000,
Wait = 10;

LargeOffset_Flux_Slip_SQUIDS_1_4
Each line applies to both SQUID 1 and SQUID 4 in turn.

Entry conditions:

SQUIDs 1 and 4 in hardware range 2.
Calibration signal off.
Science Mode Offset Adjust disabled.
Science low pass filter enabled.
Guide star valid.

This probably will need to be broken into three templates.
Steps 1 – 6 put the SQUIDs in high offset and minimizes at zero Phi_0 offset in gain 1.
Steps 7 – 18 allow data collection for a roll period at zero Phi_0 offset, slips to 100 (60) Phi_0 offset and collects data for one roll period, slips back to zero Phi_0 offset. This template will need to be scheduled multiple times to get the n times required (n=10 nominally).
Steps 19 – 25 put the SQUID back to normal offset, lock the FLL, leave it in post gain 1 and coarse zero the science low pass filter output.

Detailed Steps

Unlock FLL.
Change the Post Gain setting to 1.
Ramp the OFFSET to x8000.
Set OFFSET mode to High.
Lock the FLL's.
Perform SQUID Init Minimize SLPF with the following settings:

Increment = 128,
Avg = 4,
Niter = 1000,
Wait = 10;

Wait 180 seconds (one roll period).
Unlock the FLLs.
Use the SQUID Init Ramp_Offset_Current command with the following values to get SQUIDs 1 to 3 close to 100 $\Phi 0$ out:

Increment = 1024,
Niter = 21,
Wait = 5;

Use the SQUID Init Ramp_Offset_Current command with the following values to get SQUID 4 close to 60 $\Phi 0$ out:

Increment = 1024,
Niter = 24,
Wait = 5;

Use the SQUID Init Ramp_Offset_Current command with the following values for the final adjustment: (Note: these are the values determined form PLII data. They may change based on data collected during the Flux Slip Calibration performed on flight).

	SQUID 1	SQUID 2	SQUID 3	SQUID 4
Increment	1000	798	990	1114
Niter	1	1	1	1
Wait	2	2	2	2

Lock the FLL's.
Wait 170 seconds (one roll period).
Unlock the FLLs.
Use the SQUID Init Ramp_Offset_Current command with the following values to get SQUIDs 1 to 3 close to 0 $\Phi 0$ out:

Increment = -1024,
Niter = 21,
Wait = 5;

Use the SQUID Init Ramp_Offset_Current command with the following values to get SQUID 4 close to 0 $\Phi 0$ out:

Increment = -1024,
Niter = 24,
Wait = 5;

Use the SQUID Init Ramp_Offset_Current command with the following values for the final adjustment: (Note: these are the values determined form PLII data. They may change based on data collected during the Flux Slip Calibration performed on flight).

	SQUID 1	SQUID 2	SQUID 3	SQUID 4
Increment	-1000	-798	-990	-1114
Niter	1	1	1	1
Wait	2	2	2	2

Lock the FLLs.
Repeat steps 0 through 0 4 times.
Unlock the FLL.
Ramp OFFSET to x8000 in steps of x1000 waiting 0.2 seconds at each step.
Set the OFFSET mode to Normal.
Set the OFFSET to x7D28.
Lock the FLL.
Perform SQUID Init Minimize SLPF for all four SQUIDs with the following settings:

Increment = 128,
Avg = 4,
Niter = 1000,
Wait = 10;

LargeOffset_Flux_Slip_SQUIDS_2_3

Same as LargeOffset_Flux_Slip_SQUIDS_1_4 but replace SQUID 1 with SQUID 3 and SQUID 4 with SQUID 2

SmallOffset_Flux_Slip_SQUIDS_1_4

Each line applies to both SQUID 1 and SQUID 4 in turn.

Entry conditions:

Calibration signal off.
Science Mode Offset Adjust disabled.
Offset in normal mode.
Science low pass filter enabled.
Range 2 Gain 1.
Guide star valid.

Enable the calibration signal in DC mode on level 4 for SQUIDs 1 and 4 (off for SQUIDs 2 and 3).

Wait 25 seconds.

Decrease the offset by x1999.

Wait 25 seconds.

Disable the calibration signal in DC mode on level 4 for SQUIDs 1 and 4 (off for SQUIDs 2 and 3).

Wait 25 seconds.

Increase the offset by x1999.

Wait 25 seconds.

Repeat steps 0 - 0 15 times.

SmallOffset_Flux_Slip_SQUIDS_2_3

Same as SmallOffset_Flux_Slip_SQUIDS_1_4 but replace SQUID 1 with SQUID 3 and SQUID 4 with SQUID 2.

Chg_Temp_Setpt_for_FluxSlip

Increase the temperature setpoint on all electronics boards by 1C.

Appendix C – Alternative Flux Slipping Procedure

Using a slightly different procedure, the overall stability of the pickup loop scale factor may be determined. The procedure is similar to the one described above except that after relocking the SQUID, the offset current is reset to its original value. Then, the change in the gyroscope readout voltage, V_R, is given by

$$\Delta V_R = k_{AD} k_{LP} k_{VD} \frac{R_F}{M_F} \left\{ \frac{L}{M_I} \left(n\phi_0 + \Delta\phi_N \right) \right\} \tag{C1}$$

where n is the shift in the flux quanta at the SQUID. Note that if this measurement is repeated at intervals over the course of the mission, it is a direct measure of the stability of the scale factor defined by equation (11) as long as the inductance of the pickup loop, L, and the mutual inductance between the input coil and the SQUID, M_I, are constant. Both of these quantities depend on the mechanical properties of the pickup loop and SQUID that lie in a temperature controlled cryogenic environment and are expected to be very stable. The disadvantage of this method is that to allow for the large change in the SQUID readout voltage, it would be necessary to switch to a different range and gain different than the operating range. In this case, the stability of the voltage divider and the post-gain are not necessarily the same as those quantities for the operating range and gain.

Appendix D – Matlab Program to Calculate Covariance Matrix for Flux Slipping at Regular Intervals

Program:

```
% Fluxslip.m
% Calculation of Accuracy with which sine(annual) cosine(annual) and linear
%    terms may be determined given N measurements to accuracy sigma
%
N=12;      % Number of measurements
sigma=7;     % Fractional Error of each measurement (dimensionless)
%
interval=1/N;  % Interval between measurements (years)
time=interval/2:interval:1;    % Time vector at each measurement
%
H=[sin(2*pi*time') cos(2*pi*time') time'];  % Measurement matrix
I=H'*H/sigma^2              % Information matrix
P=inv(I)                % Covariance matrix
sine_error=sqrt(P(1,1))
cosine_error=sqrt(P(2,2))
linear_error=sqrt(P(3,3))
```

Sample Output:

```
I =12.2449   0.0000  -3.9426
    0.0000  12.2449  -0.0000
   -3.9426  -0.0000   8.1491

P =   0.0967   0.0000   0.0468
     0.0000   0.0817   0.0000
     0.0468   0.0000   0.1454

sine_error =   3.110

cosine_error =  2.858

linear_error =  3.813
```

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