W. W. Hansen Experimental Physics Laboratory
STANFORD UNIVERSITY
STANFORD, CALIFORNIA 94305 - 4085

Gravity Probe B Relativity Mission

Verification of Gas Flow Requirements in Payload Specification

S0545, Rev. -

October 16, 2001

Approved by: ______________________ 10/16/01
B. Clarke Date

Approved by: ______________________ 10/16/01
R. Brumley Date

Approved by: ______________________ 10-31-2001
R. Whelan Date

Approved by: ______________________ 10/31/01
M. Keiser Date

Approved by: ______________________ 10/16/01
B. Muhlfelder Date

ITAR Assessment Performed ______________________ 10/31/01 ITAR Control Req'd? ☐ Yes ☒ No
Tom Langenstein

Purpose:

This document provides the supporting analysis in verification of the following PLSE-12 requirements.

paragraph #	Title	Requirement	Method	Lower Level Compliance data
3.7.1.4.8.2	Spin-Up Exhaust Line Gas Conduction	The pressure drop along the spin-up exhaust plumbing line assembly from its interface with the gyroscope to its interface with the probe shall be <= 40 Pa assuming the pressure of 60 Pa at the SIA/probe interface (Section 3.7.2.3.6), the flow rate given in Section 3.2.1.7.1 of He-4 or He-3, and a temperature of <= 7 K.	A	
3.7.2.3.3	Gas Flow Rate	The flow rate of the spin-up gas at the interface shall be as per 3.2.1.7.1.	A,T,S	Probe C AR, GPB-100269 v3, 15 May 98 (N/A)

Results:

PLSE-12 3.7.1.4.8.2

The pressure drop along the spin-up exhaust plumbing line between the gyro interface and the SIA/probe interface assuming the conditions of paragraph 3.7.1.4.8.2 above is calculated to be less than **4.43 Pa** which is less than 40 Pa, thus **PLSE-12 3.7.1.4.8.2 is verified** by analysis. The spin-up/exhaust plumbing from the gyro to the SIA/probe interface is identical in design for all four gyros in the SIA (see drawings 22879 and 23184).

PLSE-12 3.7.2.3.3

PLSE-12 3.2.1.7.1 specifies the gas flow rate necessary to spin a gyroscope to 80 –180 Hz must be <= 950 scc/m. Gyro #3 (FQH44/96FH09) was spun in Probe C per P0522 (Op # 1655) to 174 Hz asymptotic using a helium flow rate of 725 scc/m. S0579 estimates the on-flight spin speed for gyro #3 to be 131 Hz. **131 Hz is in the range 80 – 180 Hz** and **725 scc/m is less than 950 scc/m** thus **PLSE-12 3.7.2.3.3 is verified for Gyro #3** by test and analysis.

Gas flow data collected during Payload Verification II per P0519 (Op #1644, 1647, 1751 and 1754) shows that all science gyros in Probe C will support a flow rate of 725 scc/m. Furthermore, the spin-up gas / SIA interfaces for Gyros #1, 2 and 4 are identical in design to those of Gyro #3 (see drawings 22886 and 23183). S0579 argues by similarity that Gyros #1, 2 and 4 will also spin to 80 – 100 Hz on-flight using a flow rate of 725 scc/m. **PLSE-12 3.7.2.3.3 is verified for Gyros #1, 2 and 4** by test, analysis and similarity.

Analysis:

PLSE-12 3.7.1.4.8.2

The spin-up exhaust plumbing line running from the gyro to the SIA/probe interface is effectively a 0.218" diameter titanium tube 3.329" in length with a 48.5-degree bend on each end. See drawings 22879 and 23184 for details. Viscous flow through this tube is given approximately by the Poiseuille equation:

$$Q = \frac{\pi d^4}{256 l \eta}(P_1^2 - P_2^2)$$

(equation I)

Q = flow rate
d = diameter of the tube
l = length of tube
P_1 = inlet pressure
P_2 = outlet pressure
η = viscosity of the gas

From the kinetic theory of gasses, the viscosity of a gas in this tube near the outlet pressure specified in PLSE-12 3.7.1.4.8.2 (viscous flow regime) is given as:

$$\eta = \frac{0.499(4mkT)^{1/2}}{\pi^{3/2} d_0^2}$$

(equation II)

η = viscosity
d_0 = molecular diameter
m = molecular mass
T = temperature
k = Boltzman's constant

Solving for $\Delta P = P_1 - P_2$ gives:

$$\Delta P = \sqrt{Q\frac{256 l \eta}{\pi d^4} + P_2^2} - P_2$$

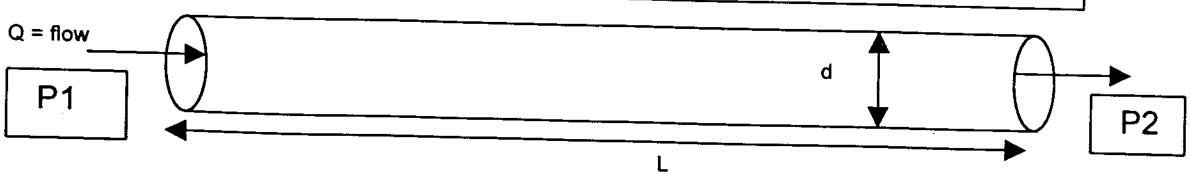
(equation III)

Using equations II and III and the ideal gas law to convert helium gas flow at STP to helium gas flow at temperature T gives ΔP (T=5K) = 2.71 Pa and ΔP (T=7K) = 4.43 Pa, where $\Delta P = P_1 - P_2$. Also note that given all other parameters, P_1 can be written as a monotonically increasing function of T, so ΔP (T=7K) sets the upper limit of ΔP in the range T=5 to 7 K. Note: expansion of gas (out of the spin-up channel) cools the gas so the 7 K estimates are worst case.

Shown below are spreadsheet entries calculating ΔP at T=7 K for helium-4. If helium-3 is used, the only parameter that will change significantly is the mass, which enters into equation III for ΔP through the viscosity. Namely, $\eta_{He-3} \approx \sqrt{3/4}\, \eta_{He-4}$, thus ΔP_{He-4} (T=7K) still sets the upper limit.

k	1.380E-23 J/K	Boltzman's constant
m	6.650E-27 kg	mass of helium-4 atom
T	7.000E+00 K	temperature
d0	2.800E-10 m	diameter of helium atom (Van der Waal's equivalent)
1 atm	1.013E+05 Pa	atmospheric pressure in Pa
max_flow	9.500E+02 scc/m	maximum flow at STP
flow	1.604E+00 Pa*m^3/s	flow equivalent to 950 scc/m in SI units
Q	4.114E-02 Pa*m^3/s	flow equivalent to 950 scc/m at temperature T in SI units
L	8.456E-02 m	length of tube
d	5.540E-03 m	diameter of tube
P_2	6.000E+01 Pa	pressure at tube outlet

η=	1.832E-06 Pa*s	viscosity of helium
P_1=	6.443E+01 Pa	calculated pressure head
ΔP =	4.431E+00 Pa	calculated pressure drop



PLSE-12 3.7.2.3.3
Gas flow data with the rotor delevitated and resting on the spin-up channel (R-half up) was collected for each gyroscope during Payload Verification II per P0519 (Op #1644, 1647, 1751 and 1754) and is summarized below. The gyroscope spin-up gas interfaces were shown to support up to 750 scc/m at the anticipated on-flight spin-up gyroscope temperature of 6.5 +/- 0.5 K. This is a more aggressive flow rate than the anticipated on-flight 725 scc/m. The spin-up pressure data (pressure head) and exhaust pressure data are very similar between gyros 1 and 2 and between gyros 3 and 4. The difference in the helium bath temperature accounts for the slight difference in pressures between the between the gyro 1and 2 data and gyro 3 and 4 data. **The spin-up gas interfaces for all the science gyros behave similarly at flight-like flow rates and temperatures with the rotors delevitated.**

The gas flow data from the asymptotic spin speed test done during Payload Verification II on gyro #3 per P0522 (Op #1655) is shown on the last line. This pressure data (*at 725 scc/m with the rotor levitated*) is very similar to the previous 'rotor delevitated' data taken at 750 scc/m. **The spin-up gas interfaces for gyro #3 behave similarly whether the rotor is delevitated or is levitated and placed in the spin-up position in the housing (off center toward the spin-up channel).**

Gyro	Flow (scc/m)	Gyro temp (K)	Filter temp (K)	$P_{SPIN\text{-}UP}$(torr)	$P_{EXHAUST}$(torr)	~ Bath temp (K)
1	750	6.82	12.92	49.11	0.1613	1.8
2	750	6.53	12.38	48.88	0.1602	1.8
3	750	6.24	8.88	45.96	0.1564	4.2
4	750	6.70	7.49	45.66	0.1559	4.2
3	725	6.58	10.47	45.53	0.1543	4.2