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S0626 Rev. –

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Verification of Telescope Pointing Noise Requirements

Prepared by

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ITAR Assessment Performed
Tom Langenstein ______ ITAR Control Required? Yes/No ______ Date 3/21/02

APPROVALS

John Turneaure	Co-PI, Preparer	3/21/02 Date
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CHANGE RECORD

LIST OF ACRONYMS AND SYMBOLS

$\theta_{N,i}$	RMS pointing noise
σ_x	Standard error of the ensemble of x_i samples
σ_y	Standard error of the ensemble of y_i samples
ATC	Attitude and Translational Control
c_2	Correlation coefficient between alternate samples along the measure charge ramp
i	Label identifying the four photodetector pairs, see Table 1-1
$I_{+/-,i}$	Current from +/- side of i^{th} photodetector pair
$n_{RMS,i}(t-t_0)$	RMS electron shot noise for photodetector pair i as a function of time along a ramp
$(dn/dt)_{L,i}$	Lower bound of summed electron current for i^{th} photodetector pair at null pointing
$(dn/dt)_{N,i}$	RMS electron current noise for the i^{th} photodetector pair at null pointing
RMS	Root mean square
$s_{+/-,i}$	Normalizing factors for +/- side of i^{th} photodetector pair. Their values are near unity.
$S_{n,i}$	Normalized pointing signal for i^{th} photodetector pair
$(dS_n/d\theta)_{Av,i}$	Weighted average of $dS_n/d\theta$ across the 400 nm to 1000 nm wavelength band for photodetector pair I
t	Time
t_0	Initial time along ramp used for estimate of pointing signal
TRE	Telescope Readout Electronics
x_i	i^{th} sample along the as measured charge ramp
y_i	i^{th} sample along a hypothetical charge ramp with no correlation among the points

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0 SUMMARY

This document contains the analyses that verify the Telescope pointing noise requirements for both the ATC system and for Science. Table 0-1 lists these two requirements, their required values and their actual values. The values in the table show that both of these requirements pass their specifications.

Table 0-1. Pointing noise requirements.

Document: Paragraph	Title (section where analyzed)	Required Value	Actual Values	Pass/ Fail
T003: 7.5.1	Telescope Pointing Noise, ATC System (section 3)	≤ 100 marcsec (τ_s=68 ms with ≤ 40 samples)	≤ 94 marcsec (τ_s=68 ms with 40 samples)	Pass
T003: 7.5.2	Telescope Pointing Noise, Science (section 3)	≤ 75 marcsec (τ_s=90 ms with ≤ 101 samples)	≤ 65 marcsec (τ_s=90 ms with 100 samples)	Pass

This document is maintained in the GP-B database in two forms: first as an Adobe Acrobat file accessible diretly in the database, and second as the set of files given in Table 0-2 in their native form available in the GP-B database directory.

Table 0-2. Set of S0626 files in their native source form.

Description	Filename	File type
Main text	S0626 Rev.-.doc	Microsoft *Word*®
Appendix 1	S0626 Apndx 1 Rev.-.nb	*Mathematica*®
Data file for Appendix 1	S0626 DataA1-100.txt	Text File
Data file for Appendix 1	S0626 DataA101-200.txt	Text File
Data file for Appendix 1	S0626 DataB1-100.txt	Text File
Data file for Appendix 1	S0626 DataB101-200.txt	Text File
Appendix 2	S0626 Apndx 2 Rev.-.nb	*Mathematica*®

1 INTRODUCTION

This report documents the analysis and supporting data that verify the two Telescope pointing noise requirements, which are stated in section 1.1 below. Section 2 documents the summed electron current for each photodetector pair, and the analysis of the electronic noise and the electron shot noise. Section 3 documents the analysis of the Telescope pointing noise for the ATC system, and the analysis of the Telescope pointing noise for Science.

1.1 T003 Requirements

7.5 Telescope Pointing Noise

The telescope pointing noise is specified as a standard deviation in the time domain in units of marcsec for a given measurement time using at most N samples along the 100 ms charge ramp. The noise on the charge ramp signal has two sources: electron shot noise due to a small photon rate and quantum efficiency of the photodetectors, and Gaussian white noise introduced by the electronics, which has some correlation due to a 2-pole low-pass filter, which is down 3 dB at 500 Hz.

Verification Method: **N/A**

7.5.1 Telescope Pointing Noise, ATC System

As input to the Science data analysis, the standard deviation of the telescope pointing noise for each of the four telescope photodetector pairs shall be <= 100 marcsec for a measurement time of 68 ms using at most 40 samples along 68 ms of the 100 ms charge ramp for each of the 8 photodiodes.

Verification Method: **A, T**

7.5.2 Telescope Pointing Noise, Science

As input to the ATC system, the standard deviation of the telescope pointing noise for each of the four telescope photodetector pairs shall be <= 75 marcsec for a measurement time of 90 ms using at most 101 samples along 90 ms of the 100 ms charge ramp for each of the 8 photodiodes. This requirement corresponds to a single-sided power spectral density of 34 marcsec/sqrt(Hz).

Verification Method: **A, T**

2 Detected Electron Current and Noise Sources

The telescope pointing signals are derived from the currents on the plus and minus sides of a photodetector pair. There are four detector pairs designated by i in this document with i having the values 1…4: two pairs for the X-axis and two pairs for the Y-axis. Table 1-1 contains the relation of this designation with the telescope axes and TRE designations, and the location of the detector package assembly on the telescope forward plate.

Table 2-1. Designations for photodetector pairs.

Photodetector Pair Designation, *i*	Detector Module	Location	Rotation Axis	TRE Designation
1	DPA-A Reflected	+Y	Y	TRE-A X
2	DPA-B Transmitted	+X	X	TRE-A Y
3	DPA-A Transmitted	+Y	Y	TRE-B X
4	DPA-B Reflected	+X	X	TRE-B Y

The normalized pointing signals $S_{n,i}$ are defined as follows, where $I_{+,i}$ and $I_{-,i}$ are the currents from the plus and minus sides of photodetector pair i, and $s_{+,i}$ and $s_{+,i}$ are normlizing factors that account for the slightly different responsivities of the plus and minus photodetectors. For the analyses in this document, it is adequate to assume that the normalizing factors are unity.

$$S_{n,i} = \frac{s_{+,i}\, I_{+,i} - s_{-,i}\, I_{-,i}}{s_{+,i}\, I_{+,i} + s_{-,i}\, I_{-,i}} \qquad \text{Eq 2-1}$$

2.1 Summed Current for Each Photodetector Pair

The upper bound for the pointing noise occurs for the lower bound of the summed currents from a photodetector pair when the telescope is null pointed toward the guide star HR8703. We reported the lower bound for the summed electron currents $(dn/dt)_{L,i}$ for each photodetector pairs in verification document S0619. The results from that document are given in Table 2-2.

Table 2-2. Lower bound for the summed currents for each of the four photodetector pairs for null pointing of the telescope.

ID	Detector Axis	Lower Bound for Summed Electron Current at Null Pointing (electrons/s)
1	Y-axis, TRE-A	40,256.
2	X-axis, TRE-A	100,362.
3	Y-axis, TRE-B	52,945.
4	X-axis, TRE-B	60,575.

2.2 Noise Sources

The telescope pointing noise has two noise sources. One is the electron shot noise due to the shot noise of photons arriving at the photodetectors and the quantum efficiency of the photodetectors. The electron shot noise is derived from the summed currents of a photodetector pair. The other is the noise introduced by the electronics.

2.2.1 Electron Shot Noise

The electron shot noise of a photodetector pair follows directly from the summed electron current $(dn/dt)_{L,i}$ for that pair. The telescope readout electronics produces a charge ramp, which is reset every 0.1 s. The charge on the ramp is sampled at the rate of 2200 samples/s, and thus 220 samples during each 0.1 s ramp. We find the current from the charge ramp by estimating the slope of the charge ramp. The first 20 samples are not included in the estimation to allow for settling of the electronics after the reset of the charge in the integrator to approximately zero. For the first charge sample of each ramp used in the estimation at time t_0, there is no electron shot noise. From that sample forward, the time dependence of the RMS electron shot noise $n_{RMS,i}(t-t_0)$ is governed by Poisson statistics according to the following equation.

$$n_{RMS,i}(t-t_0) = \sqrt{(dn/dt)_{L,i}\ (t-t_0)} \qquad \text{Eq 2-2}$$

2.2.2 Electronic Noise

R. Farley and J.P. Turneaure analyzed the TRE electronic noise for each flight detector pair in a *Mathematica*® notebook, which is listed in appendix 1.

The source data for this analysis is from data taken during Payload Verification II with the flight telescope and flight TREs[1]. The sample size for this analysis included 20,000 points for each of the eight photodetectors. The electronic noise has a common mode signal at 60 Hz. When the plus and minus sides of the photodetector pairs are subtracted, as they are in the processing for extracting the pointing signal, the 60 Hz common mode signal is removed. Calculation of the 2-sample variance as a function of separation time shows that the electronic noise is white. Table 2-3 lists the electronic noise for each detector pair found in the analysis. The electronic noise in the table refers to the measured charge noise in the measurement of the charge along the charge ramp.

Table 2-3. Electronic noise for each photodetector pair.

Photodetector Designation, *i*	TRE Designation	Electronic Noise (electrons)
1	TRE-A X	196
2	TRE-A Y	224
3	TRE-B X	194
4	TRE-B Y	224

The 2-sample variance as a function of separation time also shows that there is correlation between adjacent and alternate points, with little correlation for greater separation times. In this analysis for which we only use alternate points, we find the correlation coefficient c_2 between alternate sample points to be 0.2. The set of measured data points x_i with standard error σ_x are expressed in terms of a second set of uncorrelated

[1] The data were taken during the execution of P0490. R. Farley, private comm., 2001.

data points y_i with standard error σ_y according to Eq 2-3. We include the effect of correlation in our calculation of telescope pointing noise.

$$x_i = y_i + c_2\ y_{i-2} \quad \text{Eq 2-3a}$$

$$\sigma_y^2 = \frac{\sigma_x^2}{1+c_2^2} \quad \text{Eq 2-3b}$$

3 Telescope Pointing Noise

A determination of the telescope pointing noise requires knowledge of the lower-bound of the summed electron current for each photodetector pair $(dn/dt)_{L,i}$ given in Table 2-2, and of the derivative of the telescope normalized pointing signal with respect to angle, $(dS_n/d\theta)_{Av,i}$ across the optical band. The lower-bound values of $(dS_n/d\theta)_{Av,i}$, which are all about 0.44 /arsec, are given in Table 6-3 of verification document S0619. Using our knowledge of the electron shot noise and the electronic noise, we numerically calculate the RMS current noise $(dn/dt)_{N,i}$ on a statistical basis using 10,000 ramps (see appendix 2) for each photodetector pair. We find the RMS pointing noise $\theta_{N,i}$ in units of arcsec for each photodetector pair with the following equation.

$$\theta_{n,i} = \frac{(dn/dt)_{N,i}}{(dn/dt)_{L,i}\left(\dfrac{dS_n}{d\theta}\right)_{Av,i}} \quad \text{Eq 3-1}$$

3.1 Attitude and Translation Control

The attitude and translational control system uses the telescope pointing signal as an input to keep the telescope pointed toward the guide star. For this system to meet its requirements, the specification in requirement T003 7.5.1 must be met. The upper-bound standard deviation of the telescope pointing noise for each photodetector pair using 40 samples along 68 ms of the 10,000 charge ramps is calculated using the *Mathematica*® notebook listed in appendix 2. Table 3-1 lists the calculated values after a 1.3% adjustment to account for the slightly larger values of summed currents used in the calculations. The largest standard deviation in the table is 93.7 marcsec for photodetector pair 1, which meets the specification of ≤100 marcsec.

Table 3-1. Upper bound for standard deviation of telescope pointing noise in time domain for each photodetector pair for ATC system. The upper bounds are for 40 alternate points and a measurement time of 68 ms.

Photodetector Pair Designation, *i*	TRE Designation	Pointing Noise Std Deviation (marcs)
1	TRE-A X	93.7
2	TRE-A Y	48.6
3	TRE-B X	73.2
4	TRE-B Y	73.3

3.2 Science

The Science analysis uses the telescope pointing signal in several ways. The principal use is to subtract the telescope pointing signal from the gyroscope signal to measure the direction of the gyroscope relative to the guide star. The use of the pointing signal for Science analysis leads to the specification in requirement T003 7.5.2. The upper-bound standard deviation of the telescope pointing noise for each photodetector pair using 100 samples along 90 ms of the charge ramp is calculated using the *Mathematica®* notebook listed in appendix 2. Table 3-2 lists the calculated values after a 1.3% adjustment to account for the slightly larger values of summed currents used in the calculations. The largest standard deviation in the table is 64.8 marcsec for photodetector pair 1, which meets the specification of ≤75 marcsec.

Table 3-2. Upper bound for standard deviation of telescope pointing noise in time domain for each photodetector pair for Science analysis. The upper bounds are for 100 alternate points and a measurement time of 90 ms.

Photodetector Pair Designation, i	TRE Designation	Pointing Noise Std Deviation (marcs)
1	TRE-A X	64.8
2	TRE-A Y	35.5
3	TRE-B X	52.2
4	TRE-B Y	51.4

Appendix 1

Listing of *Mathematica*® Notebook used for Calculating the TRE Electronic Noise from Experimental Data

TRE Electronic Noise

R. Farley and J. P. Turneaure
March 8, 2002

■ Summary

The electronic noise is found to have a common mode signal at 60 Hz. When the Plus and Minus signals are subtracted as they are in the processing for extracting the pointing signal, the electronic noise per detector pair is as follows.

TRE-A X-Axis	195.8 electrons
TRE-A Y-Axis	223.9 electrons
TRE-B X-Axis	194.0 electrons
TRE-B Y-Axis	224.3 electrons

Calculation of the 2-sample variance as a function of separation time shows that the electronic noise in the ramp is white. The 2-sample variance also shows that adjacent data points are moderately correlated, every other data point is slightly correlated, and data points separated by 3 points are almost completely uncorrelated.

■ Introduction

The purpose of this document is to determine the electronic noise (in units of electrons) of each of the four telescope detector pairs. Also, the correlation between data pairs is determined and a verification that for the duration of a 0.1 s ramp the electronic noise is white.

■ Electronic Noise Data

Turn off spelling messages.

```
Off[General::spell]; Off[General::spell1];
```

Set the directory that contains the electronic noise data.

```
SetDirectory["C:/Telescope/S0626/TRE"]

C:\Telescope\S0626\TRE
```

Read the data into the lists "dataInA" and "dataInB" representing the A and B side TREs. The lists "dataInA" and "dataInB" are 4 by 44000 matrices. The first index refers to the detectors in the following order: 1) TRE-A/B X-Axis Plus, 2) TRE-A/B X-Axis Minus, 3) TRE-A/B Y-Axis Plus, 4) TRE-A/B Y-Axis Minus. The second index refers to each of the ordered measurements, which are taken at a data rate of 2200 /s. The signal consists of a charge integrator that is reset at a 10 Hz rate with the data stream starting at the initial reset period. For this reason, the first 20 points of every 220 points are not used since these points encompass the reset and settling period.

```
dataInA1 = Import["DatA1-100.txt", "Table"];
dataInB1 = Import["DatB1-100.txt", "Table"];
dataInA2 = Import["DatA101-200.txt", "Table"];
dataInB2 = Import["DatB101-200.txt", "Table"];
dataInA = Join[dataInA1, dataInA2];
dataInB = Join[dataInB1, dataInB2];
Clear[dataInA1, dataInA2, dataInB1, dataInb2];
```

The input data is in volts at the output of the Charge Lock Loop. The input data list is adjusted to convert it to electrons using the known capacitance of 0.58 pF and the electron charge of 0.1602176 aC.

$$\text{dataInA} = \text{dataInA}\,\frac{0.58\;10^{-12}}{1.602176\;10^{-19}};$$

$$\text{dataInB} = \text{dataInB}\,\frac{0.58\;10^{-12}}{1.602176\;10^{-19}};$$

Now place the input data into a more convenient list "dataT". The first index refers to the detector (i = 1 ... 8). The second index refers to the time ordered ramp number (j = 1 ... 200). The third index refers to the time ordered data point in each ramp with the first 20 points removed (k = 1 ... 200).

```
iD = 8; iP = 4; jN = 200; kN = 200;
dataT = Table[0.0, {i, iD}, {j, jN}, {k, kN}];
dataTr = Table[0.0, {i, iD}, {j, jN}, {k, kN}];
Do[
  dataT[[i, j, k]] = dataInA[[20 + 220 (j - 1) + k, i]];
  dataT[[i + 4, j, k]] = dataInB[[20 + 220 (j - 1) + k, i]],
  {i, 4}, {j, jN}, {k, kN}];
Clear[dataInA, dataInB];
```

■ Linear Regression of Data

Load linear regression package.

```
<< Statistics`LinearRegression`;
```

■ Detrend Data with Constant and Slope

Fit data to model that includes a constant "a01" and a slope "a11". These are lists of length 8 by 200 since there is a pair of constants for each of the 200 ramps and for each detector. The standard error "stdE", a list of length 8 by 200, of each ramp is calculated. The average and rms value of the slopes "slopeAv" and "slopeRMS", and the average standard error "stdEAv" is given. The unit for the slope is electron/sample period. The unit for the stdE is electron.

```
a01 = Table[0.0, {i, iD}, {j, jN}];
a11 = Table[0.0, {i, iD}, {j, jN}];
stdE = Table[0.0, {i, iD}, {j, jN}];
stdEAv = Table[0.0, {i, iD}];
slopeAv = Table[0.0, {i, iD}];
slopeRMS = Table[0.0, {i, iD}];
Do[
  regress = Regress[dataT[[i, j]], {1, x}, x];
  a01[[i, j]] = regress[[1, 2, 1, 1, 1]];
  a11[[i, j]] = regress[[1, 2, 1, 2, 1]];
  stdE[[i, j]] = √regress[[4, 2]] ,
  {i, iD}, {j, jN}];
Do[
  stdEAv[[i]] = stdEAv[[i]] + stdE[[i, j]]^2; slopeAv[[i]] = slopeAv[[i]] + a11[[i, j]];
  slopeRMS[[i]] = slopeRMS[[i]] + a11[[i, j]]^2,
  {i, iD}, {j, jN}];
Do[
  stdEAv[[i]] = √(stdEAv[[i]] / jN) ;
  slopeAv[[i]] = slopeAv[[i]] / jN;
  slopeRMS[[i]] = √(slopeRMS[[i]] / jN) ,
  {i, iD}];
Print["Average Slope   ", slopeAv];
Print["RMS Slope    ", slopeRMS];
Print["Average Std Error   ", stdEAv];
```

```
Average Slope   {0.166054, 0.235625, 0.21628, 0.312555, 0.178831, 0.270723, 0.216272, 0.240442}

RMS Slope   {0.645997, 0.639149, 0.657628, 0.803514, 0.725398, 0.702172, 0.715288, 0.758037}

Average Std Error   {263.725, 252.433, 299.235, 305.764, 265.516, 255.197, 299.189, 305.466}
```

Remove linear fit from data.

```
Do[
  dataTr[[i, j, k]] = dataT[[i, j, k]] - a01[[i, j]] - a11[[i, j]] k,
  {i, iD}, {j, jN}, {k, kN}];
```

▪ Detrend Data with Constant

Fit data to model that includes a constant "a01". This is a list of length 8 by 200 for detector pair. The standard error "stdE", a list of length 8 by 200, of each ramp is calculated. The average standard error "stdEAv" is given. The unit for the stdE is electron.

```
<< Statistics`DescriptiveStatistics`
a01 = Table[0.0, {i, iD}, {j, jN}];
stdE = Table[0.0, {i, iD}, {j, jN}];
stdEAv = Table[0.0, {i, iD}];
Do[
  a01[[i, j]] = Mean[dataT[[i, j]]];
  stdE[[i, j]] = StandardDeviation[dataT[[i, j]]],
  {i, iD}, {j, jN}];
Do[
  stdEAv[[i]] = stdEAv[[i]] + stdE[[i, j]]^2,
  {i, iD}, {j, jN}];
Do[
  stdEAv[[i]] = √(stdEAv[[i]] / jN),
  {i, iD}];
Print["Average Std Error    ", stdEAv];

Average Std Error    {265.705, 254.501, 300.9, 308.52, 268.155, 257.779, 301.295, 307.84}
```

Remove constant from data.

```
Do[
  dataT[[i, j, k]] = dataT[[i, j, k]] - a01[[i, j]],
  {i, iD}, {j, jN}, {k, kN}];
```

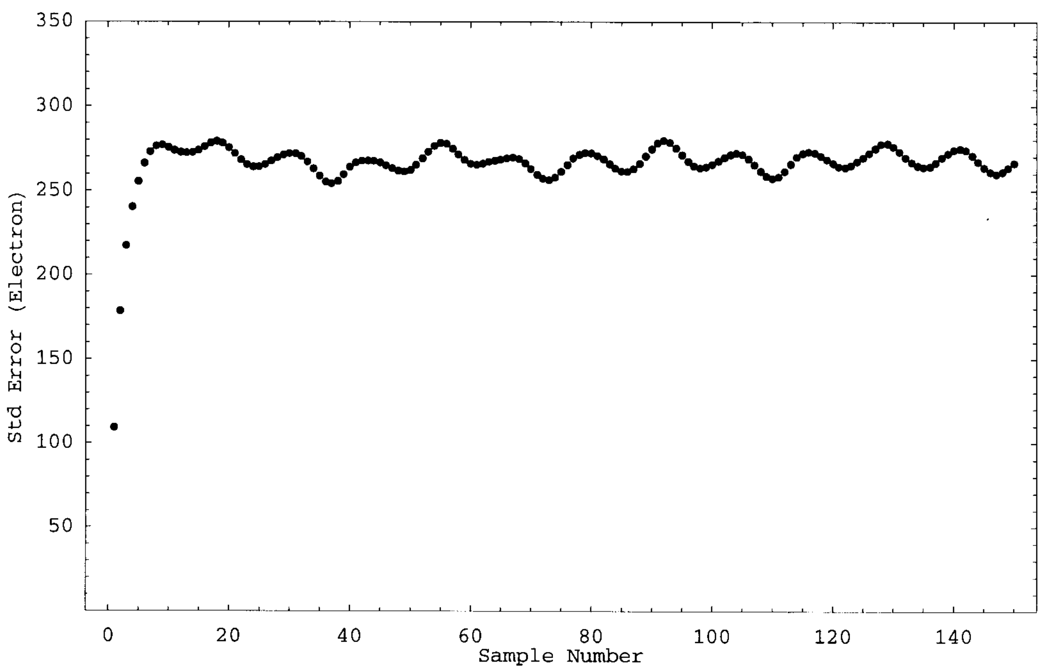
▪ Two-Sample Variance versus Time Separation without Slope Detrend

▪ For Individual Detectors

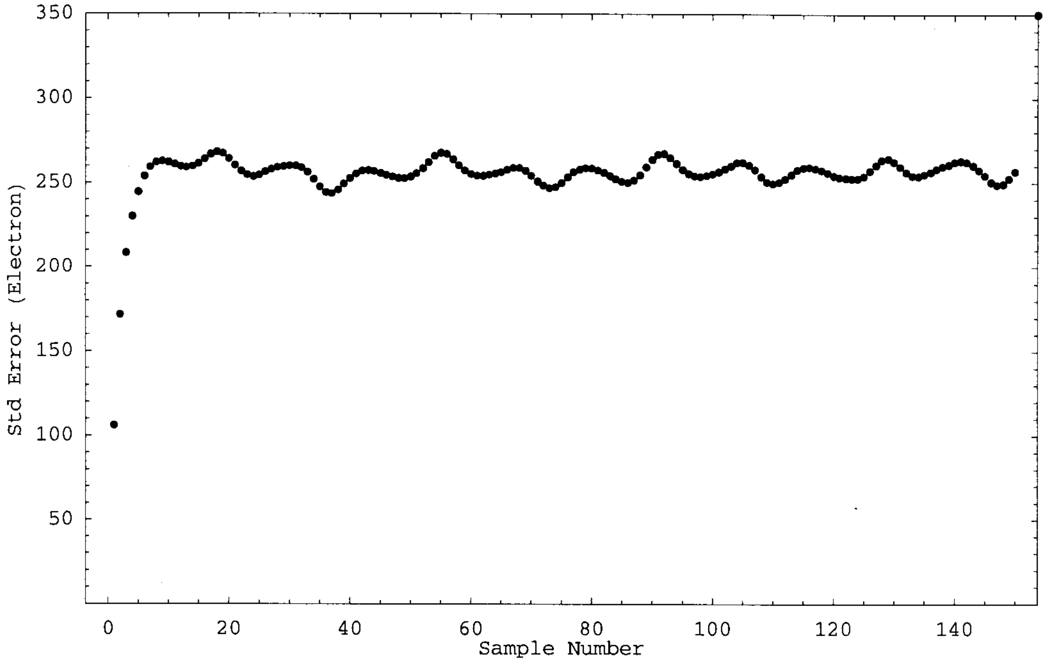
Calculate two-sample variance "stdE2" as a function of separation time measured in sample period. "stdE2" is a list of 4 by 150.

```
lN = 150;
stdE2 = Table[0.0, {i, iD}, {l, lN}];
Do[
  Do[
   stdE2[[i, l]] = stdE2[[i, l]] + 0.5 (dataT[[i, j, k]] - dataT[[i, j, k + l]])^2,
   {j, jN}, {k, kN - 1}];
  stdE2[[i, l]] = Sqrt[stdE2[[i, l]] / (jN (kN - 1))],
  {i, iD}, {l, lN}];

det = {"TRE-A X Plus", "TRE-A X Minus", "TRE-A Y Plus", "TRE-A Y Minus",
   "TRE-B X Plus", "TRE-B X Minus", "TRE-B Y Plus", "TRE-B Y Minus"};
Do[
  Print["Detector ID ", det[[i]]];
  ListPlot[stdE2[[i]], Frame -> True, PlotRange -> {0.0, 350.},
   FrameLabel -> {"Sample Number", "Std Error (Electron)"}],
  {i,
   iD}];
```

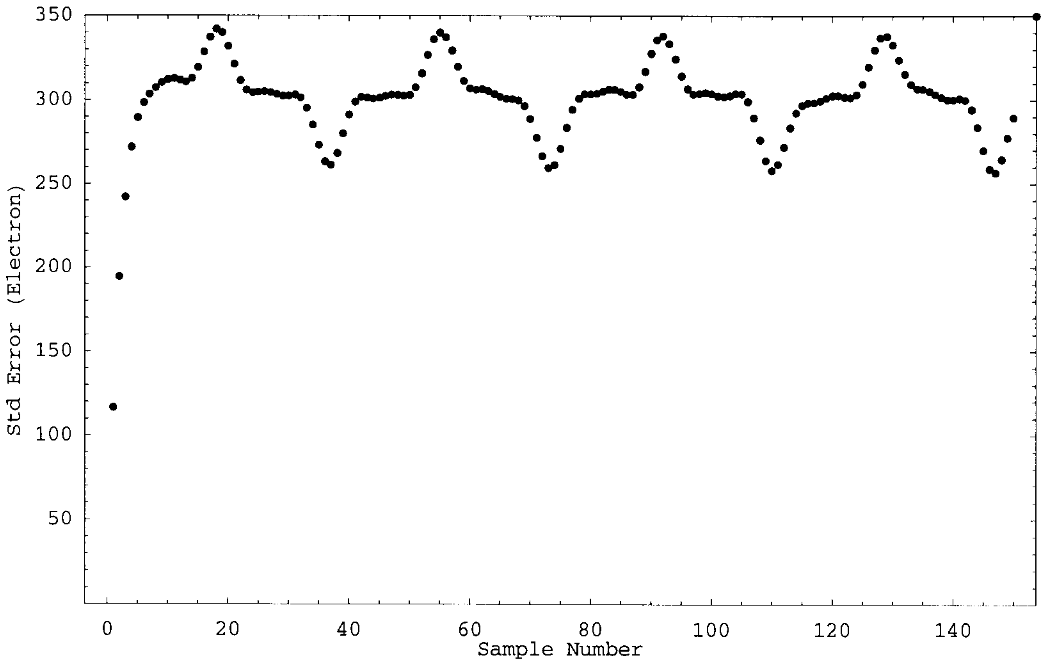
Detector ID TRE-A X Plus



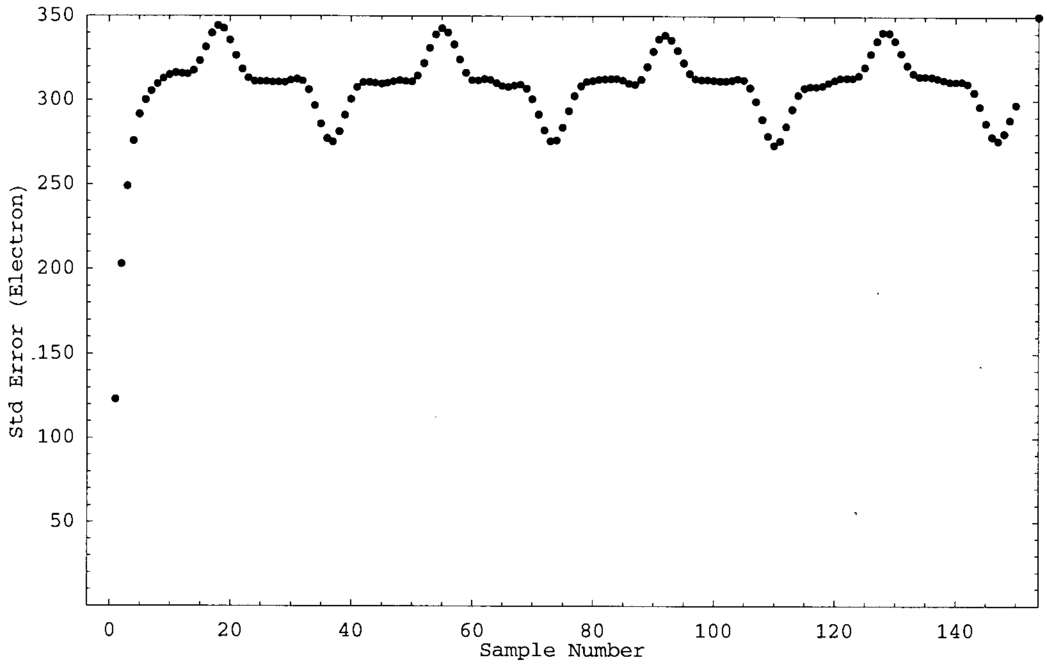
Detector ID TRE-A X Minus



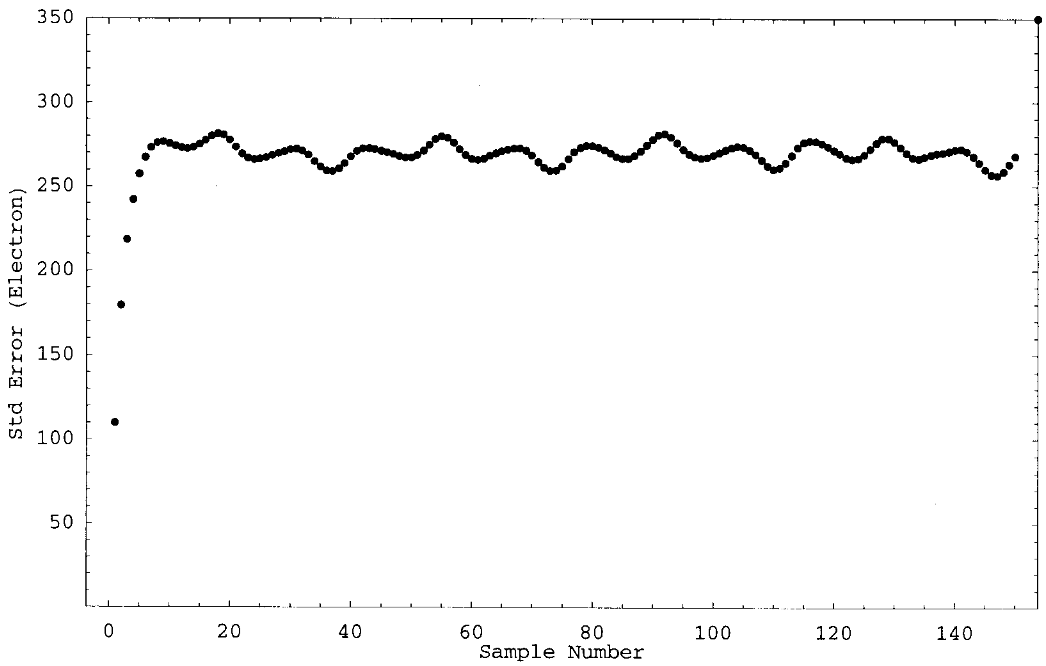
Detector ID TRE-A Y Plus



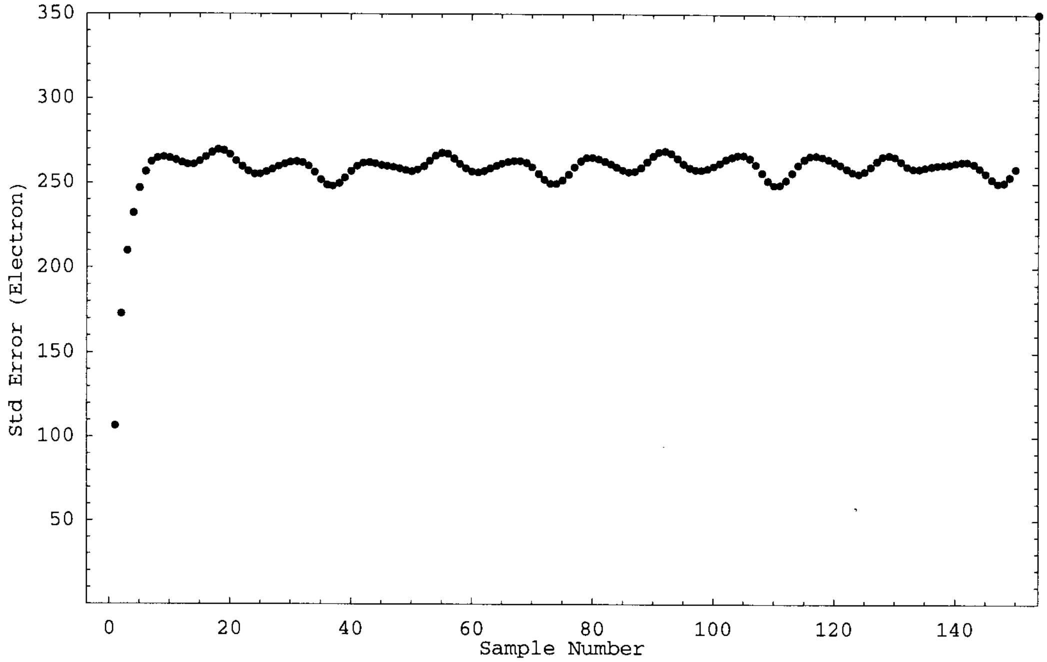
Detector ID TRE-A Y Minus



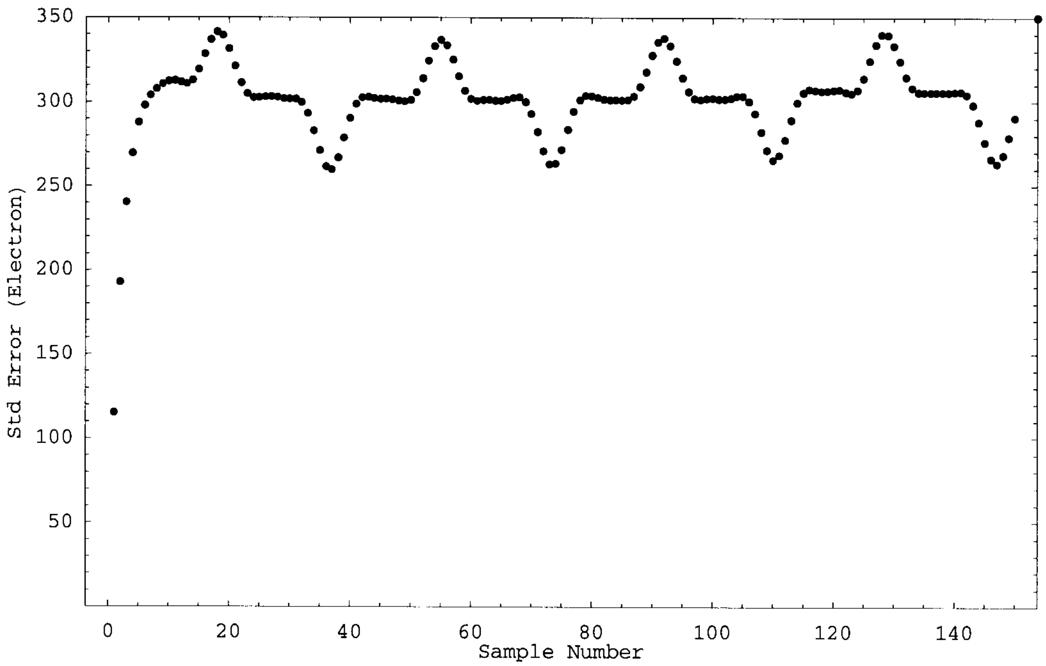
Detector ID TRE-B X Plus



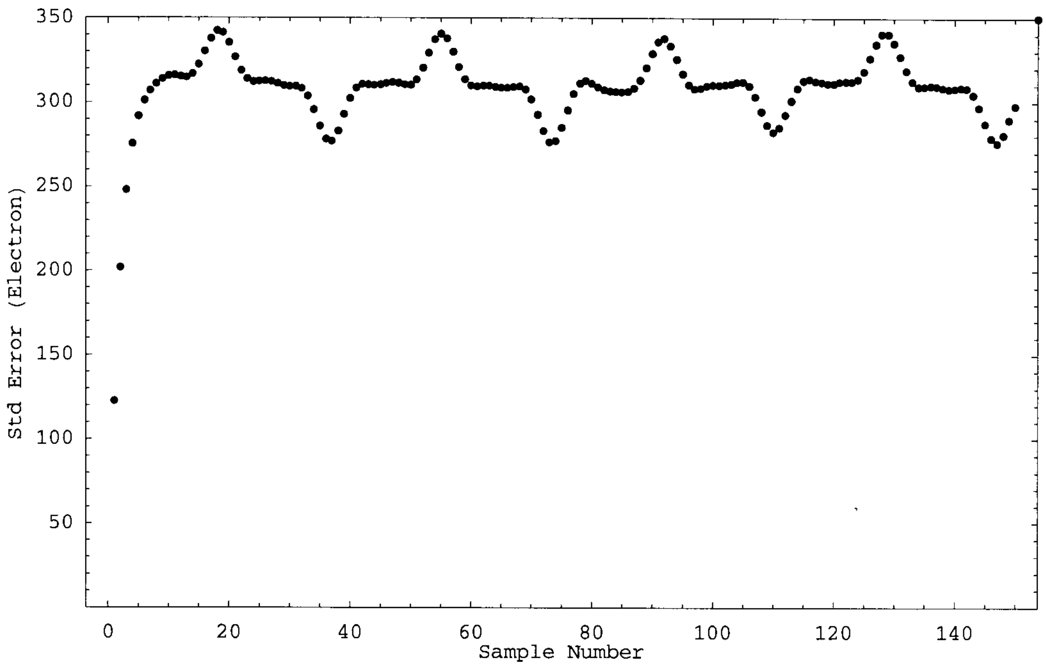
Detector ID TRE-B X Minus



Detector ID TRE-B Y Plus



Detector ID TRE-B Y Minus



■ For Detector Pairs (removes common mode noise)

Establish new list "dataTP" containing the pairs of detectors (subtracted). This is a 4 by 200 by 200 list for the detector pairs, ramp number, and data point number.

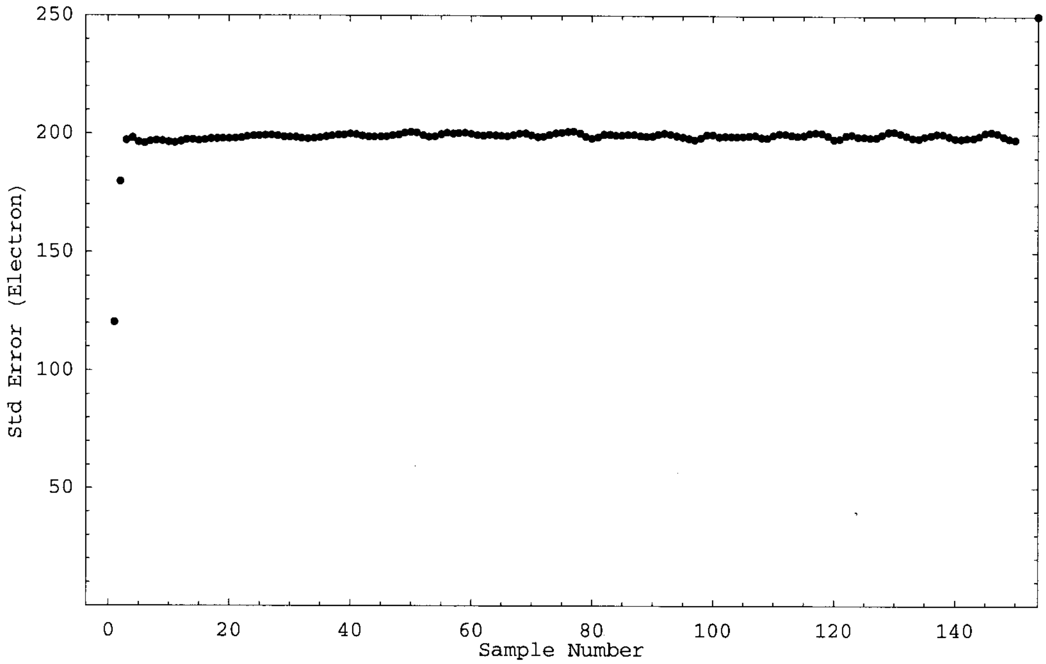
```
lN = 150;
dataTP = Table[0.0, {i, iP}, {j, jN}, {k, kN}];
Do[
  dataTP[[i, j, k]] = dataT[[2 i - 1, j, k]] - dataT[[2 i, j, k]],
  {i, iP}, {j, jN}, {k, kN}];
```

Fit data to model that includes a constant "a01" and a slope "a11". These are lists of length 4 by 200 since there is a pair of constants for each of the 200 ramps and for each detector. The standard error "stdE", a list of length 4 by 200, of each ramp is calculated. The average and rms value of the slopes "slopeAv" and "slopeRMS", and the average standard error "stdEAv" is given. The unit for the slope is electron/sample period.

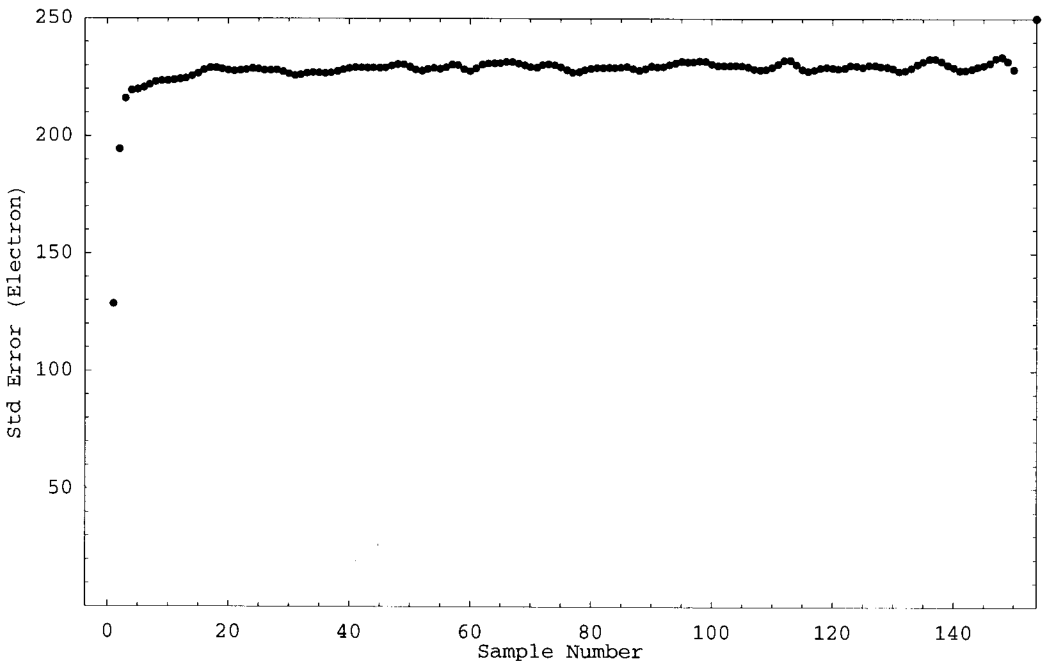
```
stdEP2 = Table[0.0, {i, iP}, {j, jN}];
stdEPAv = Table[0.0, {i, iP}];
Do[
  stdEP2[[i, j]] = stdEP2[[i, j]] + dataTP[[i, j, k]]^2,
  {i, iP}, {j, jN}, {k, kN}];
Do[
  stdEPAv[[i]] = stdEPAv[[i]] + stdEP2[[i, j]] / kN, {i, iP}, {j, jN}];
Do[
  stdEPAv[[i]] = Sqrt[stdEPAv[[i]] / jN],
  {i, iP}];
Print["Average Std Error for Pair    ", stdEPAv];

Average Std Error for Pair   {197.652, 226.686, 196.113, 227.349}
```

Calculate two-sample variance "stdE2P" as a function of separation time measured in sample period. "std2P" is a list of 4 by 150.

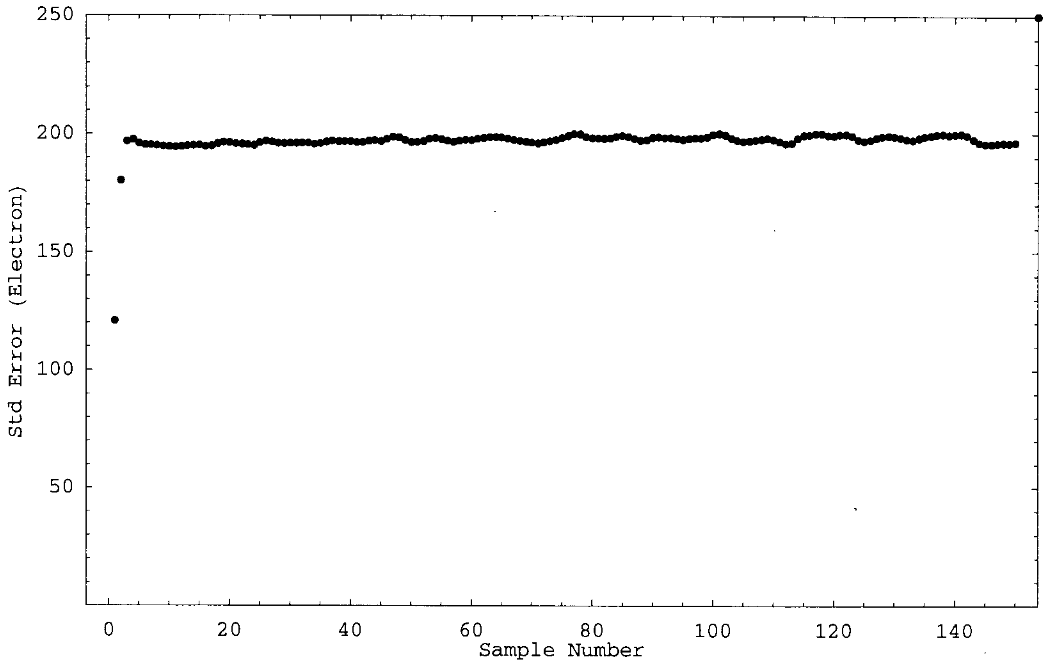
```
stdE2P = Table[0.0, {i, iP}, {l, lN}];
Do[
  Do[
   stdE2P[[i, l]] = stdE2P[[i, l]] + 0.5 (dataTP[[i, m, k]] - dataTP[[i, m, k + l]])^2,
   {m, jN}, {k, kN - l}];
  stdE2P[[i, l]] = Sqrt[stdE2P[[i, l]] / (jN (kN - l))],
  {i, iP}, {l, lN}];

detP = {"TRE-A X", "TRE-A Y", "TRE-B X", "TRE-B Y"};
Do[
  Print["Detector Pair ID ", detP[[i]]];
  ListPlot[stdE2P[[i]], Frame -> True, PlotRange -> {0.0, 250.},
   FrameLabel -> {"Sample Number", "Std Error (Electron)"}],
  {i,
   iP}];

Detector Pair ID TRE-A X
```



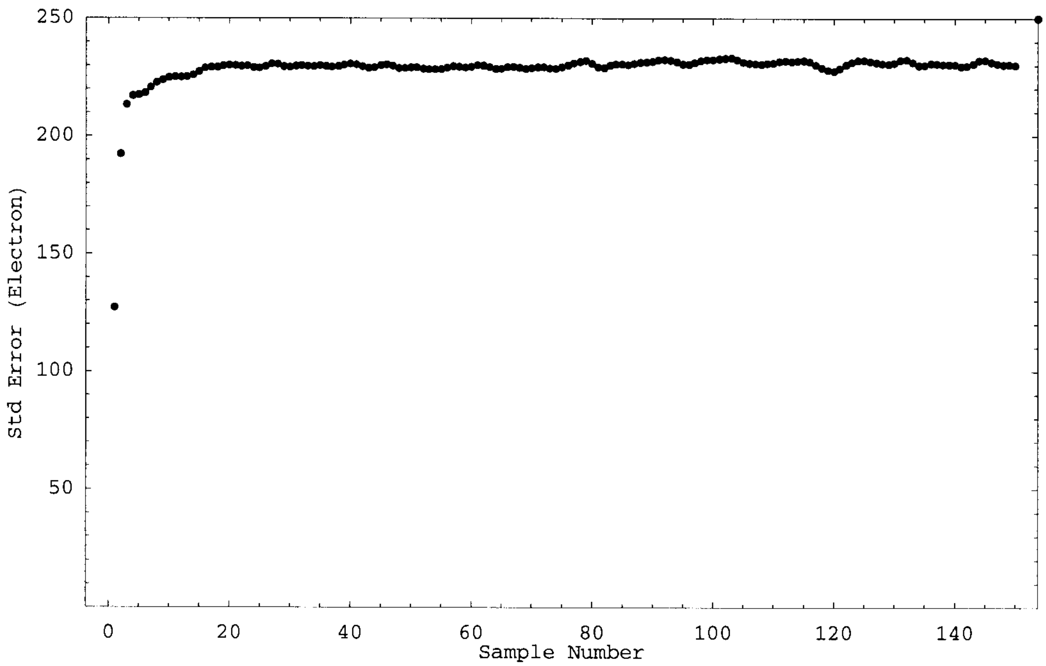
Detector Pair ID TRE-A Y



Detector Pair ID TRE-B X



Detector Pair ID TRE-B Y



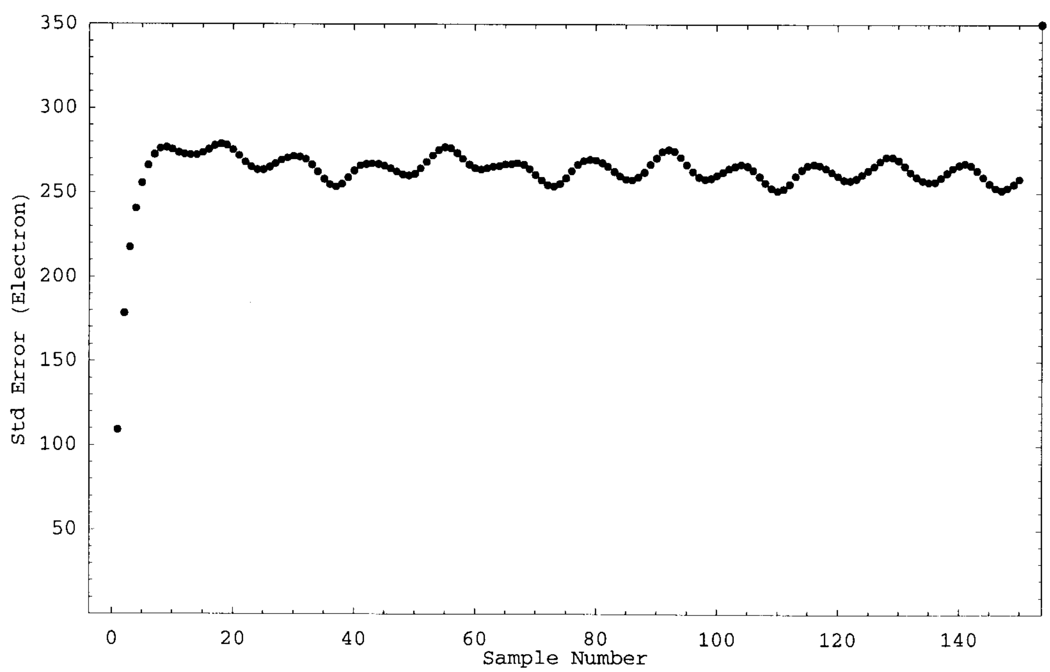
■ Two-Sample Variance versus Time Separation with Slope Detrend

■ For Individual Detectors

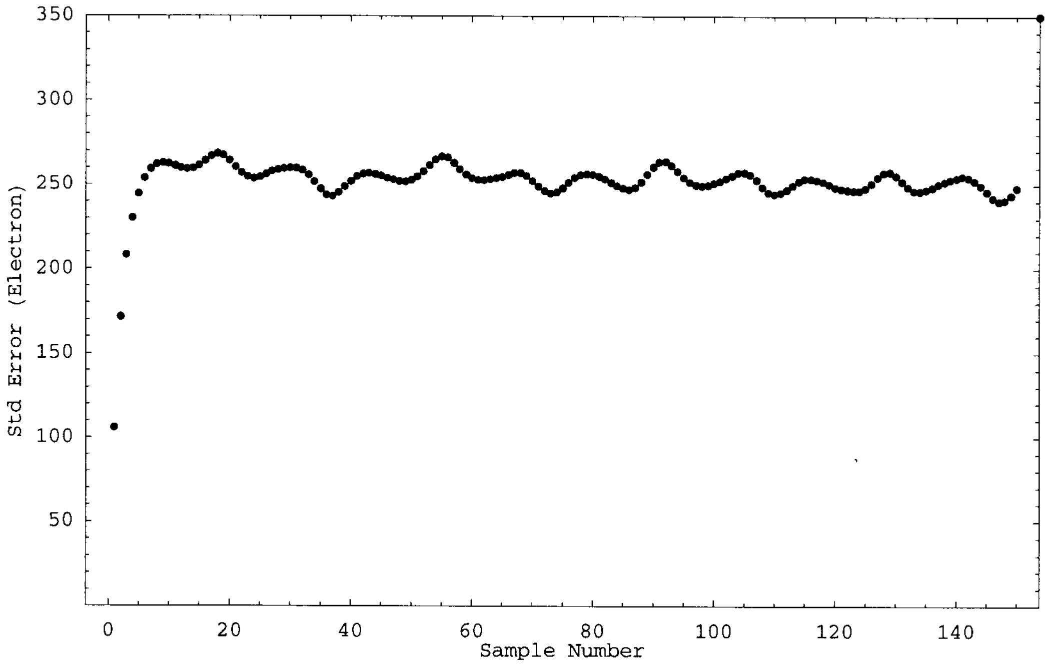
Calculate two-sample variance "stdE2" as a function of separation time measured in sample period. "stdE2" is a list of 4 by 150.

```
lN = 150;
stdE2 = Table[0.0, {i, iD}, {l, lN}];
Do[
  Do[
   stdE2[[i, l]] = stdE2[[i, l]] + 0.5 (dataTr[[i, j, k]] - dataTr[[i, j, k + l]])^2,
   {j, jN}, {k, kN - l}];
  stdE2[[i, l]] = √(stdE2[[i, l]] / (jN (kN - l))),
  {i, iD}, {l, lN}];

det = {"TRE-A X Plus", "TRE-A X Minus", "TRE-A Y Plus", "TRE-A Y Minus",
   "TRE-B X Plus", "TRE-B X Minus", "TRE-B Y Plus", "TRE-B Y Minus"};
Do[
  Print["Detector # ", det[[i]]];
  ListPlot[stdE2[[i]], Frame → True, PlotRange → {0.0, 350.},
   FrameLabel → {"Sample Number", "Std Error (Electron)"}],
  {i,
   iD}];
```

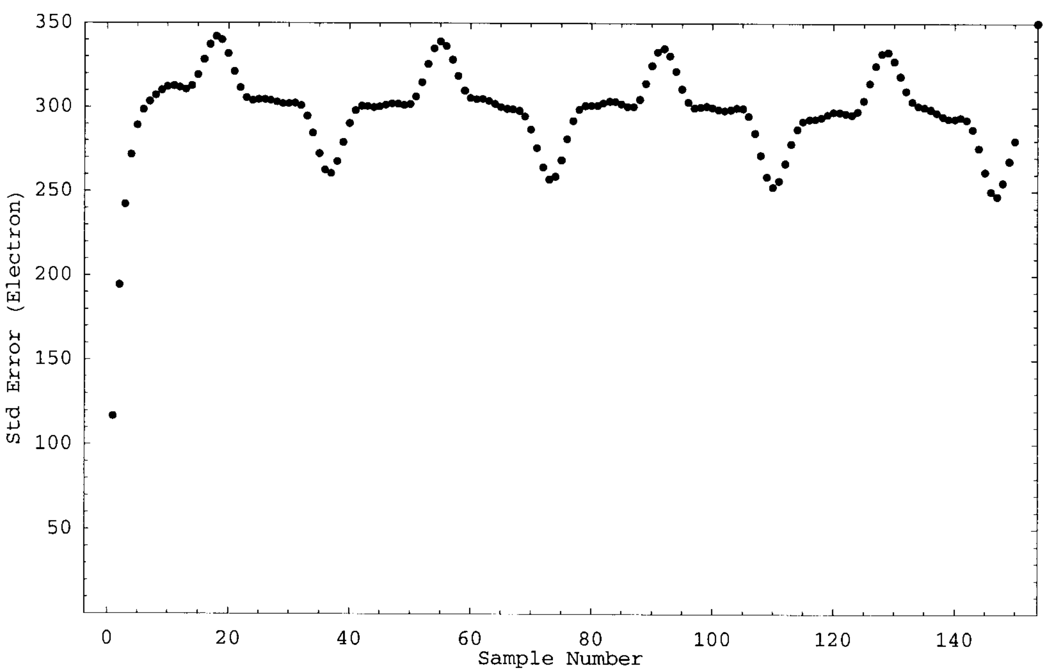
Detector # TRE-A X Plus



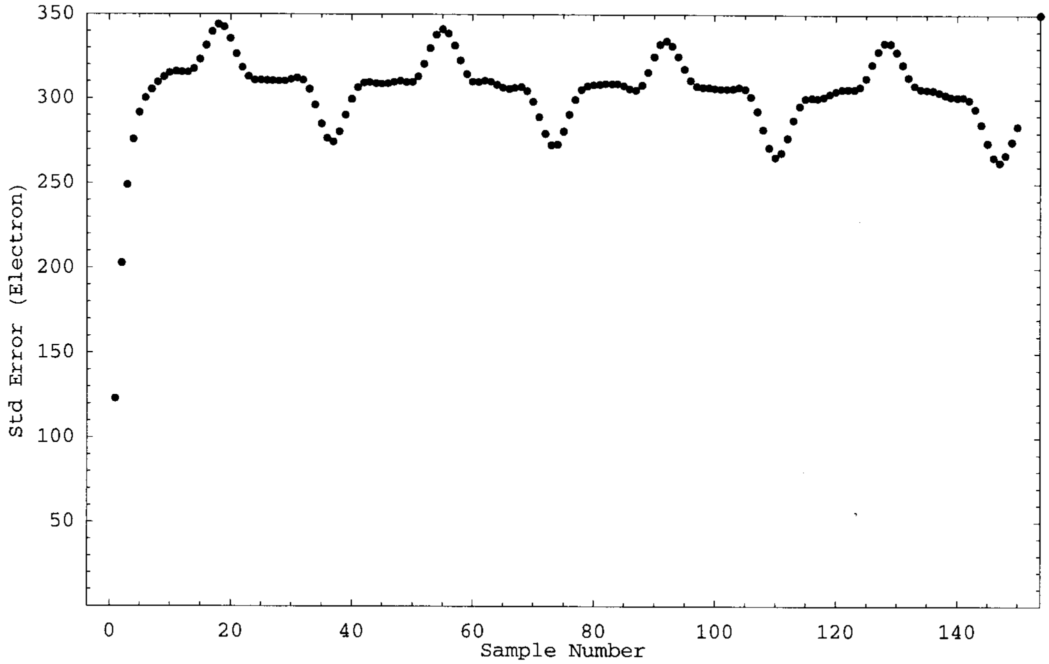
Detector # TRE-A X Minus



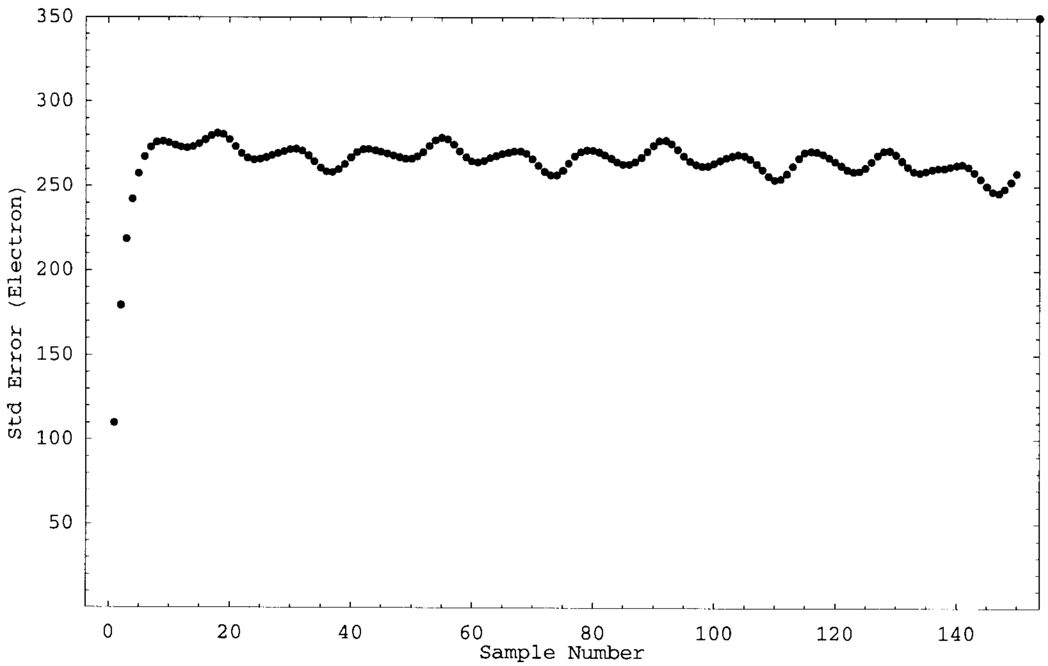
Detector # TRE-A Y Plus



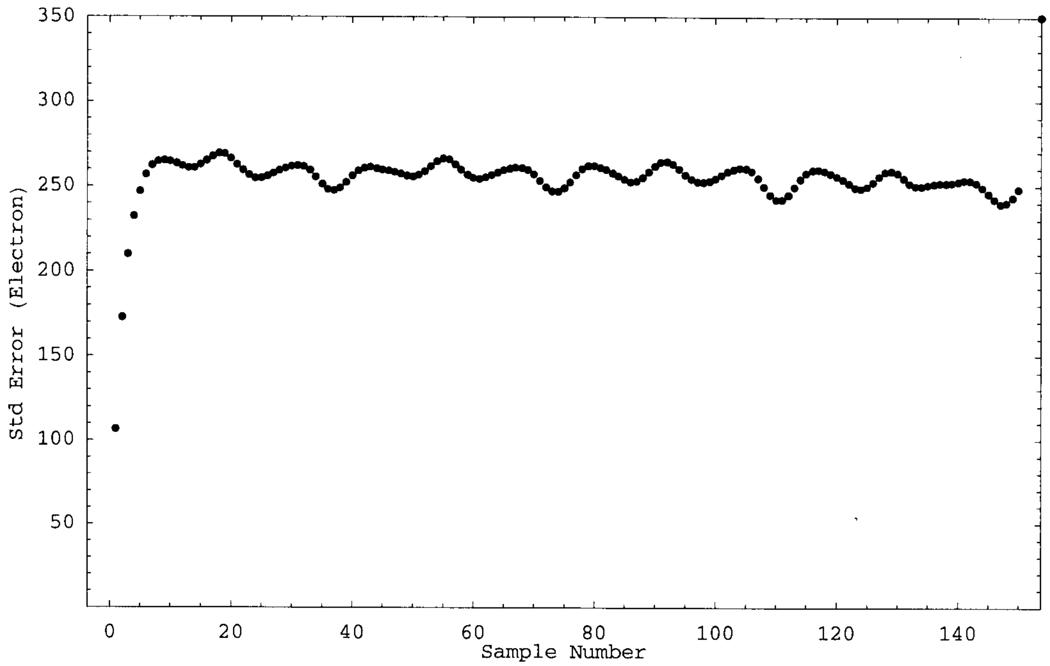
Detector # TRE-A Y Minus



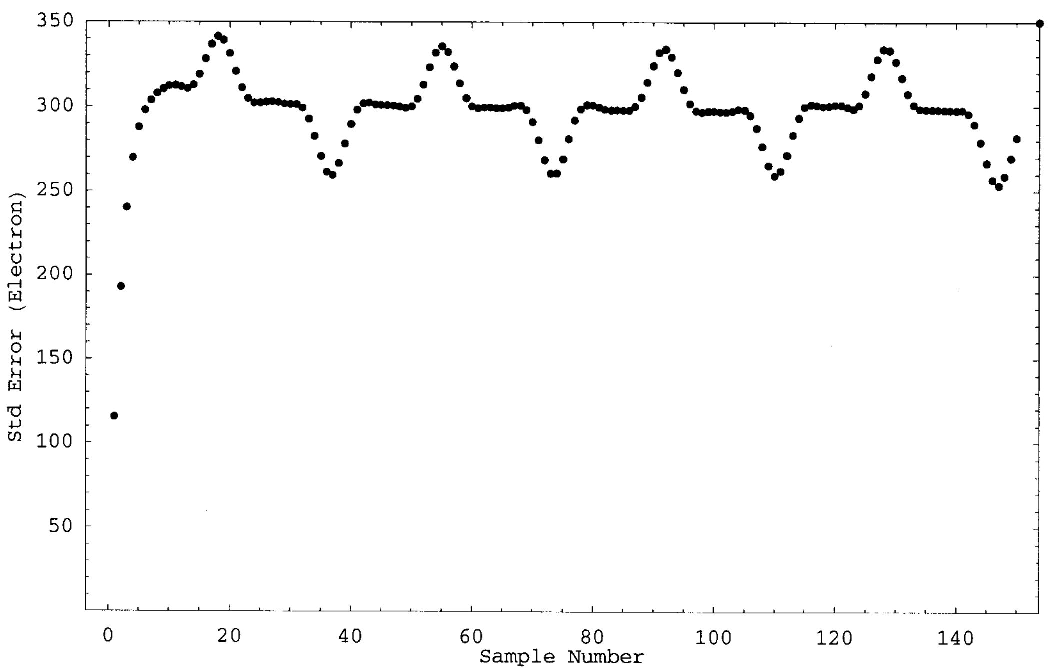
Detector # TRE-B X Plus



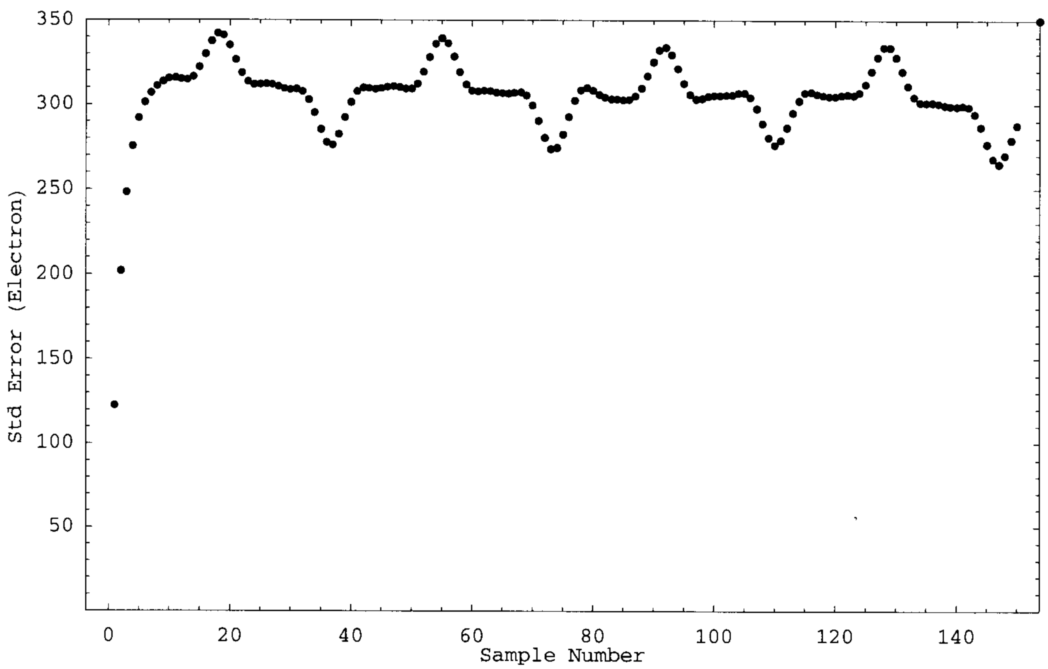
Detector # TRE-B X Minus



Detector # TRE-B Y Plus



Detector # TRE-B Y Minus



■ For Detector Pairs (removes common mode noise)

Establish new list "dataTP" containing the pairs of detectors (subtracted). This is a 2 by 200 by 200 list for the detector pairs, ramp number, and data point number.

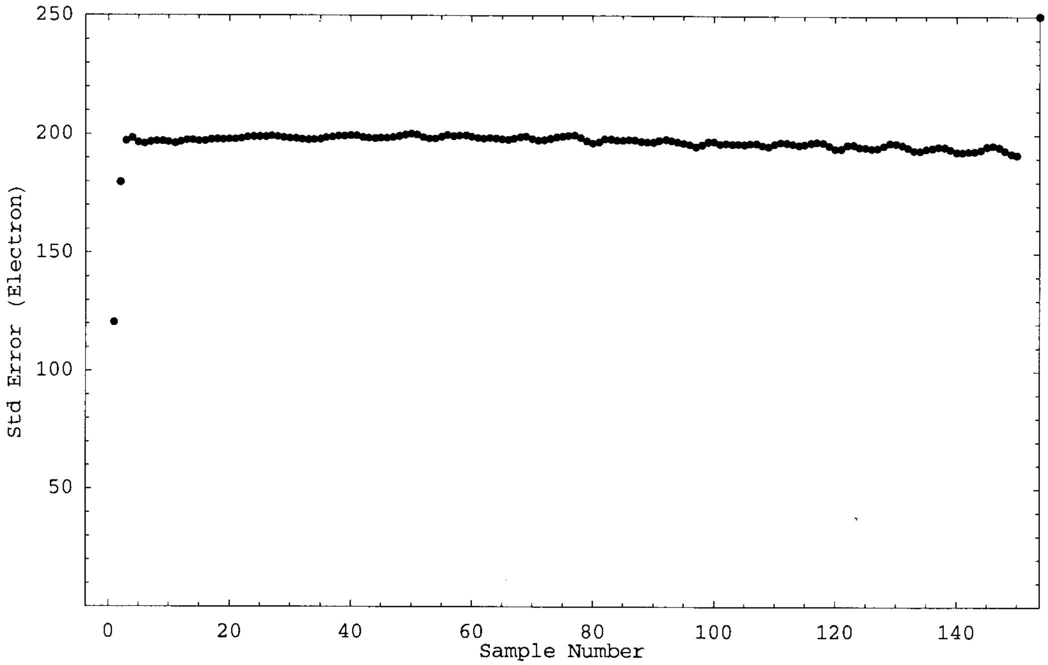
```
lN = 150;
dataTPr = Table[0.0, {i, iP}, {j, jN}, {k, kN}];
Do[
  dataTPr[[i, j, k]] = dataTr[[2 i - 1, j, k]] - dataTr[[2 i, j, k]],
  {i, iP}, {j, jN}, {k, kN}];
```

Fit data to model that includes a constant "a01" and a slope "a11". These are lists of length 4 by 200 since there is a pair of constants for each of the 200 ramps and for each detector. The standard error "stdE", a list of length 4 by 200, of each ramp is calculated. The average and rms value of the slopes "slopeAv" and "slopeRMS", and the average standard error "stdEAv" is given. The unit for the slope is electron/sample period.

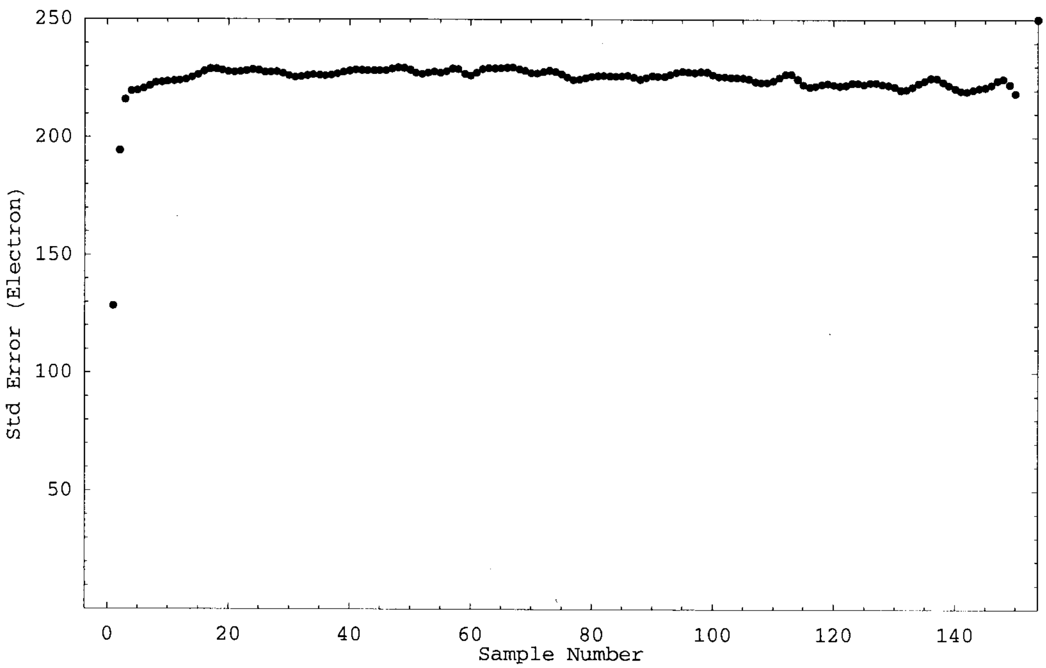
```
stdEP2 = Table[0.0, {i, iP}, {j, jN}];
stdEPAv = Table[0.0, {i, iP}];
Do[
  stdEP2[[i, j]] = stdEP2[[i, j]] + dataTPr[[i, j, k]]^2,
  {i, iP}, {j, jN}, {k, kN}];
Do[
  stdEPAv[[i]] = stdEPAv[[i]] + stdEP2[[i, j]] / kN, {i, iP}, {j, jN}];
Do[
  stdEPAv[[i]] = √(stdEPAv[[i]] / jN),
  {i, iP}];
Print["Average Std Error for Pair    ", stdEPAv];

Average Std Error for Pair   {195.834, 223.942, 194.043, 224.338}
```

Calculate two-sample variance "stdE2P" as a function of separation time measured in sample period. "std2P" is a list of 4 by 150.

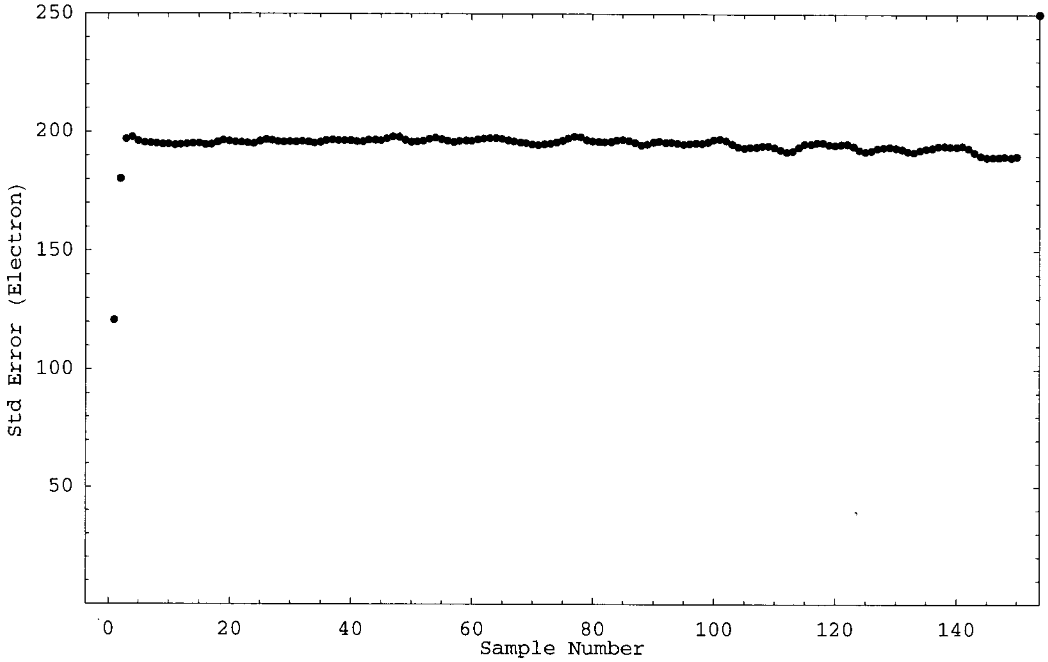
```
stdE2P = Table[0.0, {i, iP}, {l, lN}];
Do[
  Do[
   stdE2P[[i, l]] = stdE2P[[i, l]] + 0.5 (dataTPr[[i, m, k]] - dataTPr[[i, m, k + l]])^2,
   {m, jN}, {k, kN - l}];
  stdE2P[[i, l]] = √(stdE2P[[i, l]] / (jN (kN - l))),
  {i, iP}, {l, lN}];

detP = {"TRE-A X", "TRE-A Y", "TRE-B X", "TRE-B Y"};
Do[
  Print["Detector Pair ID ", detP[[i]]];
  ListPlot[stdE2P[[i]], Frame → True, PlotRange → {0.0, 250.},
   FrameLabel → {"Sample Number", "Std Error (Electron)"}],
  {i,
   iP}];

Detector Pair ID TRE-A X
```



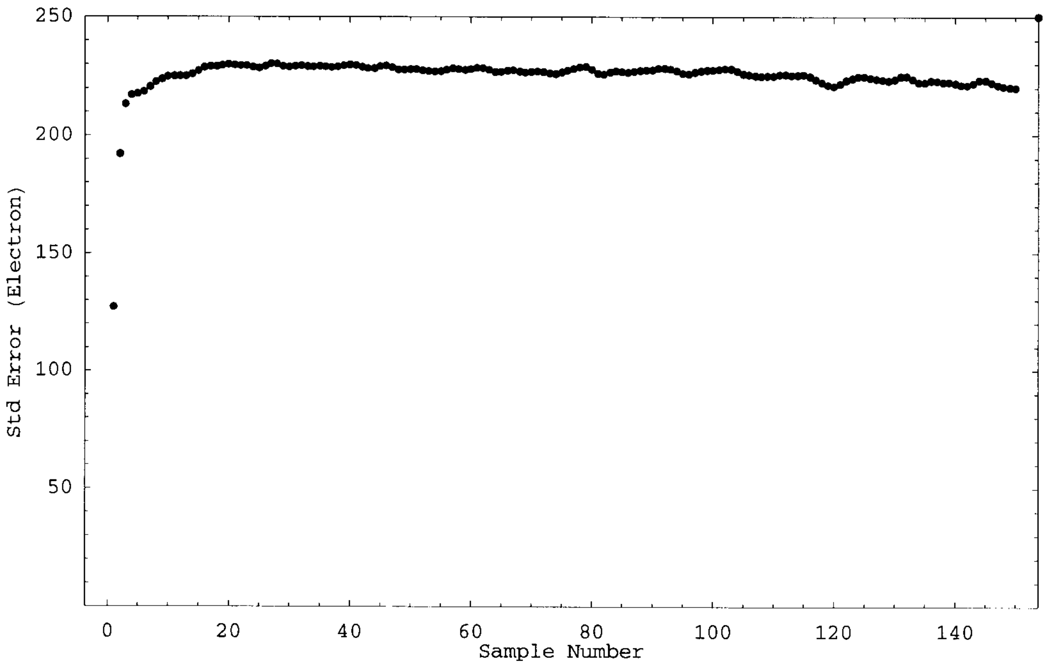
Detector Pair ID TRE-A Y



Detector Pair ID TRE-B X



Detector Pair ID TRE-B Y



Appendix 2

Listing of *Mathematica*® Notebook used for Calculating the ATC and Science Pointing Noise

Numerical Simulation of Error Signal with Gaussian Correlated White Noise and Random Walk Noise

John P. Turneaure
March 8, 2002

■ Introduction

This *Mathematica* notebook estimates the standard error of the pointing noise for both ATC and Science. For ATC, we assume the worst case conditions using 40 samples in a 68 ms section of each 100 ms charge ramp. Fore ATC, we also assume that the slope estimate is based on an average of the first 20 alternate points and the last alternate 20 points. For Science, we assume the worst case conditions using 100 samples in a 90 ms section of each 100 ms charge ramp. For Science we also assume that the slope estimate is based on a least square fit to 100 alternate points. Further we assume the lower bounds for the summed current to each of the four photodetector pairs and the lower bound for the derivative of the telescope normalized pointing signal with respect to pointing angle.

Turn off warnings for spelling.

```
In[1]:= Off[General::spell, General::spell1];
```

Here we establish the lower bounds for summed electron current to each photodetector pair (electrons/s). The order of the data is TRE-A X, TRE-A Y, TRE-B X, TRE-B Y. This data is taken from Table 5-2 of S0619.

```
In[2]:= iSumPair = {40783, 101678, 53639, 61370};
```

Here we establish measured electronic noise of flight system (flight telescope and flight TREs), which is the Gaussian correlated noise on the measured charge along the charge ramp when the photodetector current is zero (i.e., it does not include the electron shot noise). The rms electronic noise is in units of electrons. These values are found in appendix 1 of this report. The order of the data is also TRE-A X, TRE-A Y, TRE-B X, TRE-B Y.

```
In[3]:= noiseElectronics = {196.0, 224.0, 194.0, 224.0};
```

Here we establish the correlation values. The samples are spaced at 100/220 ms intervals. There are only significant correlations between adjacent points and every other point. Here we only use every other sample. Thus, we only need to account for corr2.

```
In[4]:= corr2 = 0.2;
```

Here we establish the value of the lower bounds of the derivative of the normalized telescope signal with respect to pointing angle in units of /arcsec. These values also in the same order are found in Table 6-3 of S0619.

```
In[5]:= dSndt = {0.438, 0.436, 0.439, 0.436};
```

■ Generation of Signal

The signal has the following form with units of arcsec-s. In the generation, we take a0 to be zero.

s=a0 + a1 t + nS + nE

The electronic noise nE is white Gaussian noise characterized by a standard error σE in a normal distribution. For each step, the noise is only correlated with the two previous samples.

The electron shot noise nS follows a random walk process. The noise for the first point is 0, but for subsequent points the noise is the noise of the previous point plus a random added error characterized by σS of a Poisson distribution.

```
In[6]:= << Statistics`ContinuousDistributions`;
        << Statistics`DiscreteDistributions`;
        << Statistics`LinearRegression`;
```

Define list of sample times corresponding to every other sample in units of s. There are 100 samples in each ramp, which is long enough for both the 68 ms and 90 ms periods.

```
In[9]:= t = Table[(i - 1) / 1100., {i, 100}];
```

Define correlation factor.

In[10]:= $\text{corrFactor} = \frac{1}{\sqrt{1 + \text{corr2}^2}}$;

Define module that generates simulated data. The arguments of the module are the summed current for a particular photodetector pair sI, the electronic noise for a particular photodetector pair nE, and the number n of ramps to be generated. The signal output is the list s with dimension n by 100.

```
In[11]:= f[sI_, nE_, n_] := Module[{sI0, sGn, i, j},
          (* initialize signal list *) s = Table[0.0, {i, n}, {j, 100}];
          (* electrons/time step/single photodetector *) sI0 = sI / (2 * 1100);
          (* set Poisson distribution *) pdist = PoissonDistribution[sI0];
          (* set Normal distribution *) ndist = NormalDistribution[0, nE * corrFactor];
          (* start loops for number of ramps & samples per ramp *)
          Do[
           (* add shot noise to signal *)
           Do[s[[i, j + 1]] = s[[i, j]] + Random[pdist] - Random[pdist], {j, 99}];
           (* add random noise to signal *)
           sGn = Table[Random[ndist], {j, 101}];
           Do[s[[i, j]] = s[[i, j]] + sGn[[j + 1]] + corr2 sGn[[j]], {j, 100}],
           {i, n}]];
```

■ Calculate Standard Error for both ATC and Science

```
In[12]:= (* set number of ramps for estimate *)
n = 10000;
(* start do loop for the four photodetector pairs *)
Do[
  Print["DETECTOR PAIR = ", j];
  (* call module that generates the signal *)
  f[ iSumPair[[j]], noiseElectronics[[j]], n];
  (* calc pointing noise for n ramps for 68 ms & 100 ms *)
  (* ATC 68 ms *)
  pN68 = Table[0.0, {i, n}];
  Do[
   sAv0 = Sum[s[[i, k]], {k, 20}] / 20; sAv1 = Sum[s[[i, k + 56]], {k, 20}] / 20;
   pN68[[i]] = (sAv1 - sAv0) / ((t[[57]] - t[[1]]) dSndt[[j]] iSumPair[[j]]),
   {i, n}];
  pN68Av = Sum[pN68[[i]], {i, n}] / n;
  pN68RMS = Sqrt[Sum[(pN68[[i]] - pN68Av)^2, {i, n}] / (n - 1)];
  Print["  ATC at 68 ms"];
  Print["    Average Angle (arcsec) = ", pN68Av];
  Print["    Std Error (arcsec) = ", pN68RMS];
  (* Science 100 ms *)
  pN100 = Table[0.0, {i, n}];
  Do[
   dat = Table[{t[[k]], s[[i, k]]}, {k, 100}];
   out = Fit[dat, {1, x}, x];
   pN100[[i]] = out[[2, 1]] / (iSumPair[[j]] dSndt[[j]]), {i, n}];
  pN100Av = Sum[pN100[[i]], {i, n}] / n;
  pN100RMS = Sqrt[Sum[(pN100[[i]] - pN100Av)^2, {i, n}] / (n - 1)];
  Print["  Science at 100 ms"];
  Print["    Average Angle (arcsec) = ", pN100Av];
  Print["    Std Error (arcsec) = ", pN100RMS],
  {j, 4}];
```

```
DETECTOR PAIR = 1

  ATC at 68 ms

    Average Angle (arcsec) = -0.00109976

    Std Error (arcsec) = 0.0925358
```

```
  Science at 100 ms

    Average Angle (arcsec) = -0.00103277

    Std Error (arcsec) = 0.063957

DETECTOR PAIR = 2

  ATC at 68 ms

    Average Angle (arcsec) = 0.0000402718

    Std Error (arcsec) = 0.0479738

  Science at 100 ms

    Average Angle (arcsec) = -0.000155106

    Std Error (arcsec) = 0.0349721

DETECTOR PAIR = 3

  ATC at 68 ms

    Average Angle (arcsec) = 0.000217157

    Std Error (arcsec) = 0.0723076

  Science at 100 ms

    Average Angle (arcsec) = 0.000288765

    Std Error (arcsec) = 0.0514658

DETECTOR PAIR = 4

  ATC at 68 ms

    Average Angle (arcsec) = -0.00123955

    Std Error (arcsec) = 0.0724005

  Science at 100 ms

    Average Angle (arcsec) = -0.00139837

    Std Error (arcsec) = 0.050747
```