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Gravity Probe B Relativity Mission

Justification for Gyroscope Shake Test Levels

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Part I

The Quantitative Model

1. Overview

The Gravity Probe B payload includes a caging mechanism which is designed to press the gyroscope rotor against its housing during launch. The goal is to keep the rotor from "rattling" back and forth inside the housing and protecting it from potential damage due to rotor-housing collisions. This mechanism was included in the design even though there was substantial evidence that it was probably not necessary.[1] The science gyroscope system has been fully qualified to launch in the caged configuration. There is currently a delta qualification program underway to assess whether the flight design is compatible with launching the gyroscopes uncaged. The purpose of this document is to derive an appropriate set of shake test levels for that program.

Because the rotor/housing system reacts to a shake test in an unusual non-linear way (free floating sphere bounded by a rigid wall), it is not appropriate to apply standards which were developed for normal structures or electronic boxes. Some environments which are very harsh for these standard systems have very little effect on an uncaged rotor. Conversely, other environments that are normally considered trivial for standard systems have a much larger effect on the gyroscope. Therefore it is vital that a careful analysis be performed to ensure that the qualification test environment adequately bounds the launch environment.

Note that determining the qualification levels based purely on the expected launch environment (without this analysis) is a problematic approach. The reason for this is that the quasistatic and dynamic components of the acceleration environment couple to the gyroscope in a manner different from traditional systems. It would be necessary to faithfully reproduce both quasistatic and dynamic components of the acceleration simultaneously during the test. Unfortunately, there are no facilities conveniently available that can accomplish this. The analysis in this document determines what suite of standard tests (random vibration, sine sweep, and centrifuge) subject the gyroscope to a severe enough set of environments to ensure that the gyroscope will function properly after launch.

[1] See, for example, *S0494 Vibration Test of the Gravity Probe B Gyroscope*

This work is accomplished in 3 steps:

1. Identify all potential mechanisms for damage to the flight hardware when launching uncaged. Develop a quantitative set of criteria to judge the "harshness" of an environment. This will be used to ensure that the qualification level adequately bounds the launch environment.

2. Develop acceleration models for all phases of the ascent to orbit. This is primarily an exercise in summarizing the results from the various Lockheed and Boeing engineering memos that detail the launch environment. Use the quantitative criteria to assess the impact of launch on the gyroscopes.

3. Using the same set of quantitative criteria, assess the impact of various candidate qualification test environments on the gyroscope. Determine what set of levels adequately bound the launch environment.

The resultant set of tests will be used to qualify the flight gyroscope design for launching in the uncaged configuration.

2. Referenced Documents

2.1. Lockheed Documents

SMS 214B	Spacecraft Launch and Operational Loads
SMS 233	ARP Pre-MECO Loads Analysis
SMS 260A	Dynamic Loads Model, Coupled Loads Analysis #2
SMS 304	Evaluation of Coupled Loads Analysis Results
SMS 349	Evaluation of PreMECO & MECO Loads Analysis Results
SMS 370	Integrated Payload Acoustic Test Plan
SMS 375	Shocklog Record of Test Gyro Transportation
SMS 377	Pathfinder Environment for Palo Alto to Sunnyvale Transport

2.2. Stanford Documents

S0181	Stress Analysis of a Quartz Gyroscope Housing
S0211	Gyroscope Caging Mechanism Design
S0288A	GP-B Rotor Mass Unbalance and Difference in Moments of Inertia
S0289	GP-B Rotor Shape and Torque Coefficients
S0370	Validation of Rotor Asphericity Requirement 1.4, T003
S0487	RT vs. Cold Gyro Shake Analysis
S0488	Gyro Shake Pass/Fail Criteria
S0492	Justification for Performing Acoustic Test Uncaged
S0494	Vibration Test of the Gravity Probe B Gyroscope (Thesis 10/98)
S0498	Justification for Gyro Shake Test Configuration
S0499	Justification for Gyro Shake Test Levels
S0521	Report on the Shake Test of an Uncaged Gyroscope
S0522	Justification for Launching Uncaged
S0525	Effect of Uncaged Launch on Science Data

2.3. Other Documents

- *Delta II Payload Planners Guide.* This document is in the public domain and can be downloaded electronically at http://www.boeing.com/defense-space/space/delta/delta2/guide/index.html. Section 4.2.3 details the expected flight dynamic environment, and Section 4.2.4 gives suggested programs for spacecraft qualification and acceptance testing.
- MDC Letter A3-Y935-EDAO-98-080 (11 December 1998) is Boeing's analysis detailing the environment Gravity Probe B can expect during MECO. They used Boeing's Finite Element Model (FEM) of GP-B to generate accelerations at every Degree of Freedom (DOF) in the model.
- Data from the Spacecraft Loads and Acoustic Measurement (SLAM) spacecraft, which launched August 25, 1997 on a Delta II to measure the vibration environment. Electronic copies of the data are available at http://analyst.gsfc.nasa.gov/slam.

3. Background

3.1. The GP-B Gyroscope and Flight Payload

Figure 3.1 is a drawing of the GP-B gyroscope. The gyroscope rotor is a Quartz or Silicon sphere with a Niobium coating sputtered on it. The rotor resides within the spherical inner cavity of a quartz housing. Three pairs of orthogonal electrodes are sputtered onto the housing so that the rotor can be electrostatically levitated. Helium gas flows through a spinup channel to spin the rotor to its full operational speed of approximately 100 Hz. One of the housing halves (designated the Readout or R-Half) has a pickup loop on the parting plane which is used to detect the rotor's spin axis during the science mission. The other half of the housing which contains the spinup channel is designated the S-Half.

In the nominal flight configuration a cylindrical caging rod enters the cavity through a hole in the R-Half of the housing and presses the rotor against the S-Half, firmly caging it for launch. This rod is actuated via a bellows which is pressurized with Helium gas. However, there are significant problems with the Helium supply lines that feed this caging actuation system for three of the four gyroscopes in the GP-B Payload. This leaves the sphere free to move in response to launch accelerations. Note that the rotor-housing clearance is very small, so that the rotor can move at most 39 μm (1.5 milli-inches) from one side of the housing to the other. Although this tiny space allows very little differential velocity to build up between the rotor and its housing, there is still a question as to whether it is possible that the gyroscope housing or rotor coatings could nevertheless become damaged due to rotor-housing impacts.

Although one gyroscope is sufficient to perform the relativity measurement, the GP-B design contains four gyroscope assemblies for redundancy. These gyroscopes are mated to a quartz index plate / spacer assembly, which is in turn mated to a quartz block (shown in Figure 3.2). Bonded to the quartz block is a telescope that points towards the guide star which provides the inertial reference for GP-B. This whole assembly is mounted in the probe which is evacuated and

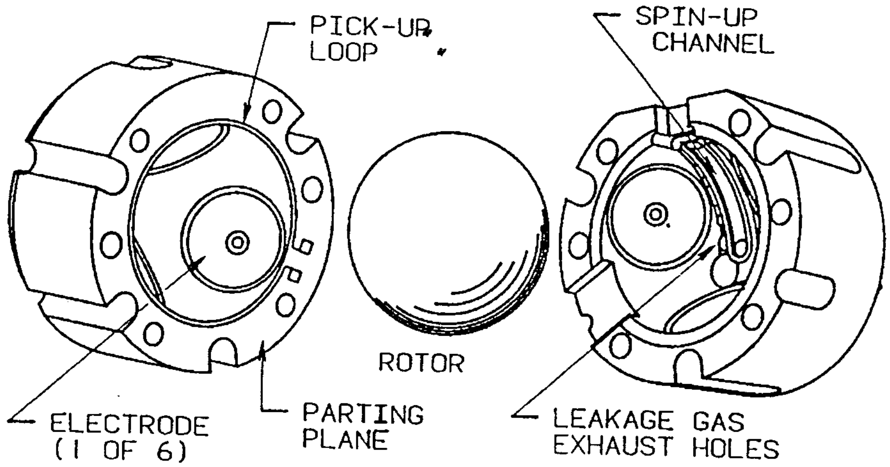


Figure 3.1: The Gravity Probe B Gyroscope

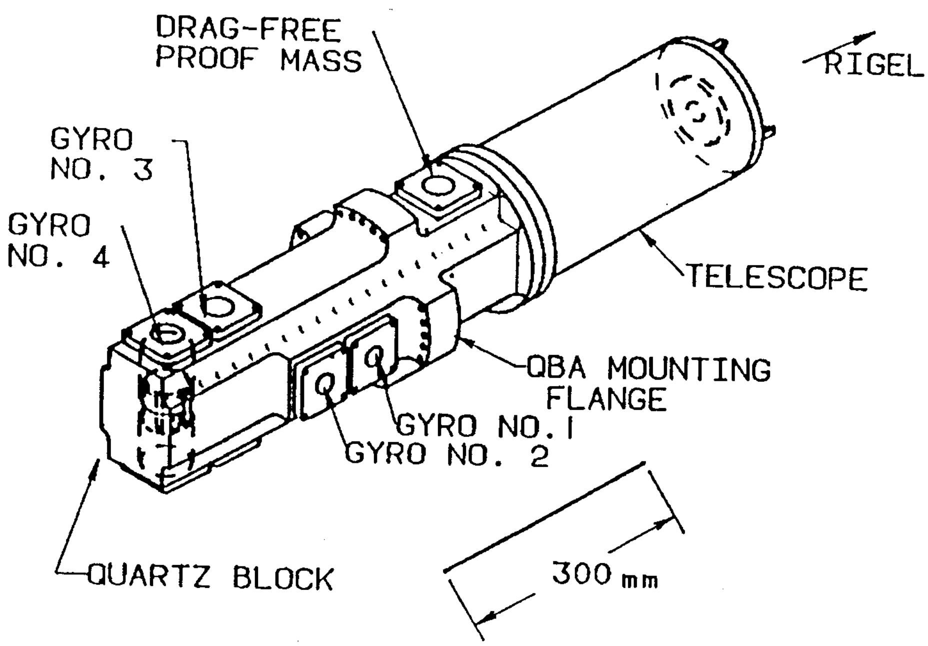


Figure 3.2: The Quartz Block Assembly

then inserted in a dewar filled with liquid Helium. Thus the GP-B gyroscopes operate in a cryogenic vacuum environment.

3.2. Coordinate Transformations

Figure 3.3 depicts two coordinate frames which are tied to the gyroscope housing. The Electrode or E-frame has its origin in the center of the housing with principle axes aligned with the electrode pairs. This is the coordinate frame which the suspension system uses to record rotor position. The X,Y, and Z axes are the same as those used by the flight suspension system, so positions calculated here should map one-to-one with observed positions. The Housing or H-frame has its origin in the center of the housing and its $+\hat{z}_H$ axis aligned with the rotor spin axis. Gyroscopes are mounted in the quartz block such that for two of the gyroscopes $+\hat{z}_H$ points towards the guide star and for the other two $-\hat{z}_H$ points towards the guide star. The Y-axis of the H-Frame points towards the spinup gas inlet and the X-axis completes the system per the right hand rule.

Transformations between the two frames are easily accomplished using the proper rotation matrix:

$$[\overrightarrow{r}]_E = {}^E\mathbf{R}^H \cdot [\overrightarrow{r}]_H$$

with

$${}^E\mathbf{R}^H = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{-2}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$$

During the science mission, Gyros 1 and 3 are oriented such that $+\hat{z}_H$ points toward the guide star, and Gyros 2 and 4 are oriented such that $+\hat{z}_H$ points away from the guide star. Practically, this means that during ascent $+\hat{z}_H$ for Gyros 1 and 3 point "up", while for Gyros 3 and 4 it points down. The payload is in this configuration during (1) the payload-level acoustic test, (2) the spacecraft-level acoustic test, and (3) the vast majority of the payload acceptance test.

It is interesting to note the position of the rotor in the electrode frame (as seen by the suspension system) during launch. Based on the transformation matrices given above and assuming the rotor sits 19.5 μm offcenter when it is all the way down (a value consistent with the thickness of rotor spinup and support lands), Gyros 1 and 3 will sit at X = -13.8 μm, Y = 0 μm, and Z = $+13.8$ μm. Gyros 2 and 4, whose spin axis points away from the guide star, will sit at X = $+13.8$ μm, Y = 0 μm, and Z = -13.8 μm.

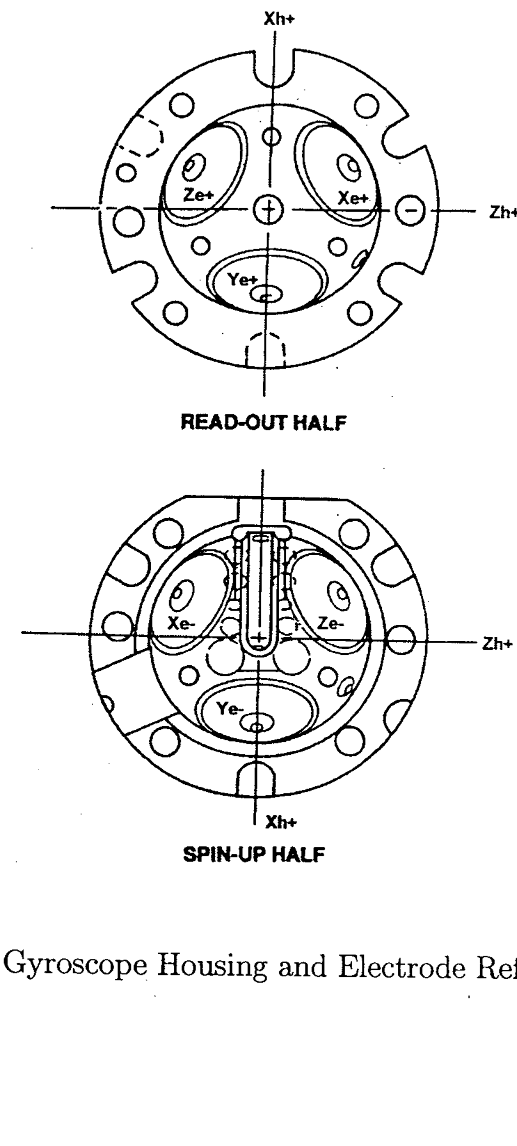


Figure 3.3: Gyroscope Housing and Electrode Reference Frames

3.3. The Rotor Support Lands

The gyroscope housing contains approximately 40 rotor support lands which serve as "bumpers" to protect the sensitive housing coatings from rotor impacts. GP-B drawings 23167 and 23168 detail the location of all support lands on the gyroscope housing. Physically, these support lands are arcs of metallic coating which are sputtered around the electrodes and spinup channel so as to keep them from being damaged by an impacting rotor. They consist of a Titanium-Copper-Titanium trilayer with an average thickness of 550 microinches. The two Titanium layers are very thin (only 25 microinches each) so that overall the support land responds to stress and impact forces like a thin piece of copper. Figure 3.4 is a drawing of the R-Half of the gyroscope housing with 1 of the 21 support lands on that half specifically pointed out for reference. Note that the hole in the center of the housing in Figure 3.4 is where the caging rod enters the housing.

Three of the support lands on the S-Half of the housing happen to be in a location where the rotor touches them when it is fully caged. These are called the caging support lands. However, physically there is no difference between the caging support lands and the multitude of other rotor support lands in the housing. Inspection of drawings 23167 and 23168 indicates that all lands are identical in composition and height.

When launching a gyroscope uncaged, the rotor will sit in one of the two positions indicated in Section 3.2. This means that the rotor will be resting against a different set of support lands than those that were designated for caging. During the ascent to orbit, the quasistatic component of the acceleration will press the rotor against these lands, providing an effective caging force for the rotor. Since the quasistatic component of the acceleration is only about 8% of the nominal caging force, and the rotor is being pressed against support lands of identical composition to the caging lands, it is clear that launching in this configuration holds no problem from a quasistatic point of view. This is examined in more detail in Section 4.2.1. The primary concern involves what happens when the dynamic component of the housing acceleration overcomes the quasistatic component so that rotor-housing impacts occur.

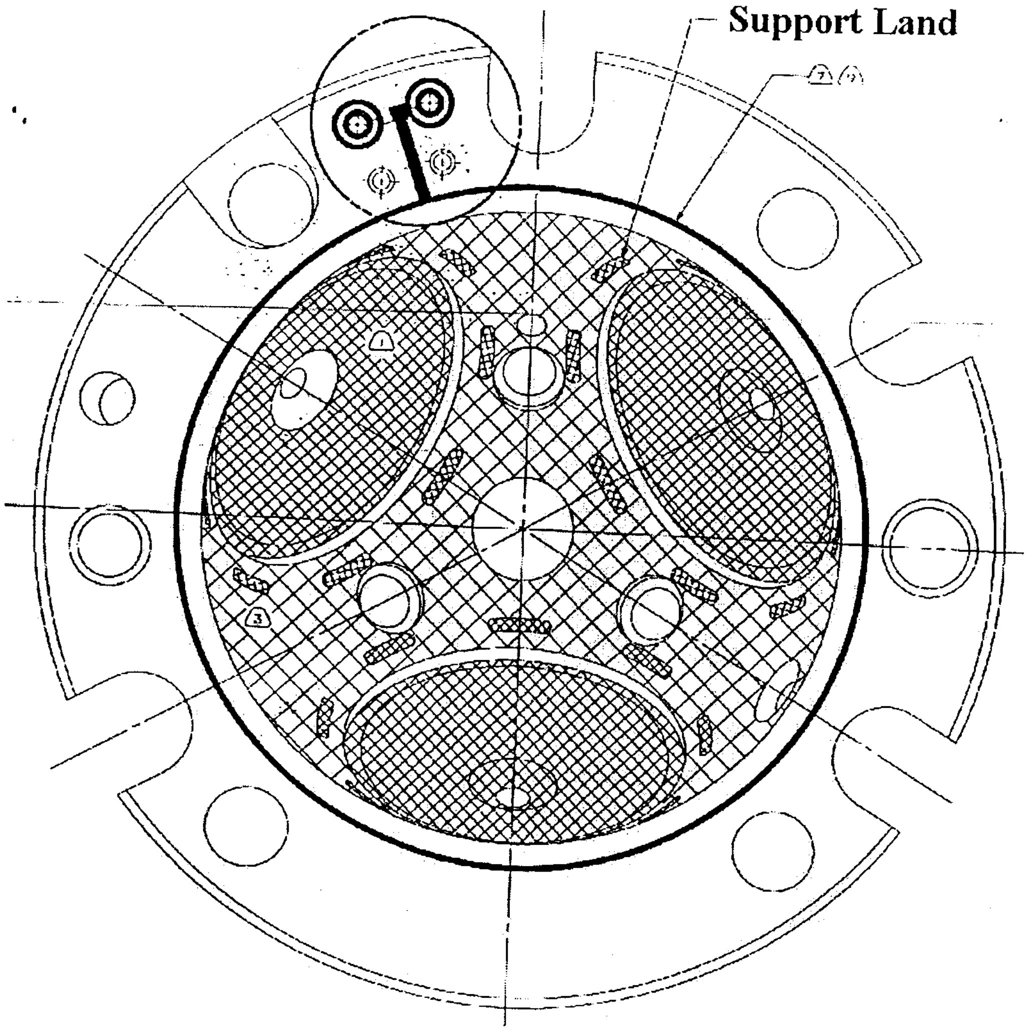


Figure 3.4: Rotor Support Land Geometry (R-Half of Housing Shown)

4. Effect of Housing Acceleration on Gyroscopes

4.1. Mechanisms for Gyroscope Damage

The mechanisms for damage when launching a gyroscope uncaged are very different than traditional mechanisms which are applied to structures or electronic boxes. Therefore it is not appropriate to blindly apply standards which were developed to qualify those types of items. The purpose of this section is to take a detailed look at the specific modes and mechanisms for damage to the gyroscope so that a reasonable test program may be developed for this specific problem. This is in accordance with NASA-Std-7001[1] which states that it is appropriate to tailor tests in certain situations, among them "Certain Fragile, one-time use items such as instrument detector elements."[2]

During the ascent to orbit, the gyroscope will experience both dynamic and quasistatic accelerations. Part 2 of this report will outline in detail the character and magnitude of these disturbances. The primary effect of the quasistatic component is to press the sphere against the housing wall, effectively caging the sphere against the support lands. Therefore it will sometimes also be referred to as the "caging component" of the acceleration. The effect of the dynamic portion is to cause impacts with the housing wall. However, for this to occur the dynamic component must first overcome any caging component which exists on the relevant axis.

The dynamics of an uncaged gyroscope are highly nonlinear. The system consists of a sphere in free fall that is bounded by a cavity. Upon impact with the wall of the housing, both sphere and housing deform slightly. This results in

[1] NASA-Std-7001 is not formally binding on the Gravity Probe B project. However, it does to a certain degree constitute industry standard, and is therefore considered in the development of this test plan.

[2] NASA-Std-7001 Section 4.3.6

a restoring force or "bounce", which pushes the sphere in the opposite direction.

During this process, the following mechanisms may result in damage to the gyroscope:

1. The quartz could fracture on either the housing or the sphere.

2. As the sphere scrapes against the rotor support lands[3], the top of the lands could create either a scratch or an indentation in the rotor coating. If the defect is bad enough, it could interfere with 1-g levitation. Enough defects could potentially interfere with the achievement of science-level performance from the rotor.

3. As the sphere scrapes against the rotor support lands, portions of the rotor support lands could rub off on the sphere, possibly interfering with one-g levitation. If enough material gets transferred, then it could potentially interfere with the science-level performance of the rotor.

The probability of any of these damage mechanisms occurring in flight depends on (1) the number of rotor-housing impacts, and (2) the momentum of the impacts. Because of the nonlinear nature of the gyroscope dynamics during an uncaged launch, it is not immediately obvious how to establish the details of a test program that will ensure the qualification gyroscope is subjected to the proper number and intensity of collisions. For this reason, it was necessary to create a detailed simulation that models rotor-housing collisions. This will contribute to the rationale for the test levels outlined in this document.

4.2. The Rotor-Housing Impact Model

In order to model the response of the rotor-housing system to launch acceleration environments, it is helpful to divide the rotor position into three distinct categories:

Case 1: The rotor is pressed against the housing wall

Case 2: The rotor is flying freely through the housing, or

Case 3: The rotor is impacting the housing wall.

These situations will be discussed separately below.

[3] Rotor support lands are Titanium ridges which are sputtered onto the ground plane and protect the electrodes and spinup channel from rotor impacts. These ridges are in the form of numerous arcs that are approximately 500 *μinches* high which bound the electrodes and the spinup channel.

4.2.1. Case 1: The rotor is in steady-state contact with the housing wall

The purpose of this section is to examine the case where the sphere is being pressed against the wall (i.e. it is not flying freely in the center of the housing or in the process of "bouncing" off the housing wall). This section will not consider how the gyroscope happens to arrive in this state, as that can not be done in a complete way until the discussion of housing impacts in Section 4.2.3.

Once the rotor is pressed against the wall, it will remain in that position until the sign of the acceleration changes. For instance, an acceleration which varies between +1 and +8 g's will not result in the rotor leaving the wall. This is a simple result of elementary dynamics. However, both rotor and housing will deform slightly in response to the force, and the magnitude of the deformation will change as the acceleration is changed between +1 and +8 g's. This will be called the "rotor-wall system", and it obeys a set of dynamics which are very different than those a sphere flying freely through the housing.

The magnitude the deflection has been calculated in S0181 and S0494 (see referenced documents). First, consider the load during the moment of 7.9 g peak acceleration (which is a factor of 1.5 over the expected maximum acceleration). At this point it is being pressed against the wall with a force of 4.9 N (1.1 lb), or approximately 8% of the force which the caging mechanism applies. Clearly, this is a very small force.

It is useful to examine the load margins for a gyroscope under these conditions. S0181 examined the load margins for a gyroscope when the rotor is caged with a 15 lb force. It defines the margin M as

$$M = \frac{\text{Tolerable stress}}{\text{Actual Stress}} - 1$$

The smallest margin of safety was for tensile stress and was found to be 2.85 under the full 15 lb caging load.[4] Scaling the actual stress obtained for the 7.9 g maximum acceleration during launch yields a margin

$$M(\text{uncaged}) = 50.5$$

The above discussion shows that the quasistatic-component of the launch loads poses no threat to the gyroscope. The primary reason for this is that the rotor is

[4] S0181 uses a finite element analysis to calculate the stress in the housing. It notes that since the analysis uses mesh elements of finite size (whereas the actual material is a continuum), the analysis will overestimate the true tensile stress.

quite light (0.0633 kg), so that even relatively large accelerations result in small forces being applied to the housing.

While the quasistatic acceleration is pressing the rotor against the housing wall, it is also being subjected to sizeable sinusoidal accelerations. This brings up the question as to whether there is some resonant frequency in the rotor-wall system which can be excited during this phase of the launch. Assuming the maximum deformation calculated in S0181 (which takes into account the metallic rotor support lands) scales linearly with applied force[5], 7.9 g's of acceleration will result in a deformation of 58 nm. Based on this data it is possible to estimate a spring constant for the interaction

$$k = \frac{F_{MAX}}{X_{MAX}} \approx 8.3 \times 10^{7}\ N/m$$

which yields a natural frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 5.8\ kHz$$

There is no appreciable housing acceleration above approximately 400 Hz[6]. Since the system is being excited well below its lowest resonant frequency, it responds in a quasistatic way to all launch input accelerations. Since launch vibrations excite no mode in this system and the maximum deformation is negligible, it is acceptable to model both housing wall and rotor as perfectly rigid during all periods where the rotor is pressed against the wall.

4.2.2. Case 2: The rotor is flying free

If the rotor is not in contact with the wall, then it behaves like a simple $1/s^2$ plant. More precisely, if the acceleration of the housing (due to launch loads) is a_H and the acceleration of the rotor (due to gravity) is a_R, then the position of the rotor *relative to the housing* at any time is

$$x(t) = \int\int (a_R(t) - a_H(t))\ dt$$

It continues to follow this trajectory until it impacts the housing $x_R - x_H = 0$.

[5]See Table 3, page 13, S0181. Note that the discussion indicates Model 3 is the most accurate. Also, although demormation is not really a linear function of force, it still provides a reasonable estimate to linearize in this small regime.

[6]See the Lockheed documents referenced in Section 2.

4.2.3. Case 3: The rotor is impacting the housing

The equations of motion for a rotor impacting the housing are developed in S0494. Although the details of this work will not be repeated here, it does draw two very important conclusions:

- The velocity v_d of the rotor (relative to the housing) when it bounces off the housing wall is proportional to its impact velocity v_i

$$v_{d,R} - v_{d,H} = \epsilon \left(v_{i,R} - v_{i,H} \right)$$

  where ϵ is a coefficient of restitution that sits in the range 0 to 1.

- The duration of the impact is very short ($< 100\ \mu s$) compared to the acceleration frequencies which shake the housing during launch.

The second item simplifies the simulation of an impact tremendously. Since the purpose of the simulation is to record number of impacts and the impact velocity, it is not necessary to simulate in detail what happens during the collision itself. Therefore the simulation can simply reset the state of the system so that the velocity of the rotor at the timestep after the impact is ϵ times the velocity of the rotor at the timestep prior to the impact.

The value of ϵ was determined experimentally to be approximately 0.24 ± 0.05. This was done by observing the size of a "bounce" during a sinusoidal shake test (see, for example, plots of rotor bounces contained in S0494). This value is the same for both quartz and silicon rotors.

4.2.4. The Integrated Impact Model

The Simulink implementation of the model outlined above is shown in Figure 4.1. The path in effect when the rotor is flying free in the housing traces through two $1/s$ integrators. The first integrator, which takes rotor-housing differential acceleration to a differential velocity shall be called the velocity integrator. The second, which takes the differential velocity to a differential position is termed the position integrator.

The position integrator has positive and negative saturation values at the location of the housing walls. When this integrator reaches its saturation point, it sends a trigger (via its lower output) to the velocity integrator. This tells the velocity integrator to reset. The velocity integrator will reset to whatever value

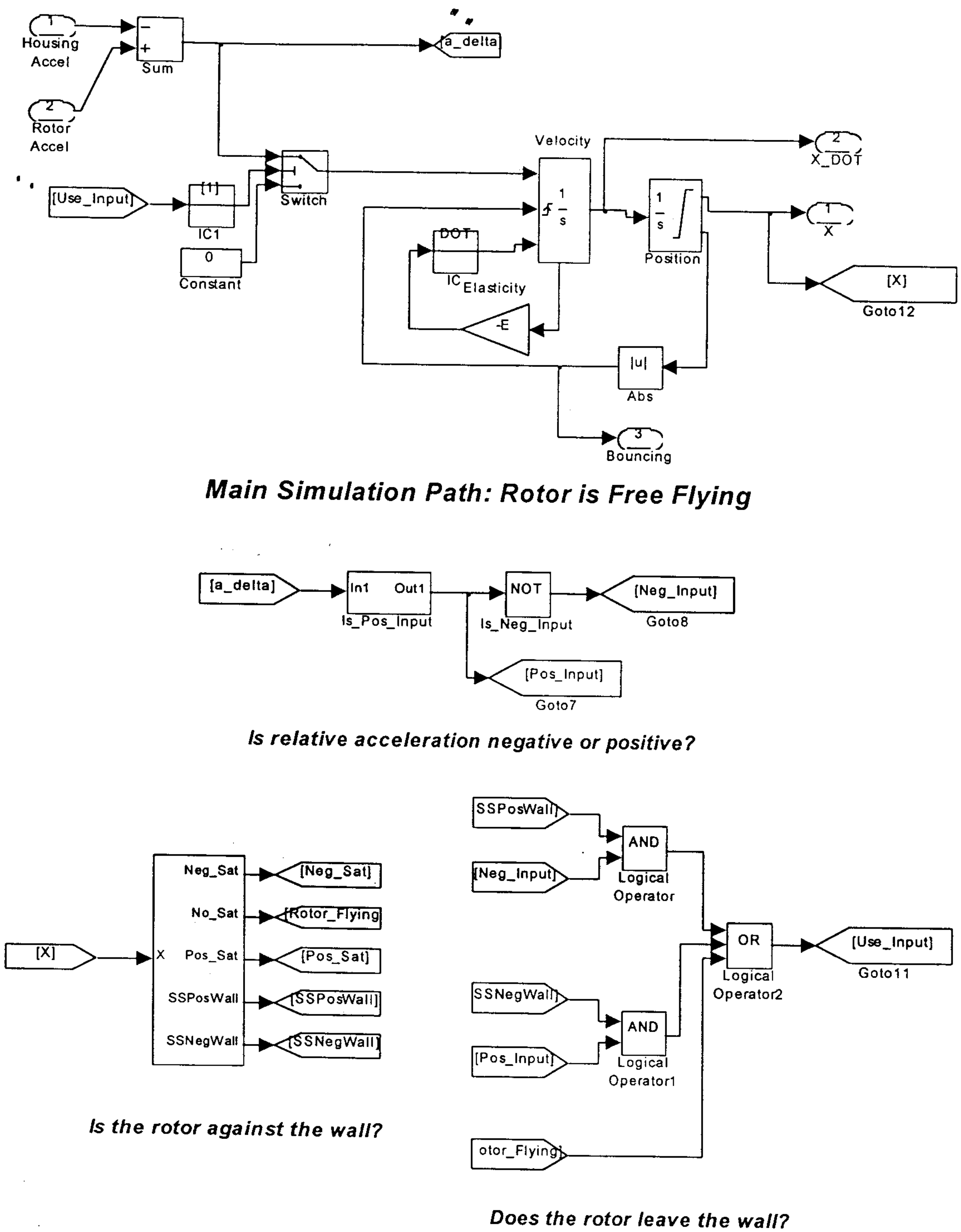


Figure 4.1: Core of the Impact Simulation

is on its third input. The small inner feedback path on the velocity integrator forces this reset value to be ϵ times the impact velocity.

If the rotor is saturated against the wall as described in Case 1 above, then it is necessary to disable the acceleration input to the velocity integrator in order to avoid the non-physical situation where the velocity continues to increase while the position stays saturated. The logic in the bottom right hand side of the figure takes the rotor position (top or bottom wall) and calculates based on the gyro housing acceleration whether the sphere will leave the wall. This event triggers the simulation to resume the main "Rotor is Free Flying" loop.

This model of gyroscope impacts was then incorporated into a larger simulation that included launch or test environments and a special subsystem which logged the time and velocity of every rotor-housing impact. Figure 4.2 shows the version which was used to model the response of the uncaged gyroscope to liftoff. This includes the low-frequency results of the coupled loads analysis for this event (called "Liftoff Data" in the figure) as well as the expected launch acoustic environment. More about the simulation of this phase of the launch will be included later.

This simulation has been tested extensively against a variety of inputs and found to behave in the desired fashion. Figure 4.3 is a plot generated of the differential rotor-housing position when the housing is subjected to a sinusoidal acceleration. It can be compared to the experimental plot found in Figure 21b of S0494. The model predictions were also compared to measured rotor position during a 1.65 grms random vibration test. Later on it will be shown that for the specific 1.65 grms considered in this document, a total of 12,576 impacts are predicted by this model. When the actual test was being performed, the rotor position was monitored and "snapshots" were recorded digitally. Counting the number of impacts in 100 ms in one of these snapshots and scaling for the full test, it was experimentally estimated that the actual number of rotor-housing impacts during this test was 12,000. Clearly, the model predictions and experiments are in excellent agreement.

4.2.5. Impact Velocity

As stated earlier, the things which should be considered when examining the issue of an uncaged gyroscope launch are number and velocity of impacts. The degree of damage which the gyroscope incurs (if any) will depend on these two variables.

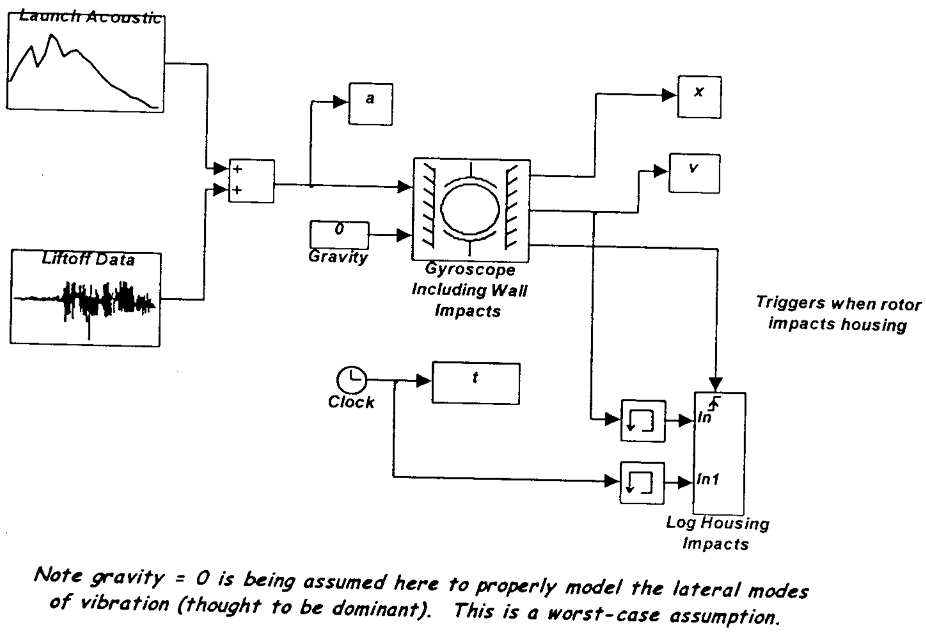


Figure 4.2: Simulation of Launch Environment Including Gyro Impacts

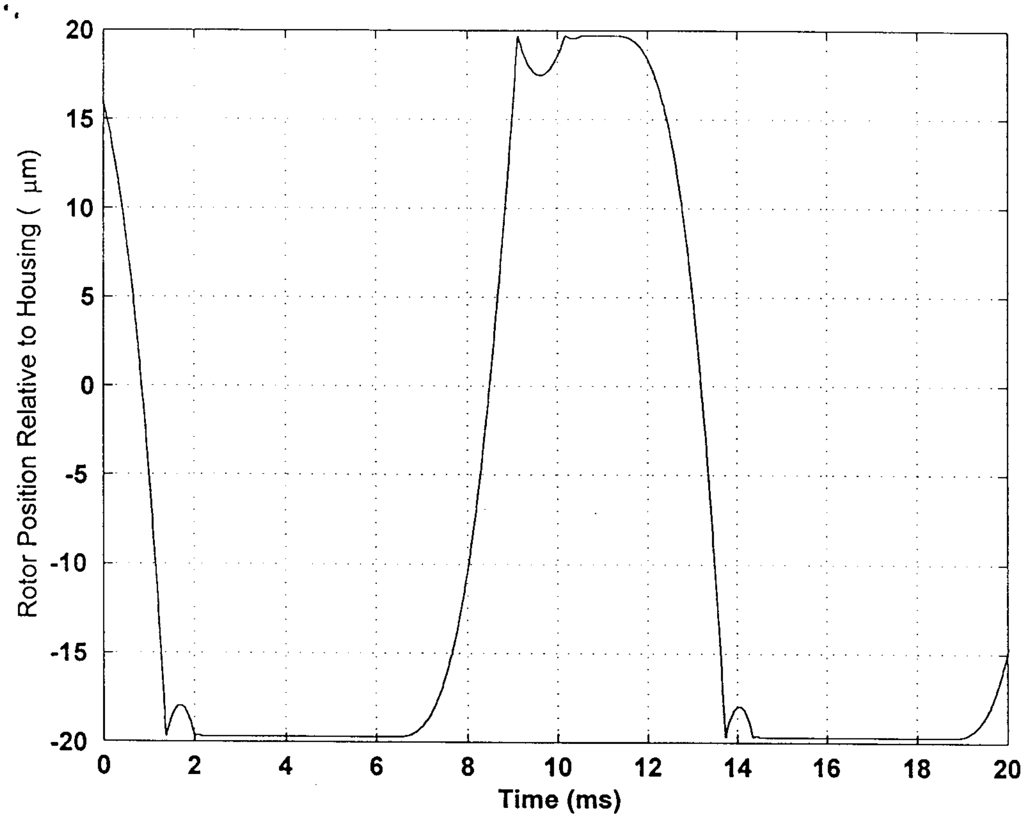


Figure 4.3: Gyroscope Modeled Response to a Sinusoidal Acceleration

At this point it is helpful to establish the proper orders of magnitude for the launch disturbances.

The spacing between the rotor and the housing is extremely small. Counting the space occupied by the support lands, the rotor only has 39 μm (1.5 mils) that it is free to move, from one edge of the housing to the other. The peak acceleration which the rotor will see during launch is 7.9 g. If the rotor were to be accelerated at this level across the entire width of the housing, then its maximum velocity would only be 7.8 cm/s (0.25 ft/s or 0.17 mph). In reality, the launch environment never produces this worst-case acceleration, with the true maximum velocity being 3-4 cm/s. Therefore the mental picture of a sphere rattling around inside a housing it incredibly misleading. In fact, the sphere is barely free to move, with less than the width of a human hair between it and the housing, and it is building up tiny velocities in response to the launch vibration environment. This fact is a significant contributor to the GPB gyroscope's robustness to vibration environments.

The following table compares velocities of impact for launch versus other events which routinely happen throughout gyroscope manufacture, test, and operation. The maximum expected collision velocity during launch is the result of the analyses which will be performed in the next section. Note that since the rotor is only free to move in a specific confined space, its velocity of impact (given a DC acceleration across the gap) goes only as the square root of the acceleration. Therefore a 4-g acceleration is only a 2× increase in impact velocity.

Event	**Impact Velocity**
Max Expected Impact Velocity during Launch	4.3 cm/s
Delevitation from Housing Center	2 cm/s
Delevitation with 0.3 Hz spinning rotor	4 cm/s (to halt surface of sphere)
Flight Gyro Transportation	$<$ 3 g peak
Qual Gyro Transportation	4.8 g peak

4.2.6. Trivial Impacts

The strict application of the model presented in this work will record impacts at velocities so small that they can be discounted. For example, consider the case where a rotor is develitated from the top of a fixed (non-accelerating) housing. It will hit the housing wall at some velocity v_i, and rebound off the wall with a velocity $\epsilon \cdot v_i$ (see Section 4.2.3). It will then go up some distance, turn around

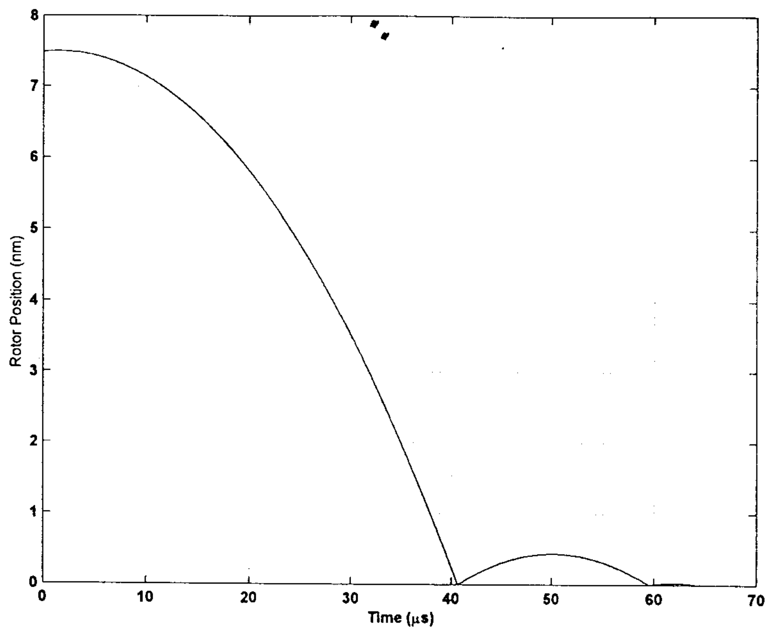


Figure 4.4: A sub-nanometer bounce

and impact the housing at $\epsilon \cdot v_i$. This will repeat, but the third time it will hit at $\epsilon^2 \cdot v_i$ and so on. In general, the first impact has velocity v_i, then the n^{th} impact has velocity $\epsilon^{n-1}v_i$. Mathematically speaking, there are an infinite number of "bounces", each reaching a smaller height and impacting with a lower velocity.

Figures 4.4 and 4.5 show plots of this example. The first bounce goes to a height of a few micrometers, the second goes a hundred nanometers. By the time of the fourth bounce (shown in Figure 4.4), the size of the bounce is less than a nanometer, or approximately the size of the atoms which comprise the sphere. By the time of this fourth bounce, the rotor is traveling at a speed of 1 cm every 20 seconds.

Mathematically, all of this is correct and does not indicate any error in the model. However, it must be acknowledged that at some point the impact velocity is so small that no harm could possibly result from the impact. In this work, any impact velocity less than 0.5 cm/s will be considered negligible and will not be shown in plots. This is 1/4 the velocity at which the rotor impacts the housing

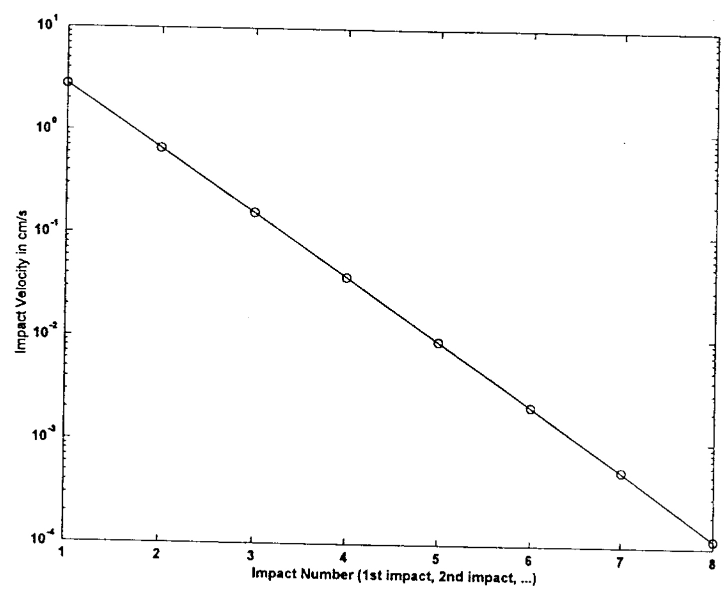


Figure 4.5: Impact Velocity for the 8th bounce is 1 micrometer per second

during a normal delevitation from housing center. It is equal to the velocity the rotor impacts the housing when the rotor is delevitated from a height 1 μm above the housing wall (1/30 the centered gap).

4.2.7. One-Dimensional versus Three-Dimensional Modeling

The model that has been presented here is a one-dimensional model. Because the rotor and inside of the gyroscope have spherical symmetry, this is suitable for modeling translational rotor-housing impacts. However, it does not model potential rotational effects. Shake tests that have been performed on gyroscopes have demonstrated that these effects do not contribute to gyroscope damage. Shake tests that have been conducted at even 5 grms (with expected peak accelerations of 20 g, and expected rotor-housing impact velocities of 3 times those expected for launch) have demonstrated that coating damage occurs in the form of microscopic indentations, not scratches. Long scratches would be indicative of damage due to rotational mechanisms, whereas a point depression is indicative of a pure translation mechanism.

Also, it should be remembered that the motivation for completing this analysis is to provide a quantitative means of comparing the "harshness" of potential qualification environments to the expected launch environment, in order to establish an appropriate set of shake test levels. The one-dimensional model presented here is sufficiently accurate to accomplish that end.

5. Criteria for Comparing Environments

Based on an investigation of the various mechanisms for damage when launching a gyroscope uncaged, it was determined that the relevant metrics for quantifying the "harshness" of an environment were (1) the number of rotor-housing impacts, and (2) the velocity of those impacts. High-velocity impacts will always be more damaging than low-velocity impacts, so when comparing two environments, it will often be helpful to compare the number of collisions above some set value.

In the next part of this work, it will be determined that the maximum rotor-housing impact velocity is expected to be 4.3 cm/s. Therefore the primary metric that will be used for assessing the harshness of a qualification program will be the number of expected rotor-housing impacts greater than 4 cm/s. However, a complete velocity histogram will also be provided so that a more in-depth comparison can be made.

Part 2 of this work will examine in detail the expected launch environment, and Part 3 will use that information to establish a set of qualification test levels.

Part II

Flight Gyroscope Environments

6. Expected Acceleration Environments

6.1. Launch

The acceleration environment to which the gyroscope is subjected during launch can be divided into three main phases: Liftoff through Transonic and Max Q, Pre-MECO events, and MECO and PostMECO events. Disturbances outside these phases are of much smaller magnitude and are therefore bounded by the above events. Figures 6.1 and 6.2 show a qualitative assessment of the acceleration which the payload will see during launch. The data is the measured acceleration during the launch of the Spacecraft Loads and Acoustic Measurement (SLAM) spacecraft. This experiment launched August 25, 1997 on a Delta II, so this environment may be considered typical, not worst case. Therefore it should only be used to provide qualitative insight into the acceleration environment, not to set test levels.

The SLAM data indicates that the quasistatic portion of the acceleration is in general increasing throughout the ascent to orbit (with a few exceptions). This quasistatic acceleration has a tendency to "cage" the gyroscope against the support lands. The larger the quasistatic component, the less chance that a dynamic component will become large enough to overcome it and cause the rotor to move. The effect of the overall load of the rotor being pressed against the housing wall was analyzed in Section 4.2.1 and found to be insignificant (the design margin is 50.5 when compared to the maximum expected peak acceleration multiplied by 1.5).

Detailed discussions of these three different phases of the ascent to orbit and their impact on an uncaged gyroscope are included in the following sections.

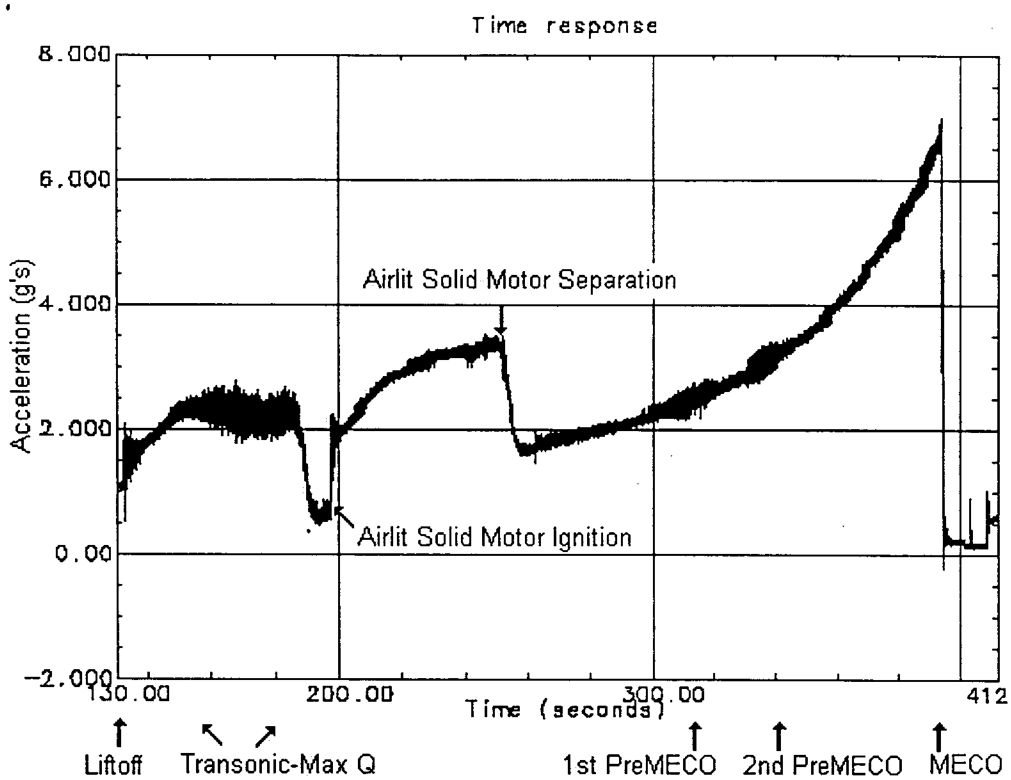


Figure 6.1: Axial Acceleration Measured during SLAM Launch (for qualitative indication only)

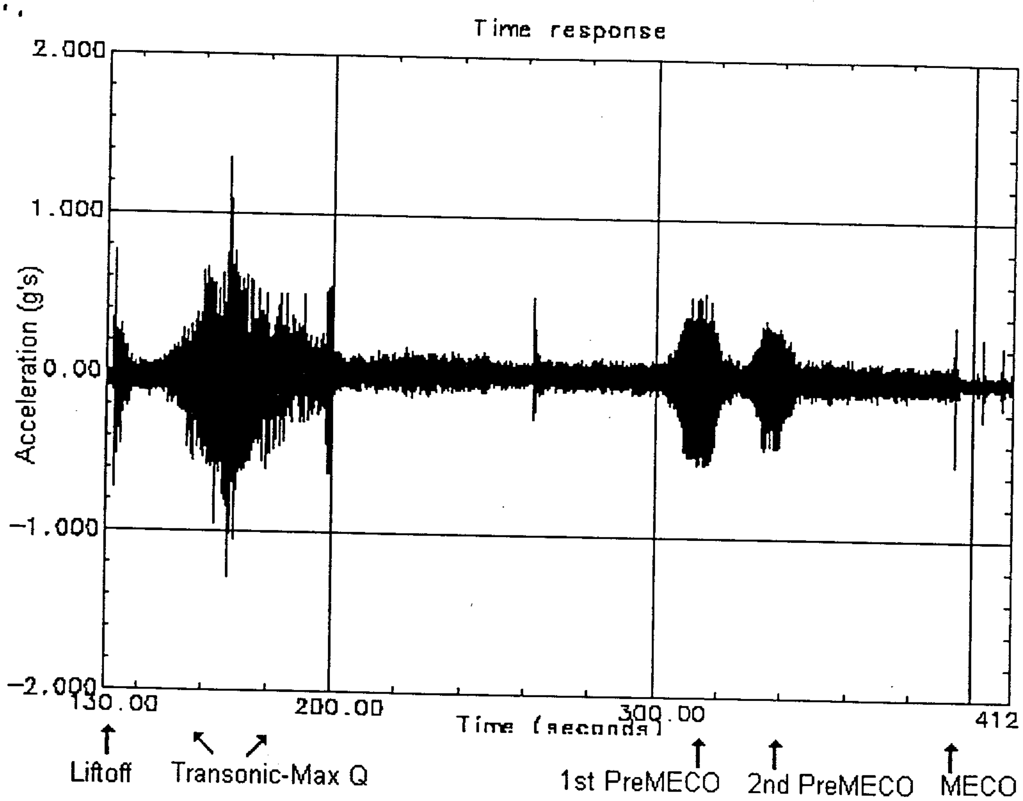


Figure 6.2: Axial Acceleration Measured during SLAM Launch (for qualitative indication only)

6.1.1. Liftoff through Transonic and Max-Q

Description of the environment

A two second liftoff acceleration time history has been supplied by Boeing, and the resultant acceleration at the quartz block has been calculated by Lockheed using the GP-B Finite Element Model (FEM). The result is shown in Figure 6.3. Since the gyroscope housing is held in rigid (quart to quartz) contact with the quartz block, this is also the response of the housing.

In order to get a complete picture of the acceleration environment during liftoff, the acoustic and airloads contributions need to be included as well. Since airloads are low-frequency disturbances that couple in a manner similar to the liftoff disturbances, the airload contribution can be modeled by scaling the above response by the appropriate factor. Analysis by Lockheed indicates that the airload correction factor $K_{acf} = 1.23$.[1] This factor has already been included in Figure 6.3 and all other representation of liftoff loads in this work. With this correction, the lateral acceleration is 2.5 g peak and 0.47 grms.

To be conservative, it is typical in the industry to include a Model Uncertainty Factor (MUF) of 1.50 when presenting calculated loads. This results in a lateral acceleration of 3.7 g peak and 0.71 grms. An overall net acceleration was obtained by performing an RSS combination of the lateral and axial accelerations. The results are tabulated below.

	Lateral Acceleration (g)		Axial Acceleration (g)		Net Acceleration (g)	
	peak	*rms*	*peak*	*rms*	*peak*	*rms*
MUF = 1.0	2.5	0.47	2.0	0.38	3.2	0.60
MUF = 1.5	3.7	0.71	3.0	0.58	4.8	0.92

Lockheed analysis indicates that this calculated acceleration bounds (at the quartz block) all launch loads through transonic and Max Q, which is the first 60 seconds of flight. The environment model assumes that this *worst-case* signal occurs continuously for the first 60 seconds of flight. This is probably much more severe than the actual flight scenario.

Two different approaches were tried for creating the 60-second acceleration time history from the 2 seconds of available data which has been calculated to bracket the first 60 seconds of flight. The first (and most obvious) method was

[1] This was obtained by performing the RSS sum of the liftoff loads and the air loads, then deriving the factor by which acceleration increased.

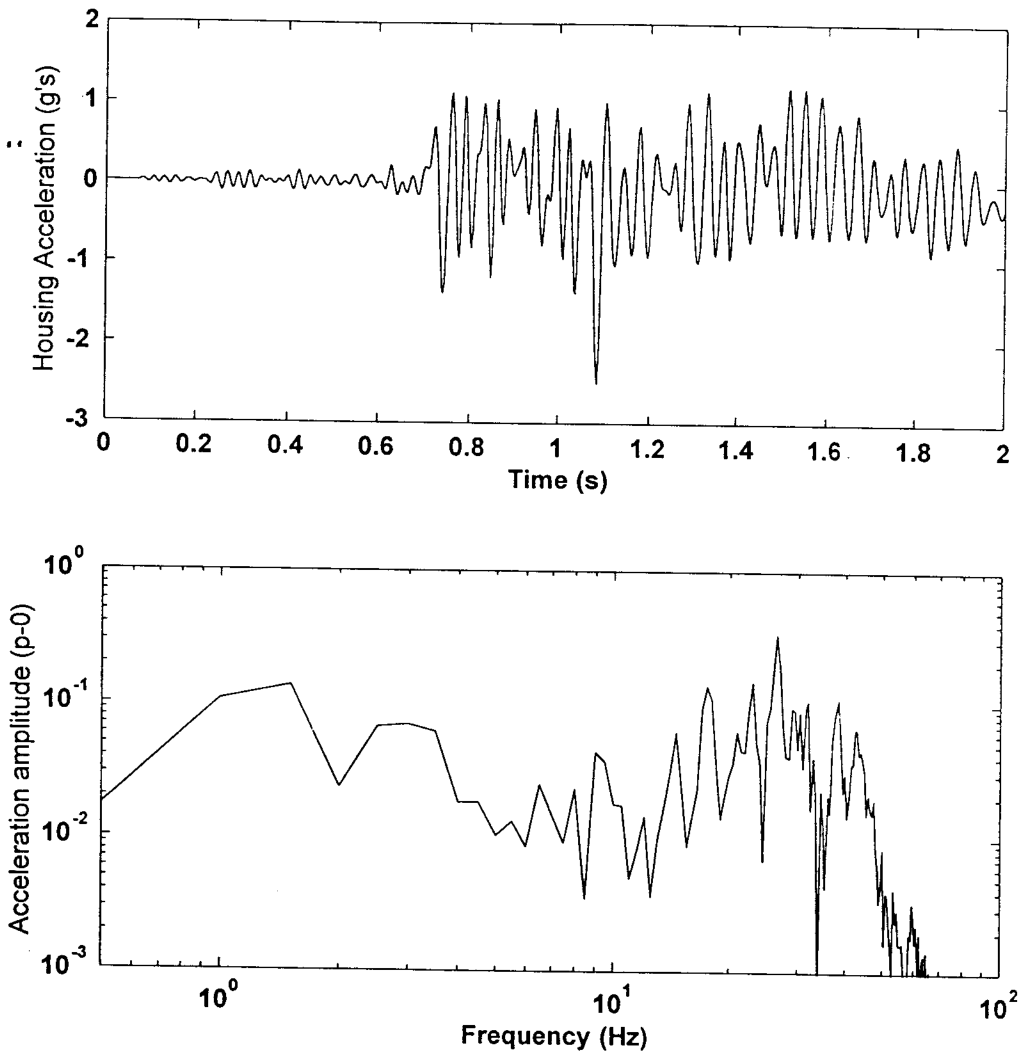


Figure 6.3: Time History and Fourier Transform of Quartz Block Response to Liftoff (MUF = 1.0)

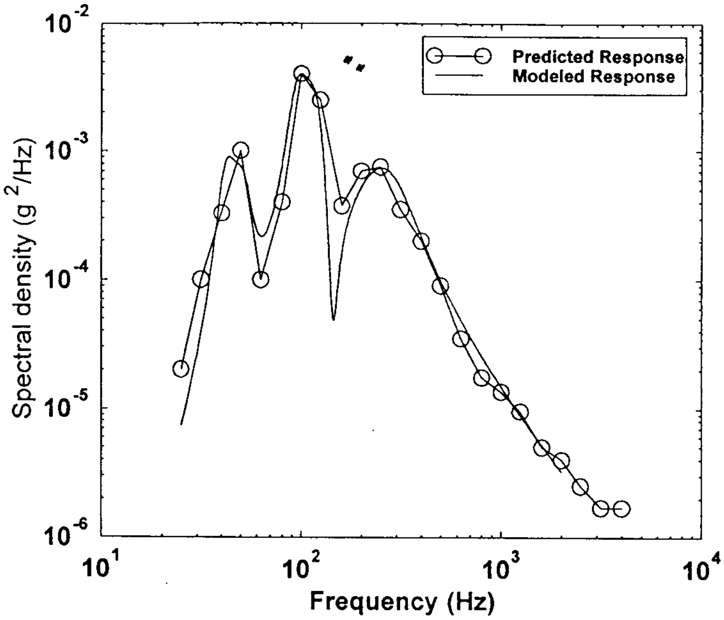


Figure 6.4: Power Spectral Density of Launch Acoustic Environment (0.56 grms)

to repeat the two-second snapshot 30 times. A better approach is to analyze that two-second snapshot and determine the statistical properties of the random process (e.g. its power spectral density). A random process can then be created in the simulation that has these same properties. This has the advantage that a full 60-second snapshot of the random process is obtained, instead of just repeating a 2-second snapshot again and again. Both approaches were tried and found to yield comparable results. However, the random process approach was slightly worst case, and was therefore chosen for the following analysis.

The above discussion provides a reasonable model of the low-frequency (<100 Hz) launch loads. However, there is also acoustic energy which couples to the gyroscope as high frequency acceleration of the housing. The expected launch acoustic spectrum and the filter used to model it (discussed below) are shown in Figure 6.4. Integrating this curve yields a net acceleration of 0.56 grms.

To model this, a filter was designed which yielded the proper power spectral density. White noise was then input to the filter to generate a random process with the same statistical properties as the launch acoustic acceleration. The resulting rms value of the sequence was verified both in the frequency domain (by integrating under the filter's curve) and computing the rms value directly in the time domain using numerical integration.[2] This sequence was then summed into the liftoff and airloads discussed above to yield the overall acceleration environment. The model is depicted in Figure 4.2.

The *Delta II Planners Guide* states that the duration of the maximum acoustic environment is less than 5 seconds.[3] To add margin and to cover any unmodeled uncertainty, the full acoustic environment was applied in the simulation for 10 seconds.

To summarize, the liftoff through transonic and Max Q (the first 60 seconds of flight) were modeled by creating a random process with the same statistical properties as the two-second snapshot provided by Lockheed. In addition to this, the maximum acoustic level was applied for the first 10 seconds of flight. The acceleration was applied to the gyroscope impact simulation and the resulting rotor-housing impacts recorded. This was then repeated with the acceleration environment increased by a factor of 1.5.

Before discussing the results of the simulation, it is useful to summarize the factors which make this a worst-case simulation:

1. The worst-case liftoff and airloads were applied continuously for the full 60 seconds of flight. Obviously, in the actual launch the worst-case environment will only be applied for some fraction of that period.

2. The maximum acoustic environment was applied for 10 seconds instead of the 5 seconds guaranteed by the *Delta II Planners Guide.* Also, this is the maximum level in each frequency bin guaranteed by Boeing. The true levels are likely to be less.

3. The simulation was repeated a second time incorporating an additional 1.5 factor on all accelerations.

Effect on the Gyroscope

The gyroscope impact spectra for the calculated liftoff, air loads, and acoustic acceleration are shown in Figures 6.5 (MUF = 1.0) and 6.6 (MUF = 1.5). These

[2] For a signal $x(t)$, the RMS value over a time frame $[0,T]$ is defined as $\sqrt{\frac{1}{T}\int_0^T [x(t)]^2\,dt}$

[3] See Table 4-6 and Figure 4-11. Note that the GPB launch vehicle has a 10 ft fairing. This is also the duration cited in the GP-B Mission Specification.

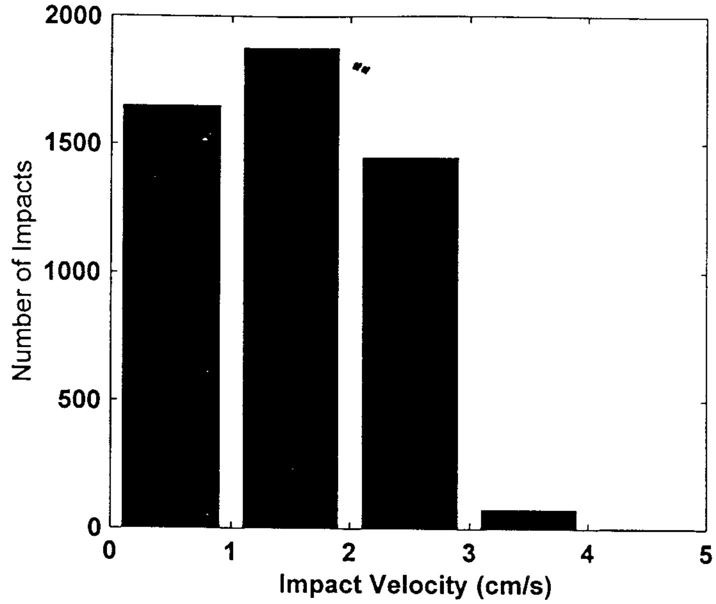


Figure 6.5: Impact Spectrum for Liftoff through Transonic and Max Q (MUF=1.0)

plots shown the number of impacts which occur in different ranges of velocity (e.g. how many rotor-housing impacts occurred where the differential velocity was between 1 and 2 cm/s). Recall that the relevant parameter in assessing damage due to rotor-housing impacts is the velocity. Therefore lower-velocity impacts have less probability to damage the gyroscope than higher-velocity impacts.

The following table summarizes the effect of this phase of the ascent to orbit.

	Total Impacts	**Max Impact Velocity**	**Total Energy**
MUF = 1.0	5048	4.3 cm/s	43.8 mJ
MUF = 1.5	6099	5.2 cm/s	63.7 mJ

The **Total Impacts** column shows the total number of rotor-housing impacts that occurred during the first 60 seconds of flight. The **Max Impact Velocity** column records the maximum velocity of all rotor-housing collisions during this period. Finally, the **Total Energy** column records how much energy was dissipated into the rotor-housing system during all the collisions. Since the rotor impacts with some energy $E_i = \frac{1}{2}mv_i^2$ and rebounds at some reduced energy due

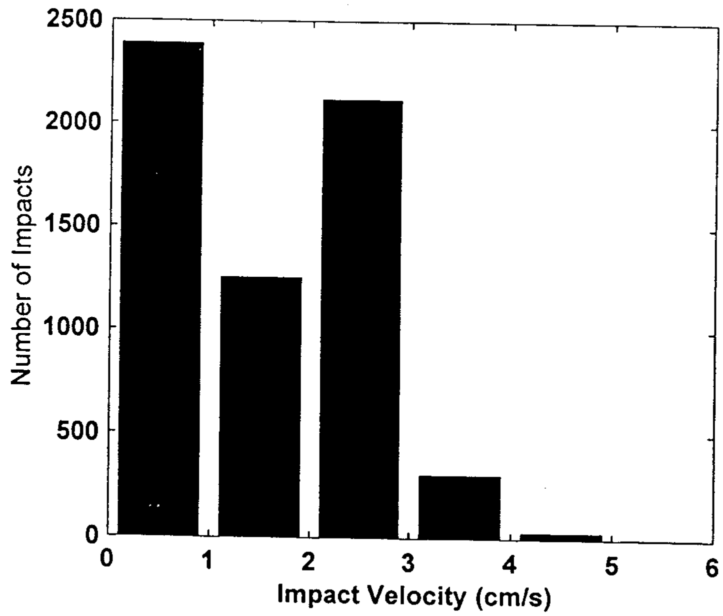


Figure 6.6: Impact Spectrum for Liftoff through Transonic and Max Q (MUF=1.50)

to the reduced velocity $E_r = \frac{1}{2}m\left(\epsilon v_i\right)^2$ (See Section 4.2.3), this means that some energy E_d is dissipated in the collision, where $E_d = \frac{1}{2}m\left(1-\epsilon^2\right)v_i^2$. The summation of this value over all the collisions is the value recorded in the **Total Energy** column.

To calibrate the numbers in Figures 6.5 and 6.6, a delevitation of the rotor from the center of the housing (something which is performed routinely in GP-B) results in a 2 cm/s collision. A delevitation of the rotor when it is spinning at 0.3 Hz means that the surface of the sphere is moving at 4 cm/s when it comes to an abrupt stop. Therefore there are no impacts in the MUF=1.0 that are more strenuous than events which are performed routinely on all GP-B gyroscopes. As a final note, the maximum impact velocity of 5.2 cm/s (for MUF=1.5) corresponds to 0.116 mph (0.17 ft/s). Although there are numerous collisions, they are at extremely low velocity.

6.1.2. Pre-MECO

Description of the Environment

After the launch vehicle goes through Max Q, the next significant events are the first and second PreMECO (Pre Main Engine Cutoff) events. These occur approximately 3 minutes after liftoff. For the GP-B spacecraft, the second Pre-MECO event is much more strenuous than the first as it can excite the spacecraft mode at 30.865 Hz.[4] Lockheed EM SMS 349 details the response of all nodes the GP-B dynamic model to both PreMECO events. For this analysis, the following inputs were applied:

Event	**Frequency**	**Acceleration Input**
Pre-MECO#1	26-28 Hz	.15g axial & .7g lateral sinusoid with 2.5g axial quasistatic
Pre-MECO#2	28-32 Hz	.20g axial & .3g lateral sinusoid with 2.5g axial quasistatic

The response of the quartz block (which is rigidly tied to the gyroscope housing) is listed on Page B-3 of this analysis as

[4] See McDonnel Douglas Aerospace (currently Boeing Aerospace) memorandum A3-L214-LEAL-DELTA-95-041 pg 1.

Node	Description	Min Acceleration	Max Acceleration
130	Quartz Block (lateral)	-0.84	+0.84
131	Quartz Block (lateral)	-0.53	+0.53
132	Quartz Block (axial)	+0.84	+6.66

As for the liftoff case, the response of the gyroscope to both the calculated environment (MUF=1.0) and the calculated environment increased by 50% (MUF=1.50) will be examined.

The MUF = 1.50 Model The numbers listed in the previous table already include a Model Uncertainty Factor (MUF) of 1.50, and are therefore the basis for this model. However, special attention must be paid to the axial acceleration numbers in order to yield a true worst case. Note that the axial response is not symmetric about the 2.5 g quasistatic load, which may cause some confusion. The reason for the asymmetry lies in how the MUF is applied. The calculated response (without margin) is +2.5 g quasistatic with a 1.94 g peak-0 sine wave superimposed on top. This leads to a minimum g-load of +0.56 and a maximum g-load of +4.44 g. Multiplying these numbers by 1.50 yields the response indicated in the above table.

This method of accounting for uncertainty yields a worst-case maximum load, and is therefore overall the most conservative approach for the majority of the spacecraft. However, it is not worst-case for gyroscopes. Section 4.2.1 examined the rotor-housing response to its peak acceleration and found a load margin of 50.5. The real damage mechanisms for an *uncaged* gyroscope are tied to accelerations which cause the rotor to impact the housing wall. As long as the sign of the acceleration does not change, the sphere does not move and there are no impacts on the housing wall. For this reason, the minimum acceleration (not the maximum) will dominate the potential for gyroscope damage in this case. If it falls below zero, the sphere will leave the housing wall and rotor-housing impacts will result.

A worst-case analysis of the Pre-MECO event would be one that results in a change in the sign of the acceleration. Therefore in this analysis a Dynamic Uncertainty Factor (DUF) will be applied only to the 1.94 g p-0 sinusoidal portion.[5]

[5] The Lockheed dynamicst who wrote EM SMS349 has concurred with this approach for this particular system.

This yields the following model for the axial acceleration

$$a(g's) = 2.5 + 2.9 \sin(2\pi f_0 t)$$

where f_0 is the 30 Hz second Pre-MECO frequency. This model indicates that there will be a 0.4 g peak differential axial acceleration between the rotor and the housing. Performing an RSS combination of this and the two lateral modes yields an overall dynamic component of 1.1 g peak-zero, which is the value used for the MUF=1.5 simulation.

The SLAM mission measured a duration of 16 seconds for Delta II 2^{nd} Pre-MECO event. It has been difficult, however, to find any documentation as to a *guaranteed* duration limit for this event, despite a concerted search of engineering memoranda from Lockheed and Boeing, as well as the Delta Payload Planners Guide. Given the uncertainty in this duration, the model applied the event as occurring at full intensity for 32 full seconds (twice the expected duration).

The MUF = 1.00 Model For this model, the MUF of 1.5 was simply removed from the calculated responses. Note that this means the axial acceleration never changes sign, therefore resulting in zero net differential acceleration between the rotor and housing. The two lateral components were combined to yield an overall lateral acceleration of 0.66 g peak to zero. Again, this is applied as a 30 Hz sinusoidal acceleration of the gyroscope housing for 32 seconds.

Impact on Gyroscope

Figures 6.7 and 6.8 show the impact histograms for both models. For a sinusoidal housing acceleration, impacts typically fall in two velocity bins. The first corresponds to the initial impact, and the second is the first bounce. From the second bounce onward, the impacts usually occur at less than 0.5 cm/s and are therefore considered trivial. Figure 6.8 shows this type of behavior. For the MUF=1.0 result (Figure 6.7), even the second bounce is trivial, so all impacts fall in one velocity bin.

The results from the PreMECO trials are summarized below. Note that all factors are a factor of 2 or more below those for the liftoff-Max Q region. In general, PreMECO is much less strenuous on a gyroscope than the liftoff through Max Q.

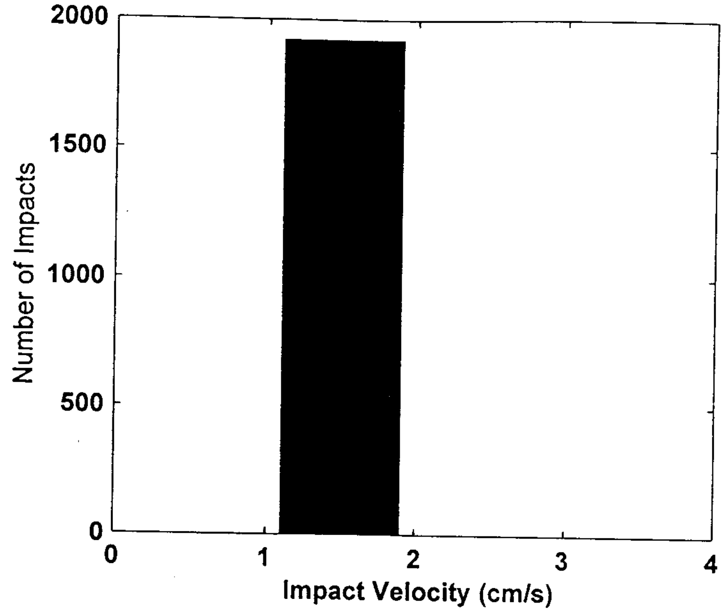


Figure 6.7: Impact Spectrum for the Pre-MECO period (MUF=1.0)

	Total Impacts	**Max Impact Velocity**	**Total Energy**
MUF = 1.0	1921	1.9 cm/s	22.1 mJ
MUF = 1.5	3841	2.3 cm/s	32.3 mJ

6.1.3. MECO and Post-MECO

Description of the Environment

The MECO event is traditionally defined as an approximately 5 second period just prior to Main Engine Cut Off. It consists of a large quasi-static acceleration (6.4 g) plus small oscillations in the 17-18 Hz range. These frequencies do not excite any spacecraft mode. The response of all nodes in the GP-B finite element model were calculated in Boeing memo A3-Y935-EDAO-DELTA-98-080 which gives the following responses:

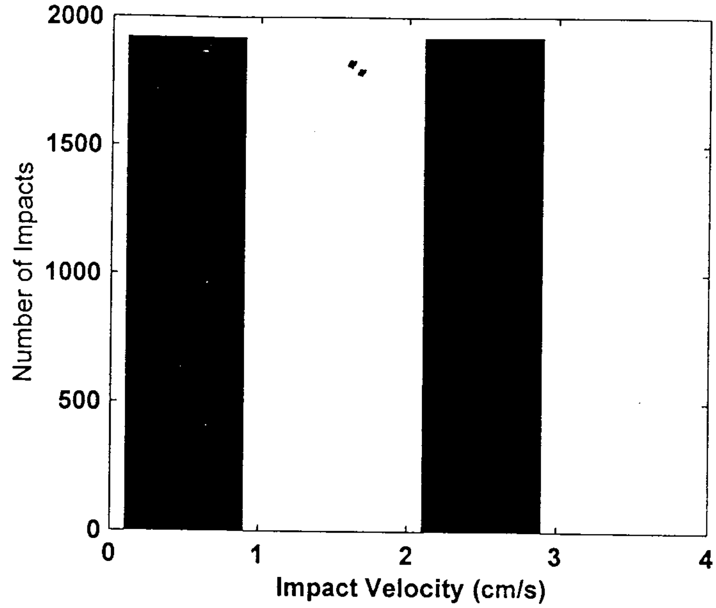


Figure 6.8: Impact spectrum for the PreMECO period (MUF=1.5)

Uncertainty Factor = 1.1 quasistatic and 1.5 dynamic

Node #	DOF	Min	Max
10950	XT	-0.2232	+0.2232
10950	YT	-0.2608	+0.2608
10950	ZT	+4.9073	+7.8927

Uncertainty Factors Removed

Node #	DOF	Min	Max
10950	XT	-0.1488	+0.1488
10950	YT	-0.1739	+0.1739
10950	ZT	+4.8230	+6.8133

Since the axial acceleration never changes sign, the sphere never leaves the housing wall and no rotor-housing impacts are produced. The lateral acceleration if very small and is negligible compared to the Liftoff through Transonic and Pre-MECO loads. Therefore the entire MECO event has negligible effect on an

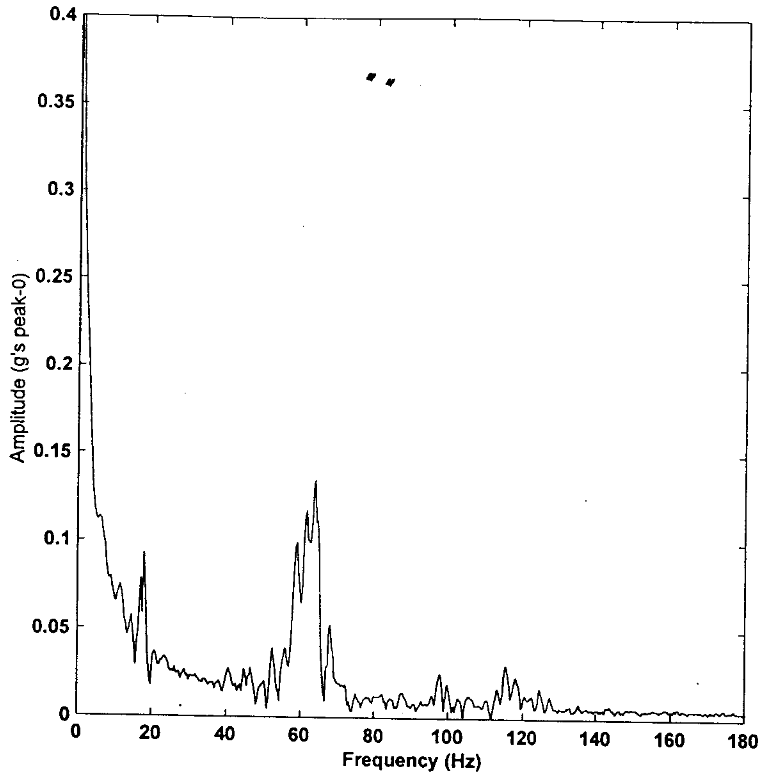


Figure 6.9: Post-MECO Event #1 Frequency Response

uncaged gyroscope compared to the other regimes already analyzed.

However, just after the engine cuts off, the quasistatic acceleration falls to near zero, and there is the potential for rotor-housing impacts. After main engine cutoff, the launch vehicle relaxes from the compression it was in while the main engine was firing. This results in a brief period of oscillations, which shall be referred to as Post-MECO Event #1. After this, there is the Stage 1/2 Separation (Post-MECO Event #2), and the Stage 2 Ignition (Post-MECO Event #3). All of these occur as independent discrete events over the course of approximately 20 seconds. The SLAM data indicates that the Post-MECO Event #1 lasts approximately 2 s. The other two events last for less than 1 second each. Careful measurement of the SLAM data shows that the total duration of all these events combined is approximately 3 seconds.

The acceleration at the gyroscope housing due to these events has not been computed as of the revision date of this document. To be sure that the analysis in this document properly bounds the problem, an extraordinarily conservative approximation will be used. Figure 6.9 shows the frequency response of the SLAM experiment to the first post-MECO event. The low-frequency portion of the plot (less than 20 Hz) is due to the fact that the launch vehicle acceleration is reduced dramatically at MECO. This does not create rotor-housing impacts as it is just the vehicle going from large positive g's to less-large positive g's. Note that the dominant dynamic component (which could cause rotor-housing impacts) is at 60 Hz (as opposed to the MECO event just prior to the cutoff, which is 17-18 Hz).[6] Later on in this document (see, for example, Figure 7.5) it is shown that for sinusoidal excitations of the housing, impact velocity has a very weak dependence on frequency of excitation in this range. However, the faster the frequency, the more impacts are obtained. Therefore, even if some of the post-MECO events occur at a lower frequency, it is still worst-case to model them as 60 Hz excitations.

A worst-case magnitude of the events can be obtained as follows. The SLAM data (see Figures 6.1 and 6.2) shows the intensity of the post-MECO events to be much smaller than the intensity of Pre-MECO1 and Pre-MECO2. Also, the Pre-MECO2 event actively drives a mode of the spacecraft, where as the post-MECO events do not. Therefore it is certainly worst case to model the magnitude of the response at the quartz block due to these small post-MECO events as equal to the response of the quartz block to Pre-MECO2, a 2 g peak-zero sine wave. This assumption probably radically over-estimates the size of the events, but such conservativism is necessary given the lack of concrete analysis.

As noted above, the total expected duration of these events is 3 s. As an added degree of conservativism, an extra factor of two will be added in duration, so that the sum of these minor post-MECO events will be modeled as a 6-second sine wave

$$a(g) = 2.0 \cdot \sin(120 \cdot 2\pi \cdot t) \text{ for } t \in [0, 6] \text{ seconds}$$

This same model of for the MECO/PostMECO acceleration was used for both the MUF = 1.0 and MUF = 1.5 cases. The reason is that this is really a worst-case bound on a problem (the only bound identified so far), not a model for the event with some uncertainty.

[6] Private email from Boeing to Gaylord Green dated 3/7/01

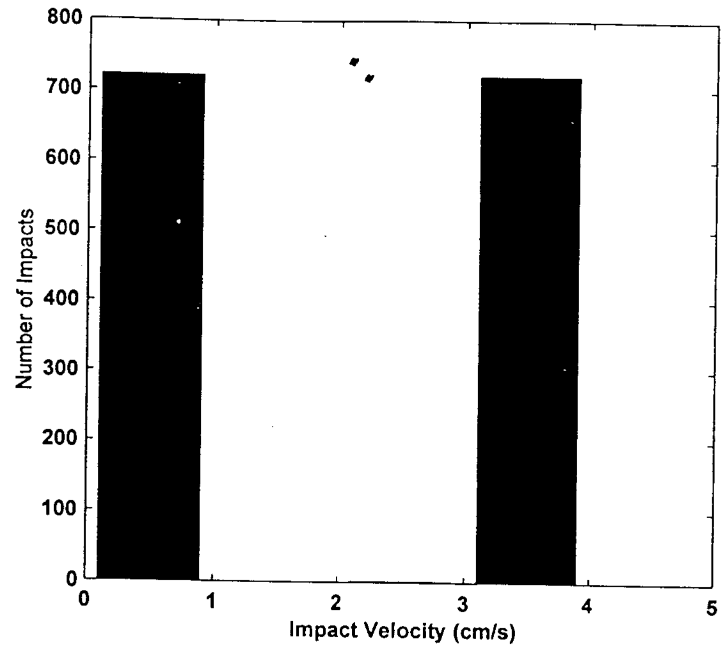


Figure 6.10: Rotor Impacts due to MECO and post-MECO model

Impact on the Gyroscope

The impact spectrum for the MECO/PostMECO events are in Figure 6.10 and summarized in the table below.

Total Impacts	Max Impact Velocity	Total Energy
1442	3.75 cm/s	26.7 mJ

Although this region appears to be in many ways comparable or even worse than liftoff-Max Q, this is highly unlikely. It is merely a product of the fact that such an extreme worst-case model was chosen for this regime of flight.

6.1.4. Integrated Launch Impacts

By combining the results of the previous three sections, the total spectrum of expected rotor-housing impacts during the ascent to orbit is obtained. The following table and Figures 6.11 and 6.12 summarize these results. It is important to remember that a large amount of conservativism was used when arriving at even the MUF = 1.0 numbers, including:

1. Maximum liftoff loads were applied for the entire first 60 seconds of flight.

2. Maximum acoustic energy was applied for 10 seconds instead of the <5 guaranteed by the *Delta II Planners Guide.*

3. PreMECO acceleration was applied for 32 seconds instead of the expected 16.

4. The maximum PreMECO acceleration was applied for the entire 32 seconds (instead of building up, peaking, and coming down)

5. The MECO acceleration was chosen to be 2.0 g peak-0, which is greater than the number obtained in the PreMECO event where a spacecraft resonance was being directly excited. The acceleration was applied for twice the expected duration. The excitation frequency was chosen to be 60 Hz so as to maximize the number of impacts (while not reducing the velocity of those impacts).

MUF = 1.0

	Total Impacts	Max Impact Velocity	Total Energy
Liftoff - Max Q	5048	4.3 cm/s	43.8 mJ
PreMECO	1921	1.9 cm/s	22.1 mJ
MECO/PostMECO	1442	3.75 cm/s	26.7 mJ
Total	*8,411*	*4.3 cm/s*	*92.5 mJ*

MUF = 1.5

	Total Impacts	Max Impact Velocity	Total Energy
Liftoff - Max Q	6099	5.2 cm/s	63.7 mJ
PreMECO	3841	2.3 cm/s	32.3 cm/s
MECO/PostMECO	1442	3.75 cm/s	26.7 mJ
Total	*11,382*	*5.2 cm/s*	*122.6 mJ*

6.2. Acoustic Tests

The flight payload undergoes two acoustic tests. The first is at the payload level (probe, science instrument, dewar, and sunshade), and the second at the space

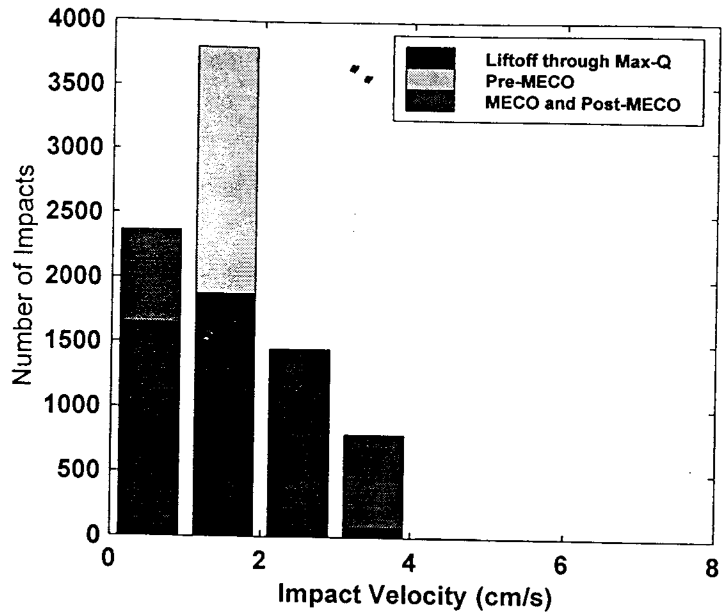


Figure 6.11: Integrated Impact Spectrum for Entire Launch (MUF = 1.0)

vehicle level (payload plus spacecraft). Because of differences in the way the acoustic energy couples to the gyroscope in the two configurations, the payload acoustic test (0.56 grms) is slightly more benign than the system-level acoustic test (0.80 grms).

6.2.1. Payload-Level Acoustic Test

Figure 6.13 compares rotor-housing impacts during the payload acoustic test to those expected during launch. Note that the maximum impact velocity of 3.8 cm/s is 88% of the maximum expected impact velocity during launch.

There are substantially fewer impacts in the 3-4 cm/s bin, however, than is expected during launch. Examination of the previous sections indicates that the vast majority of these impacts are due to the extraordinarily conservative model of the post-MECO events. Further analysis of the gyroscope acceleration during this phase of the ascent to orbit would certainly reduce the number of high-energy impacts. Recall that these impacts are due to the relaxation of the launch vehicle after MECO, the first/second stage separation, and the fairing separation.

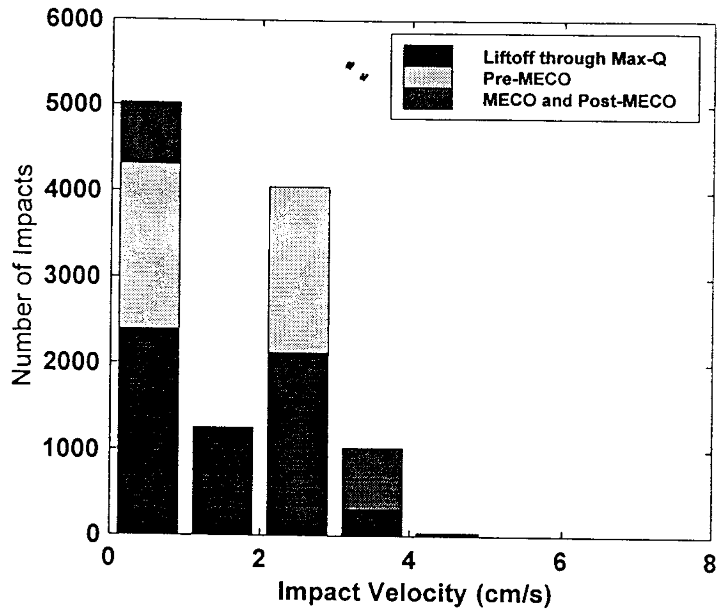


Figure 6.12: Integrated Impact Spectrum for Entire Launch (MUF = 1.5)

In all likelihood these events are probably insignificant compared to other stages of launch. If this is true, then the payload acoustic test provides a much more representative test for launch than is indicated in these graphs.

It can not be proven that the payload acoustic test bounds launch in terms of expected rotor-housing impacts. It is, however, clearly a significant test, with rotor-housing impacts of 88% max expected during launch. Since the coating quality requirements are orders of magnitude more stringent for one-g levitation than is necessary on orbit, if a gyroscope can levitate in one g after the payload acoustic test, it will certainly perform within specification on orbit after launch.

Total Impacts	Max Impact Velocity	Peak Acceleration	Total Energy
6118	3.8 cm/s	2.5 g	23.0 mJ

6.2.2. System-Level Acoustic Test

The maximum impact velocity expected for the system acoustic test (4.5 cm/s) is comparable to the maximum expected impact velocity during launch (4.3 cm/s). Furthermore, the number of intermediate velocity impacts (1 - 3 cm/s) is greater than is expected during launch. Although there are more launch impacts in the

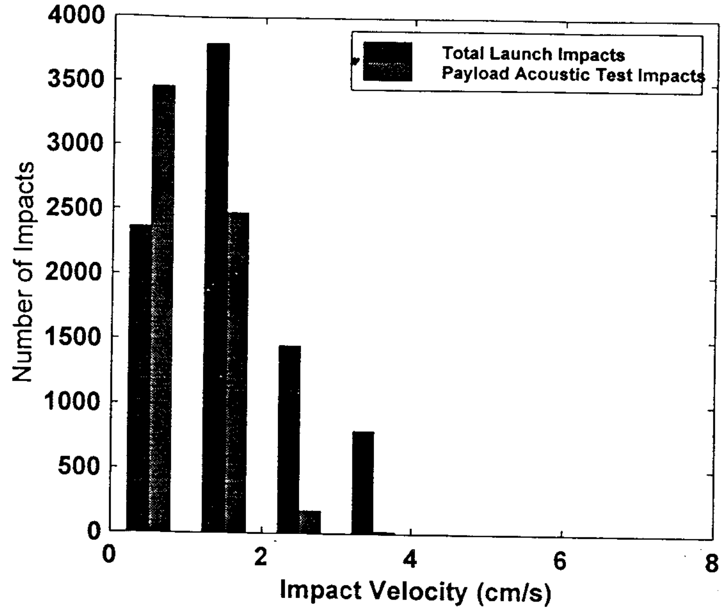


Figure 6.13: Comparison of Payload Acoustic Test to Launch Loads (MUF=1.0)

3-4 cm/s range, as discussed above the launch model almost certainly dramatically overestimates the number of launch impacts in this range.

Taking all this into consideration, it is not unlikely that the system-level acoustic test is in fact more severe than launch with respect to potential gyroscope damage.

Total Impacts	Max Impact Velocity	Peak Acceleration	Total Energy
12,824	4.5 cm/s	3.7 g	159.5 mJ

6.3. Transportation

Transportation is in general very benign (< 1 g of dynamic disturbance). However, occasionally there is an occasional "bounce" resulting in a rotor-housing collision. Data from the transportation of the payload to and from the acoustic test has shown that this is expected to be less than 3 g. Also, note that Lockheed EM SMS375 documents that the qualification test gyroscope (which is not as well protected from a vibration point of view as the payload gyroscopes) received bounces of up to 4.8 g. Transportation clearly makes a negligible contribution to

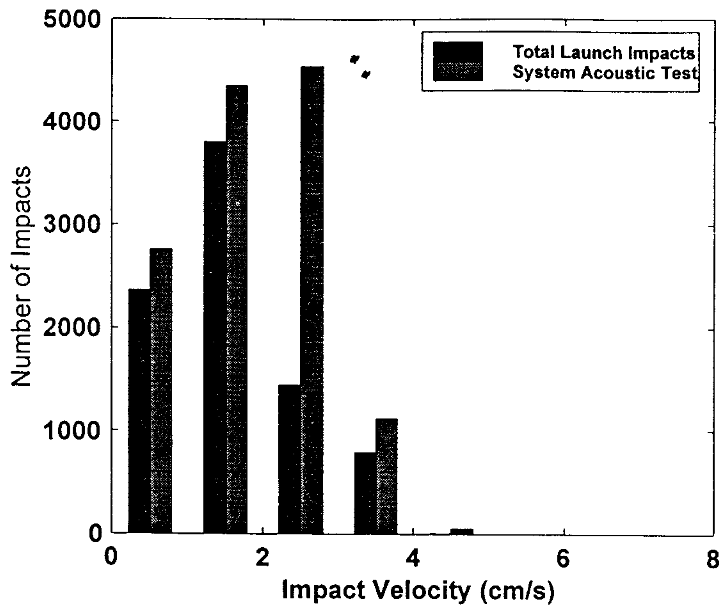


Figure 6.14:

lifetime rotor-housing impacts, and will not be considered further in this work.

6.4. Total Lifetime Impacts

As stated above, the criteria for successful one-g levitation is much more stringent than the criteria for successful on-orbit operation, at least with respect to potential damage mechanisms due to housing acceleration. The reason is that the primary effect of shaking the gyroscope (as determined by post-shake tests) is tiny microscopic indentations in the rotor coating. Gyroscope torques (which are the primary concern for on-orbit operation) are not sensitive to variations that occur over very small areas with respect to the size of the electrodes.[7] This statement is analyzed more carefully in S0525, where a rotor that has undergone a severe shake test is analyzed with respect to science mission torque requirements. Therefore the damage mechanism which occurs due to launch or acoustic test accelerations does not couple strongly to on-orbit requirements.

[7] For instance, a density variation across the entire rotor of one part in 10^6 is worse from a torque point of view than a 1 micrometer sphere on the outside of the coating.

However, the damage mechanism does couple extremely strongly to one-g levitation requirements. Tiny imperfections in the coating orders of magnitude smaller than what would couple to science torques can serve to locally enhance the electric field due to the suspension voltages, resulting in field emission and gyroscope charging. Maximum field emission due to on orbit voltages is typically at least a factor of 10^{20} less than what is obtained with ground levitation voltages. Practically what this means is that a gyroscope which charges too rapidly on the ground to maintain a stable levitation can easily still meet all on-orbit requirements.[8]

Because the one-g levitation is such a dramatic overtest with respect to science levitation, a gyroscope which displays a stable levitation in one-g may be considered to be completely healthy, and it is not necessary to log cumulative rotor-housing impacts up to that point. This is entirely consistent with standard GP-B operational practice that has been proven over many years. Although care is taken in the handling of gyroscopes, there are in practice many rotor collisions that occur in standard operation which are not logged. The maximum impact velocity during launch of 4.3 cm/s is equal to the velocity of a rotor dropped in one-g from only 0.1 mm high. This can easily occur during cleaning and gyroscope assembly, for example. Delevitating a gyroscope at the end of a performance test results in a rotor-housing impact of 2 cm/s, and almost all the energy of the acoustic tests lie in this range. Delevitating a rotor spinning at 0.3 Hz (the standard spin frequency at the end of a performance test) causes an impact with the surface of the rotor moving tangential to the housing (maximizing the probability of a scratch) at 4 cm/s. All of these events are of comparable magnitude to launch, but no attempt is made to keep a cumulative log. This is because the one-g levitation is such a dramatic overtest with respect to coating quality that successfully passing it means the gyroscope is completely suitable for on-orbit operation. The gyroscope is clearly orders of magnitude away from any cumulative lifetime impact requirement that might exist.

Finally, as noted above, the high-velocity impacts are those which have the most potential to result in gyroscope damage. When establishing an appropriate qualification test program, it is not necessary to bound the number of impacts in each frequency bin, or even to bound the total number of impacts (since there is some degree of ambiguity in how to define a "significant" impact, see Section 4.2.6 for details). What is important is providing a healthy bound on the number of high-velocity impacts that could potentially result in rotor or housing damage.

[8] See S0513 for more details

This is the philosophy that will be chosen here.

Part III

The Qualification Test Levels

7. The Qualification Test Sequence

7.1. The Severity of Various Tests

The following sections compute the impact of various candidate qualification tests.

7.1.1. Random Vibration Test

The first test to be examined is a 60-s 1.65 grms random vibration test intended to envelope the system-level acoustic test acceleration environment. The spectrum definition was

Frequency	**Acceleration Spectral Density**
20 Hz	0.0005 g^2/Hz
20 - 50 Hz	+12 dB/octave
50 - 110 Hz	0.02 g^2/Hz
110 - 200 Hz	-12 dB/octave
200 Hz	0.0018 g^2/Hz
200 - 300 Hz	0.0018 g^2/Hz
300 - 2000 Hz	-10 dB/octave
Total:	**1.65 grms**

A random process was generated in the gyro impact simulation with the same power spectral density as that used in the actual shake test. A two-second snapshot of this acceleration is shown in Figure 7.1. It is important to note that although the rms value of the signal is 1.65 grms, the peak acceleration is much more than this. The simulation was run for 60 seconds and all rotor-housing impacts were recorded. The following table and Figure 7.2 summarize the results of this test.

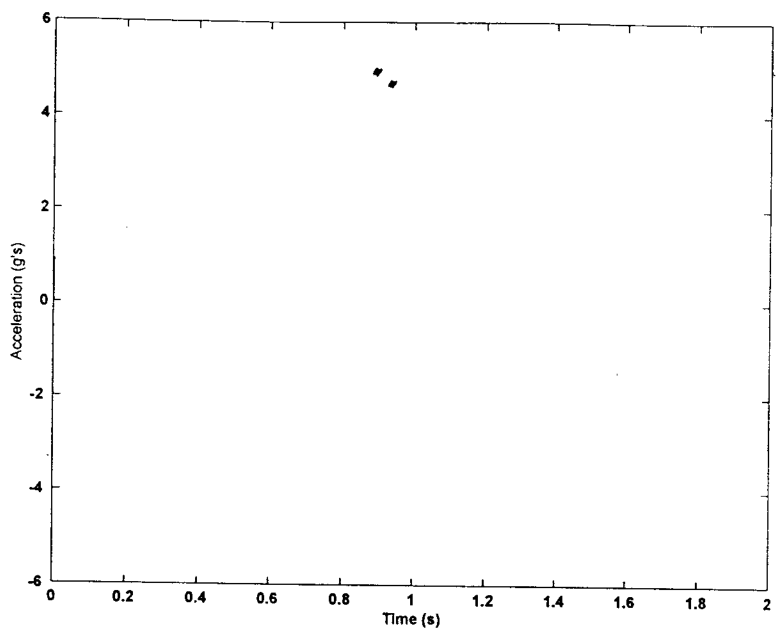


Figure 7.1: Two-second snapshot of acceleration during random vibration test

Total Impacts	Max Impact Velocity	Peak Acceleration	Total Energy
12576	6.84 cm/s	7.5 g	297.0 mJ

Note that this alone already reasonably brackets the entire launch sequence. The maximum impact velocity of 6.8 cm/s is 1.6 times the maximum expected during the entire launch sequence (4.3 cm/s for MUF=1.0). The total number of impacts is bounded (110% of the total expected during launch for MUF=1.5, and 150% for the MUF=1.0 model). More importantly, the energy of the random vibration collisions is much larger than that expected for launch. The total expected collision energy during launch is 92.5 mJ (MUF=1.0), but the random vibration test has already subjected the housing to 297 mJ.

What all this means is that the random vibration test yields much more high-energy collisions than does the expected launch environment. For example, the MUF=1.5 launch model has a total of 31 collisions of velocity greater than 4 cm/s. The random vibration test has 2,365 such collisions, or 76× as many. Since these are the collisions that hold the most potential to damage the gyroscope, *this sequence alone constitutes a significant overtest for the entire launch environment.*

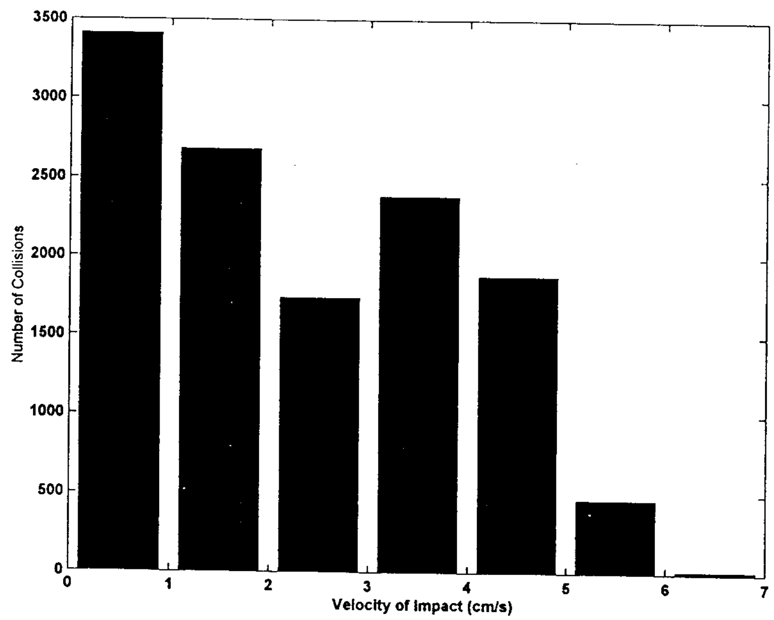


Figure 7.2: Collision Spectrum for 1.65 grms Random Vibration Test

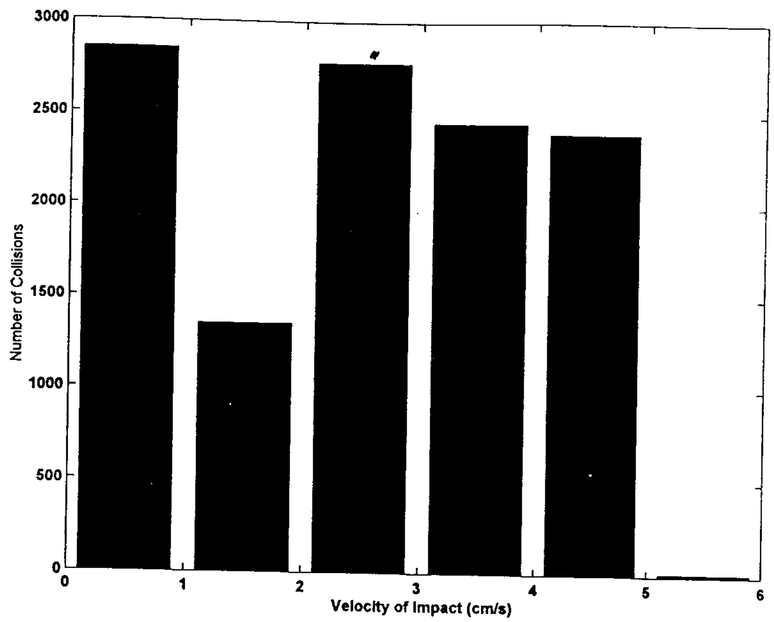


Figure 7.3: Collision Spectrum for 2.5 g p-0 sine sweep

7.1.2. Sine Sweep #1

The second candidate environment is a 2.5 g peak-0 sine sweep test from 10 to 100 Hz. The total duration of the sweep is 50 seconds. To understand the effect this had on the gyroscope, an identical sine sweep was fed into the gyro impact simulation, and the resulting collisions logged. The following Table and Figure 7.3 summarize the results.

Total Impacts	Max Impact Velocity	Peak Acceleration	Total Energy
11,836	5.03 cm/s	2.5 g	306.5 mJ

Although the maximum impact velocity was less than that obtained in the random vibration test, the effect of the sine sweep is to focus the energy on both low-energy and high-energy bins (but not intermediate energy). In this case, there are 2,414 collisions greater than 4 cm/s (78 times the number expected during launch using the MUF=1.5 model). The total energy of all the collisions is 2.5 times larger than what the MUF=1.50 model predicts during launch. Therefore this test also represents a significant test of the gyroscope's ability to survive the launch environment.

7.1.3. Sine Sweep #2

The final candidate environment is a sine sweep test 4.0 g peak-0 from 11 to 100 Hz for 50 seconds total. The results of this test are summarized in the table below and Figure 7.4.

Total Impacts	Max Impact Velocity	Peak Acceleration	Total Energy
15,010	5.95 cm/s	4.0 g	645.1 mJ

The results from the two sine sweeps can be used to make some mathematical guidelines for the response of the rotor/housing system to sinusoidal impacts. If a constant acceleration a is applied across some distance x (in this case x would be the distance the rotor could move from wall to wall, or 39 μm), then the velocity at impact is

$$v_{\max} = \sqrt{2ax}$$

For 4 g's, this corresponds to an impact velocity of 5.5 cm/s. Comparing this to the observed maximum impact velocity shows that this equation does a reasonable job of approximating the maximum impact velocity resulting from a sinusoidal impact. Also, note that as the peak acceleration of the sine sweep in increased from a_1 to a_2, the peak impact velocity only goes up (approximately) as $\sqrt{a_2/a_1}$, which is what one would expect from the $v_{\max}$ calculation. Finally, the impact energy will go up as a_2/a_1.

Figure 7.5 shows the impact velocity as a function of the instantaneous frequency during the sine sweep (where the peak acceleration of the housing is held constant). As can be seen, there is a very broad peak in the impact velocity around 45 Hz. However, the dependence is very mild. The point of Figure 7.5 is to show that there is no resonance-type behavior within the frequency regime excited by launch. The frequency of excitation affects the number of impacts (since the housing is shaken more times in a given unit of time for a larger frequency), but does not do much to the velocity of impact. In general, when the actual frequency of excitation is unknown (for instance, during some transient during launch), a worst case analysis is one where a relatively high frequency is assumed. The impact velocity changes very little, but the number of impacts (and therefore the total energy of the event) will go up linearly with frequency.

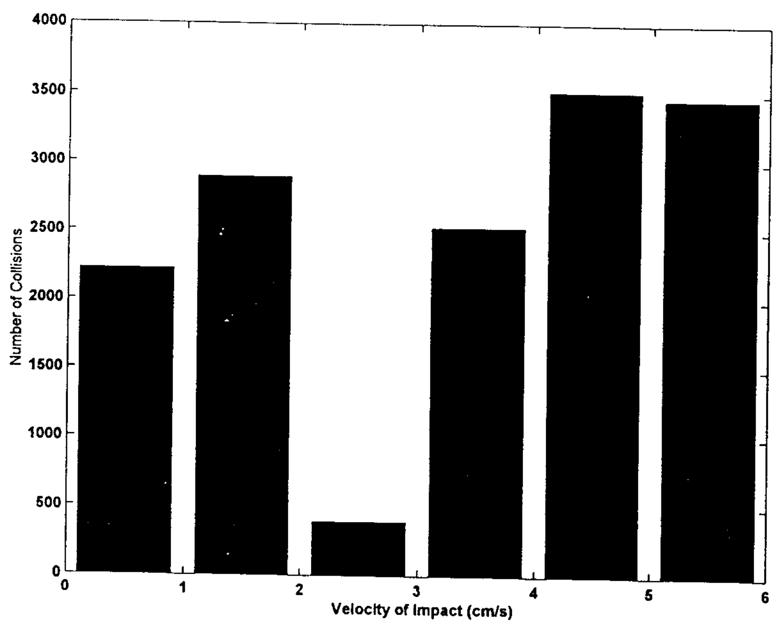


Figure 7.4: Impact Spectrum for 4.0 g p-0 Sine Sweep

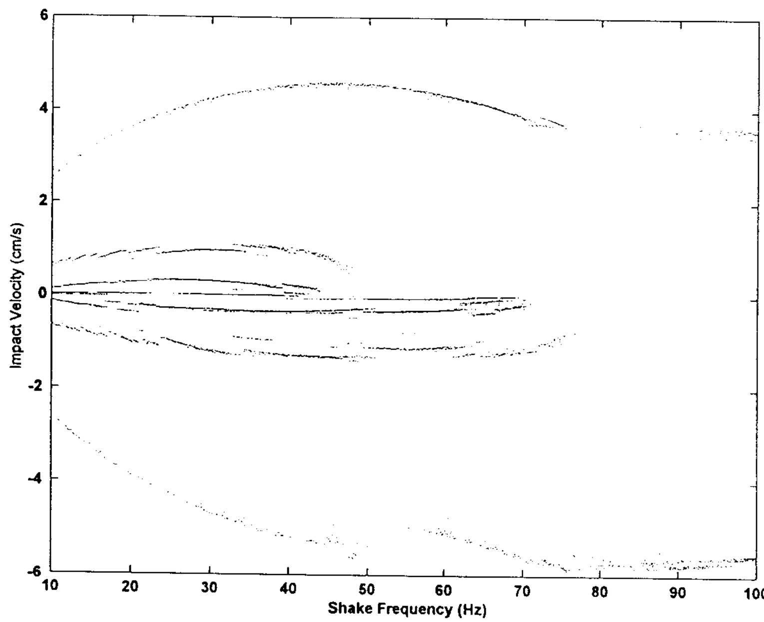


Figure 7.5: Impact Velocities for Various Shake Frequencies (4.0 p-0 sine wave)

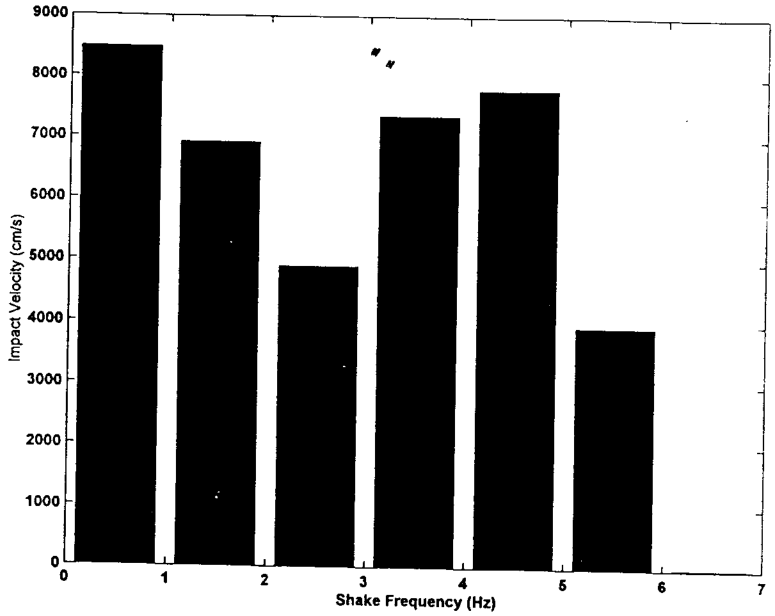


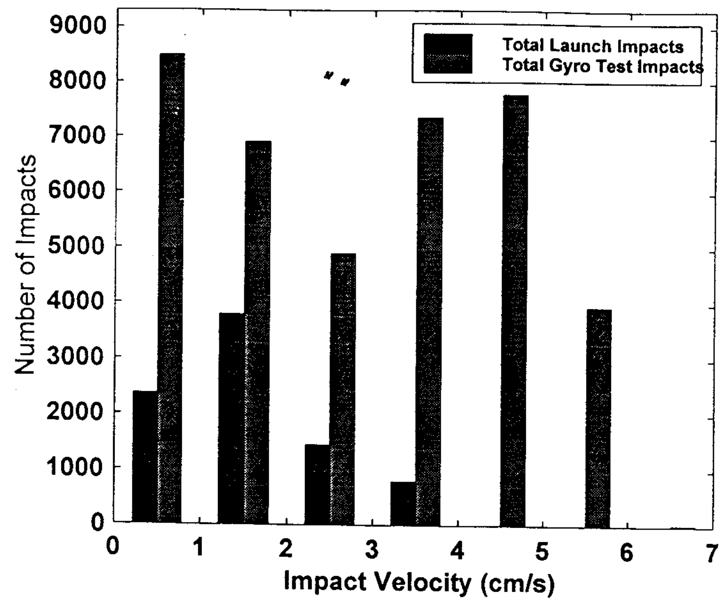
Figure 7.6: Impact Spectrum Composite for All Vibration Tests

7.1.4. Integrated Impact on the Housing

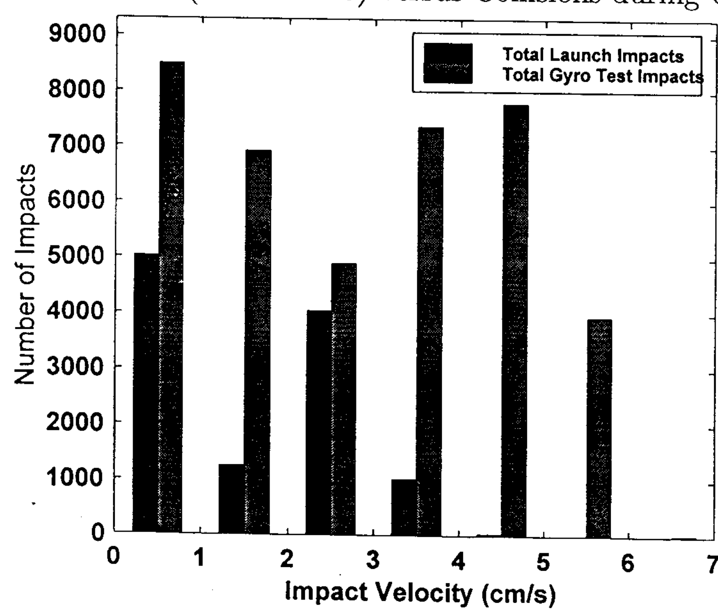
The effect of performing all three tests described thus far on the same gyroscope is summarized in Figure 7.6. These results are compared to the total expected launch impacts in the following table.

	Total Impacts	Max Impact Velocity	Impacts > 4 cm/s	Total Energy
Launch (MUF=1.5)	11,382	5.2 cm/s	31	122.6 mJ
Launch (MUF=1.0)	8,411	4.3 cm/s	3	92.5 mJ
Test Gyro	**39,422**	**6.8 cm/s**	**11,756**	**1,248.6 mJ**

There were 11,756 impacts with velocity greater than 4 cm/s. Note this is approximately equal to the *total* number of impacts (all velocity bins) for the entire launch in the MUF=1.5 case. Qualitatively, this is like taking all of the expected collisions for the MUF=1.5 case (most of which are occurring at very low impact velocity) and applying them all at velocities *greater* than any expected during the launch.



Expected Launch Collisions (MUF = 1.0) versus Collisions during Qualification Test



Expected Launch Collisions (MUF = 1.5) versus Collisions during Qualification Test

Figures ?? and ?? compare the collisions experienced by this test sequence to collisions experienced during launch. Expected launch impacts were bound by tremendous margins. Also, because the energy of the impact goes as the square of the velocity, the collisions which reside at the higher impact velocities (which occurred during the test sequence but will not occur during launch) have a much more destructive power than the low-energy impacts seen during launch.

This set of experiments represented a significant overtest of the hardware with respect to the launch environment. First, recall from Section 6.1.4 that even the MUF=1.0 has several factors that make it a very conservative estimate of the launch environment. Then, a MUF=1.5 scenario was constructed where the MUF=1.0 accelerations were multiplied by 1.5. Then it was subjected to an environment where the number of expected high-energy collisions (greater than 4 cm/s) were increased by a factor of 379 (11,756/31), and the total net impact energy was increased by factor of 10.2. Even under these extraordinary conditions, the gyroscope still met all requirements in its performance test.

7.2. Setting the Qual-Test Environment

7.2.1. Criteria Used to Determine Qual-Level Environments

The previous sections have (1) Summarized the launch environments, (2) Identified the mechanisms by which those environments might result in gyroscope damage, and (3) Developed metrics to evaluate how severe the environment is from a gyroscope safety point of view. This section takes these results and uses them to establish a set of qualification criteria that will be used to verify whether gyroscopes can be safely launched uncaged. In doing this, the following guidelines will be used:

1. The energy dissipated in rotor-housing collisions should be greater than the energy expected to be dissipated during launch by at least a factor of 2.

2. The maximum impact velocity for the qualification test should be at least a factor of 1.5 over expected. As noted above, impact velocity scales only as the square root of the acceleration, so bounding impact velocity by a factor of 1.5 is roughly equivalent to bounding acceleration by a factor of 2.25. Since it is not very sensitive to model or environment variations, a factor of 1.5 is sufficient to accommodate model uncertainties.

3. The number of rotor-housing impacts with velocity over 4 cm/s should be greater than the expected number of collisions with velocity over 4 cm/s by at least a factor of 2. These collisions represent the core of what could potentially damage the gyroscope.

4. Although it is important to establish a test environment that is severe enough to give adequate confidence that gyroscopes will survive launch uncaged, it is also important to not establish an environment so severe that a *false* negative is likely. Due caution must therefore be exercised not only in figuring what environments would be sufficient to justify launching the gyroscope uncaged, but also in setting the limit whereby the program would be forced to conclude that the gyroscope can *not* be launched uncaged.

These criteria address the gyroscope response to dynamic input. The response to quasistatic components was examined in Section 4.2.1. In this case the design margin was found to exceed a factor of 50 with response to the peak expected acceleration (MUF=1.5). Furthermore, all flight gyroscopes have been caged to loads which exceed by more than a factor of 10 the peak launch load (including MUF=1.5). Although this load is applied on a different axis of the gyroscope, it is still pressing the rotor against lands of identical construction as the caging lands inside a cavity with spherical symmetry. Finally, the magnitude of the expected load is small (1 lb including a MUF of 1.5). For these reasons, it is not believed that a new direct measurement of the gyroscope's robustness to a quasistatic load is necessary.

It should be noted that there are no resonances of the gyroscope or the gyroscope retention hardware which could be conceivably excited by specific frequencies. The lowest resonance of the housing itself is >10 kHz. The resonance of the "spring" caused by the sphere pressing against the housing wall was calculated to be on the order of 6 kHz, well above anything present in the excitation spectrum. The gyroscope is mounted in the quartz block by mating it to optically contact with a quartz index plate / spacer assembly, which is, in turn, in optical contact with a flat on the quartz block. This arrangement is then preloaded to hold everything in contact with a force that exceeds all launch loads.[1] Therefore the system which mounts the gyroscope into the quartz block is quartz-quartz-quartz, with every piece being held in optical contact with every other piece. That means there are no low-frequency resonances in the system which could conceivably be excited by anything in the input acceleration spectrum.

In addition to this, simulation of the nonlinear dynamics of a rotor impacting the housing cavity indicates that the impact velocity has only a very mild dependence on input acceleration frequency (see for example Figure 7.5). Therefore there is no strong dependence of the overall system on the particular frequency of

[1] This is documented in the qualification test program for the gyro retention hardware.

the input acceleration, save that a higher frequency will result in more collisions.

With these factors in mind, the following sections set an appropriate qualification-level test for the gyroscope.

7.2.2. Random Vibration

The random vibration environment used in the Qual Test Gyro #1 (see S0521) and described in Section 7.1.1 is found to be sufficient to address the random-vibration environment. It should be performed according to the spectrum used in Qual Test Gyro #1:

Frequency	**Acceleration Spectral Density**
20 Hz	0.0005 g^2/Hz
20 - 50 Hz	+12 dB/octave
50 - 110 Hz	0.02 g^2/Hz
110 - 200 Hz	-12 dB/octave
200 Hz	0.0018 g^2/Hz
200 - 300 Hz	0.0018 g^2/Hz
300 - 2000 Hz	-10 dB/octave
Total:	**1.65 grms**

This acceleration environment shall be applied for 60 s total. The acceleration direction shall be in the +z axis of the launch vehicle when the gyroscope is mounted in a flight-like orientation. A three-axis shake is not necessary because the nonlinear dynamics of the gyroscope excite rotor-housing impacts roughly equally in all axes regardless of the primary axis of housing excitation. S0521 provides experimental proof of this.

7.2.3. Sine Sweep

The test gyroscope shall also be subjected to a sine sweep at 2.5 g p-0 from 10 to 100 Hz (50 s total) in the vertical axis (+z for the launch vehicle with the gyroscope mounted in the same orientation as it will be in the launch vehicle). Although the random vibration test cited above does adequately bound the launch environment from a rotor-housing impacts point of view, it does not excite the housing at low frequencies (less than 50 Hz). Although there is no expected damage mechanism due to these frequencies, as a matter of completeness the gyroscope should still be exposed to a low-frequency sine sweep test.

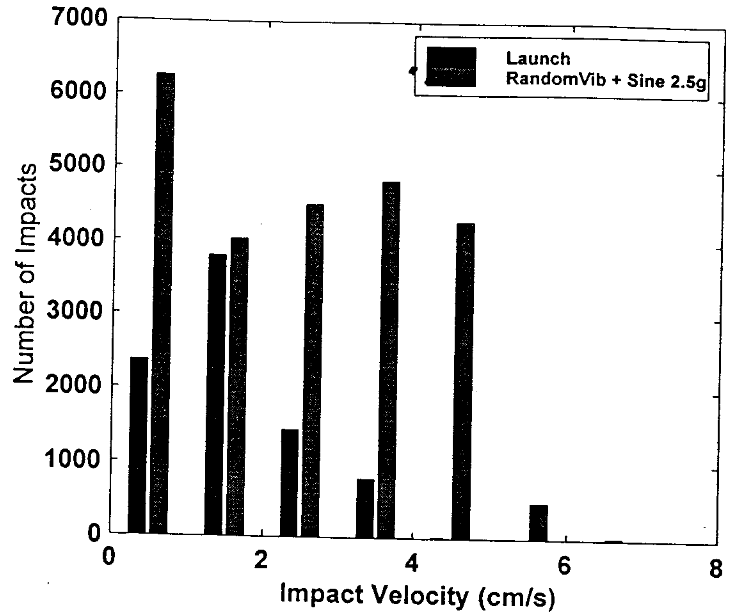
7.2.4. Integrated Test Impacts

The following table summarizes the impacts due to this test program and compares them to flight levels.

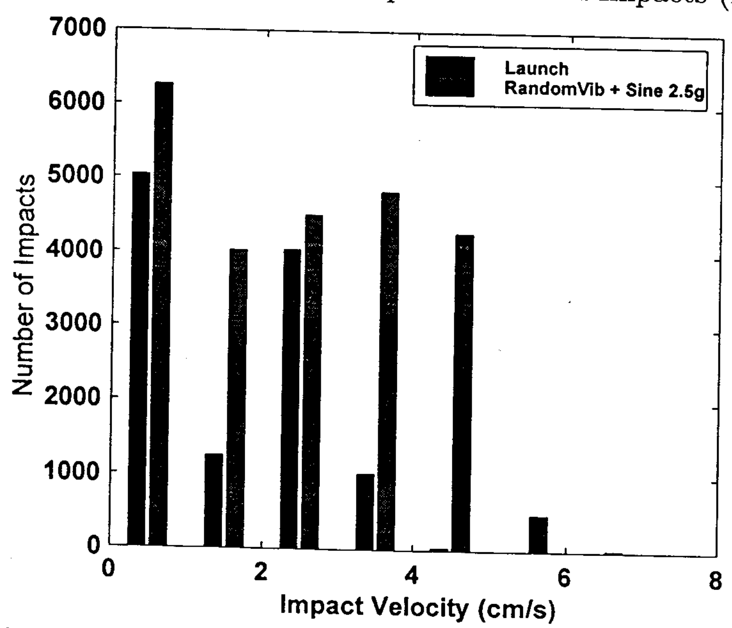
	Total Impacts	**Max Impact Velocity**	**Impacts > 4 cm/s**	**Total Energy**
Launch (MUF=1.5)	11,382	5.2 cm/s	31	122.6 mJ
Launch (MUF=1.0)	8,411	4.3 cm/s	3	92.5 mJ
Test Gyro	24,412	6.8 cm/s	4,779	603.5 mJ

Figures ?? and ?? show that this test also bounds the expected flight environment with significant margin. This will put 5 times the dissipative energy in the rotor/housing system as the amount expected in flight (according to the MUF=1.5 model). It also has a substantial number of impacts in the 4-7 cm/s range, whereas the expected flight environment does not.

Stanford's belief is that the current launch model significantly over-estimates the effect of the post-MECO events. However, the lack of analysis forced the adoption of an extremely conservative model. If additional analysis is performed which allows a re-evaluation of this model, it may allow the adoption of a more conservative qualification environment.



Qualification Test Impacts versus Expected Launch Impacts (MUF=1.0)



Qualification Test Impacts versus Expected Launch Impacts (MUF=1.5)

8. Conclusions

This document examined the possible damage mechanisms that could occur when a gyroscope is launched in the uncaged configuration. For those mechanisms it was established that potential damage to the gyroscope depended on (1) the number of rotor-housing impacts, and (2) the velocity of those impacts. Clearly, high-velocity impacts tend to cause more damage than low-velocity impacts. Therefore the number and velocity of rotor-housing impacts was established as the relevant criteria for quantifying the potential damage to the gyroscope in different acceleration environments.

The various phases of launch as well as transportation were examined to compile a composite histogram of total rotor-housing impacts expected during transportation and launch. Transportation was found to be negligible compared to launch. The magnitude of the maximum rotor-housing impact expected during launch was found to be relatively small, and comparable to impact velocities obtained in normal gyroscope operation such as delevitation of a slowly-spinning gyroscopes.

Various candidate qualification acceleration environments were also analyzed for rotor-housing impacts and the result compared to expected launch impacts. In the end, it was shown that two tests provided an adequate qualification test sequence: (1) a random vibration test at 1.65 grms, and (2) a sine sweep from 10 - 100 Hz at 2.5 g peak-0. This test sequence dissipates in the housing approximately 5 times the energy expected during launch, and provides 154 times the high velocity impacts (greater than 4 cm/s). This test sequence is not only adequate for qualification tests of the gyroscope's robustness to launching uncaged, it is a dramatic over-test.

Note that any testing in excess of these levels (e.g. adding additional shake tests) would only further strengthen the qualification test. For instance, a gyroscope has already been through the sequence of tests proposed above plus a 4 g p-0 sine sweep.

9. Acronyms

DOF	Degree of Freedom
FEM	Finite Element Model
GP-B	Gravity Probe B
MECO	Main Engine Cut-Off
MUF	Model Uncertainty Factor. Multiplies calculated acceleration.
SLAM	Spacecraft Loads and Acoustic Measurement (a NASA mission)
SIA	Science Instrument Assembly
QBA	Quartz Block Assembly (a subset of the SIA)