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W. W. Hansen Experimental Physics Laboratory

STANFORD UNIVERSITY
STANFORD, CALIFORNIA 94305 - 4085

Gravity Probe B Relativity Mission

DATA REDUCTION AND SIMULATION ALGORITHMS

END TO END TEST #5: SUMMARY

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G. M. Keiser, Chief Scientist — Date: 4/9/01

M.I. Heifetz — Date: 04/07/2001

A.S. Silbergleit — Date: 04/09/01

A. S. Krechetov — Date: 04/09/01

I. V.Mandel — Date: 4/9/01

ITAR Assessment Performed (Tom Langenstein) **ITAR Control Req'd?** ☐ Yes ☑ No

Test Description

The End-to-End Test #5 of the GP-B Data Reduction and Simulation Algorithms has been completed by December 15, 2000, as scheduled.

The aim of the test was to design and check the performance of the analytical tools for calibration and verification using magnetometer and thermometer signals. That requires the design and implementation of the corresponding Generator, Preprocessing and Analysis algorithms, as well as proper modification of the Generator Manager G0, and, most importantly, the SQUID signal generator G3 (see the usual charts at the end of the document).

Due to a large amount of work, some replanning has been done; as a result, handling of the temperature signals was moved to a separate Test 5b, so only the magnetometer signals have been generated (using the newly designed generator algorithm G8) and treated in the Test 5. The format of all other data was the same as in the Tests 1 and 4, with now trapped flux and no telescope signal (perfect pointing) involved. As usual, 12 hrs of data have been generated, preprocessed, and analyzed. Here is the list of algorithms involved in the test (compare to the similar list in [1]):

Generator algorithms: G0 (manager program, **new version**); G1 (orbit generator); G11, G10, G15, G4, G3 **(new version)** (all for SQUID LF signal); G12, G13, G14 (GPS/SRE signals); **G8 (magnetometer signal generator)**

Preprocessing algorithms: PM0, PM1 (manager programs); P11 (SQUID signal preprocessing); P12 (roll phase data preprocessing); P0, P1, P1A, P5, P2-P3, P4, P6 (GPS/SRE data preprocessing and aberration computation); **P17 (magnetometer signal preprocessing)**

Analysis algorithms: A0 (manager program); **CV1 (short-term analysis with magnetometer signals)**

The goal of verification and calibration is, by using additional sensors, such as magnetometers, thermometers, etc., to find out whether the corresponding external factors influence the SQUID signal, and correct the results, if needed. In other words, we need to find the correlation between the signal from such a sensor, and the SQUID signal, or to put an upper bound on the correlation which would show that the influence is below the desired measurement accuracy.

In particular, there will be 4 magnetometers on the GP-B S/C whose 12 scalar signals (3 cartesian components of external magnetic field from each magnetometer) will be sent down as 12 telemetry monitors. The objective is thus, using this telemetry, to detect an external field leakage into the SQUID signal, that is, to place some bounds on the attenuation coefficients.

Same as in all the previous tests, one team generated the data, the other team preprocessed and analyzed them in a "blind" manner, not knowing the true values of parameters they were to estimate. The values of parameters not estimated in the analysis that were known to the analysis team are given in Table 1. These parameters remained fixed in all the **three runs** of the test which have been carried out; the same is true for the standard for our 12 hr analysis 9 states (SQUID scale factor, etc., see [1]). This was done intentionally, to check how the external magnetic field leakage affects the

basic state estimates. So the only parameters that were changed from run to run were attenuation coefficients (att1, att2, …, att12 in Tables 2-4); they were all set to zero (no external field leakage) in the sample run 1, which thus was just another repetition of the end-to-end
Test 1. The difference between runs 2 and 3 was not only in the values of the attenuation coefficients (which were close to or larger than required), but also in the number of independent magnetometer signals (3 in run 2 and 6 in run 3) available, as explained in detail in the next section.

Methods of Simulation and Analysis

The main source of an external magnetic field in the GP-B S/C environment is magnetic field of the Earth. If all magnetometers measure the field produced by this one common source only, then the time signatures of their respective signals are identical, up to the measurement noise (which is rather small). This is true because the Earth's magnetic field gradient at the S/C size scale is negligible to the desired accuracy. In other words, in such case only 3 orthogonal components of the measured field out of 12 available signals from 4 magnetometers will be linearly independent functions of time.

However, there might be some additional local sources of magnetic field; one example is given by the S/C magnetic torquers, when thy are turned on. In this latter case up to 12 magnetometer signals might be linearly independent. These different possibilities should be reflected in the methods of analysis (described below), and their implementation should be provided by magnetometer signal generator algorithm. The analysis, therefore, should include determination of the number of independent magnetometer signals, and only then estimation of their amplitudes as present in the SQUID readout.

a) Generation of the magnetometer signals (see [2]).

The first of the described situations was implemented in run 2 of Test 5: Earth's magnetic field was assumed the only external source, and, accordingly, only 3 independent signals were generated. Their linear combination with the corresponding nonzero attenuation coefficients was added to the SQUID signal.

In the run 3, 6 signals have been made linearly independent, and their combination with 6 nonzero attenuation coefficients has been put into the SQUID signal. In both cases, all 12 magnetometer signals were preprocessed and then properly analyzed.

b) Analysis (see [3])

Combined analysis of the SQUID readout science signal and magnetometer signals is organized in three steps.

Step 1: Determination of all linearly independent magnetometer measurements.

Originally there are available 12 signals from four Payload Magnetometers: B_{1x}, B_{1y}, B_{1z}, B_{2x}, B_{2y}, B_{2z} , B_{3x}, B_{3y}, B_{3z} , B_{4x}, B_{4y}, B_{4z} . (each of them is Nx1 vector). The special iterative procedure performs determination of the maximum number (L) of the linearly independent magnetometer signals. The "full" matrix B (N x 12), that includes all twelve magnetometer signals (vectors), is formed, observability matrix $B_{ob} = B^T * B$ (12x12) is created, and eigenvalues with corresponding eigenvectors are calculated.

Step 2: Selection of the "most observable" linear combinations of the magnetometer signals.

Upon analyzing all twelve eigenvalues of the observability matrix B_{ob}, we choose L largest eigenvalues (e.g. L=rank (B_{ob})), then find the corresponding eigenvectors V=[`VectorsChosen`] and perform the coordinate transformation to the main axes (M = B*V). Eigenvectors V define the observable subspace and the transformation to the main axes of this subspace defines L functions $M_1(t),\ldots, M_L(t)$ that will be used in the two-step estimator as the corresponding elements of the meauserement matrix H.

Step 3: Estimation.

For the the structure of the SQUID signal we explore now the following model:

$$\begin{aligned} z(t_k) = C_{tot}\ \{&[NS_0 + R_{NS}\,(t_k - t_1) - AberNS(t_k)] \cos\,(Roll(t_k) + \delta\varphi) + \\ &[EW_0 + R_{EW}\,(t_k - t_1) - AberEW(t_k)] \sin\,(Roll(t_k) + \delta\varphi) + \\ &C_{tot}\, M \cos\,[\omega_{cal} t_k + \varphi_{cal}] + \sum_{i=1}^{L} \alpha_i M_i + b + noise, \end{aligned} \qquad (1)$$

where parameters C_{tot}, $\delta\varphi$, R_{NS}, R_{EW}, NS_0, EW_0, b, M, φ_{cal}, $\alpha_1,\ldots \alpha_L$ constitute the above described state-vector; AberNS(t), AberEW(t), Roll(t) are known (measured) functions of time; ω_{cal} is the known frequency of the SQUID calibration signal; $\alpha_{1.....L}$ – are the attenuation coefficients; $M_{1....L}$– are the linearly independent combinations of the magnetometer signals (see Step 2).

The difference between the model (1) and the baseline model (used in the previous End-to-End Tests) is that the former includes the magnetometer signals with corresponding attenuation coefficients.

Two-Step Batch Filter algorithm in this case consists of the following steps:

1. Introducing the first-step variables **y**:

$y_1 = C_{tot} \cos(\delta\varphi)$; $y_2 = C_{tot} \sin(\delta\varphi)$;

$y_3 = C_{tot}\,[R_{NS} \cos(\delta\varphi) + R_{EW} \sin(\delta\varphi)]$; $y_4 = C_{tot}\,[-R_{NS} \sin(\delta\varphi) + R_{EW} \cos(\delta\varphi)]$;

$y_5 = C_{tot}\,[NS_0 \cos(\delta\varphi) + EW_0 \sin(\delta\varphi)]$; $y_6 = C_{tot}\,[-NS_0 \sin(\delta\varphi) + EW_0 \cos(\delta\varphi)]$;

$y_7 = b$; $y_8 = C_{tot}\, M \cos(\varphi_{cal})$; $y_9 = -C_{tot}\, M \sin(\varphi_{cal})$;

$y_{10\ldots L+9} = \alpha_{1..L}$ (2)

For these variables the measurement equations (1)) becomes linear:

$$z_k = h_k\, \mathbf{y} + noise;$$

where h_k are (1x 9) matrices of the following structure:

h_k=[-AberNS$_k$ cos(Roll$_k$) - AberEW$_k$ sin(Roll$_k$); -AberEW$_k$ cos(Roll$_k$)+AberEW$_k$ sin(Roll$_k$);
t_k cos(Roll$_k$); t_k sin(Roll$_k$); cos(Roll$_k$); sin(Roll$_k$); 1; cos($\omega_{cal} t_k$); sin($\omega_{cal} t_k$) $M_1,\ldots,M_L$];

2. Form the Nx(9+L) batch measurement matrices (N is a number of measurements):

$$H = [h_1 , h_2 , \ldots , h_N]^T ;$$

and the batch measurement vector

$$\mathbf{Z} = [z_1, z_2, \ldots , z_N]^T.$$

3. Least-square batch (B) estimator:
first-step state-vector estimate: $\mathbf{y = (H^T H)^{-1} H^T Z;}$

4. Transformation to the original variables **x**:

C_{tot}= sqrt($y_1^2 + y_2^2$) ;
$\delta\varphi$ = arctang(y_2/y_1) if ($y_1 > 0$ and $y_2 > 0$) or ($y_1 > 0$ and $y_2 < 0$);
$\delta\varphi$ =π + arctang(y_2/y_1) if ($y_1 > 0$ and $y_2 > 0$) or ($y_1 > 0$ and $y_2 < 0$);
(Matlab standard function atan2(y_1,y_2) performs this calculation).
$R_{NS} = (y_3\, y_1 - y_4\, y_2)/ C_g^2$; $R_{EW} = (y_3\, y_2 + y_4\, y_1)/ C_g^2$;
$NS_0 = (y_5\, y_1 - y_6\, y_2)/ C_g^2$; $EW_0 = (y_5\, y_2 + y_6\, y_1)/ C_g^2$
$b = y_7$;
M = sqrt($y_8^2 + y_9^2$);
φ_{cal} = -atan2(y_8/y_9);
$\alpha_{1..L} = y_{10\ldots L+9}$;

5. Post-fit residuals: $\mathbf{ResX = Z - Z_{model}(x)}$.
Vector $\mathbf{Z_{model}}$ (**x**) is calculated as the right-hand side of the equation (1) at the corresponding estimates **x.**
6. The standard deviation (σ_{noise}) of the measurement noise is calculated as the residuals' standard deviation:

$$\sigma_{noise} = \text{std (} \mathbf{ResX} \text{);} \quad \text{(standard Matlab function)}$$

7. Gradient (**dy/dx**) is calculated explicitly according to the transformation (2).

8. Covariance matrix **Px** for the original **x** state-vector:

$$\mathbf{Px = [(dy/dx)^T (H^T H) (dy/dx)]^{-1}} \sigma_{noise}.$$

As it was described above the short-term filter, designed as a Matlab function, produces the values of the state-vector estimates, covariance matrix, and the vector of residuals.

Test Results

One of the important results of end-to-end tests is a comparison of the actual test results with the expected test results. The expected results for the baseline data analysis can be obtained analytically from [4].

First, the standard deviation of the SQUID noise may be compared with its estimate value based on the post-fit residuals. Second, the covariance matrix may be compared with the analytical solution of the covariance matrix, and the standard errors derived from each of these covariance matrices may be calculated. Finally, the estimated values of the nine states may be compared with the true values for each of these states. These estimated values should agree with the true values to within several standard deviations. Each of these comparisons serves as a check on the data processing.

For the further clarification we want to notice that the overall gyroscope scale factor, that we denote here as C_{tot}, and in the "Analytical Solution…"[4] is refered as C_g, actually is a product of several partial scale-factors (which reflects the more realistic structure of the SQUID signal generator):

$$C_{tot} = C_g * C_s * C_{g0},$$

where C_g(d-less), C_s(d-less), and C_{g0}(Volts/arcsec) are constant parameters assigned in each run but only C_g is estimated but the Data Analysis filter.

The <u>expected</u> standard deviation (in volts) of the post fit residuals is given by

$$\sigma = C_S C_g \sqrt{\frac{G}{2\ \Delta t}}$$

where C_S and C_g are components of the total scale factor, G is the <u>single-sided</u> autospectral density of the SQUID noise (in volts^2/Hz), and Δt is the time interval between the level 2 data points. This relation assumes that the cut-off frequency of the preprocessing anti-aliasing filter lies at the Nyquist frequency of the level 2 data.

For each T hours run (during those intervals where the guide star is valid) , the standard deviation of the post-fit residuals σ_{res} was calculated. Using the relation given above, the expected standard deviation σ based on the SQUID noise was also calculated.

For all three runs: $C_S = 4$, $C_g = 1$, $G = 8.073*10^{-8}$ volts^2/Hz (From PRMU****.txt); $\Delta t = 2$ sec,

Run number	T(hours)	σ_{res} (volts)	σ (volts)
5T01	12	5.5386E-4	5.6826E-4
5T02	12	5.5358E-4	5.6826E-4
5T03	12	6.3908E-4	5.6826E-4

This table shows the good agreement between the predicted value of the residuals standard deviation and its value determined from test results.

The standard errors of each of the nine estimated parameters may be calculated using this standard deviation of the post-fit residuals. Then, these standard error may be compared to the standard error of each of these nine parameters calculated from the analytical solution to the covariance matrix. The analytical expressions for these standard errors in the relative scale factor and the roll phase offset are

$$\frac{\sigma_{C_g}}{C_g} = \frac{\sigma_{C_{tot}}}{C_{tot}} = \sigma_{\delta\varphi} = \frac{2\sigma}{C_{tot}A_O}\sqrt{\frac{1}{N_0 g_1(f_O)}}$$

where C_{tot} is the total scale factor, A_O is the magnitude of the orbital aberration signal, N_0 is the number of data points and $g_1(f_0)$ is a function which depends on the fraction, f_0, of the orbit which the guide star is visible. Assuming that the guide star is visible for 63% of the orbit, we obtain $g_1(0.63) = 0.386$.

The analytical expression for the standard error in the measured drift rate is given as

$$\sigma_R = \frac{\sigma}{C_{tot}T}\sqrt{\frac{24}{N_0}}$$

where the T is the time interval over which the measurements are made. The expected value of the standard error in the relative amplitude and phase of the calibration signal is

$$\frac{\sigma_M}{M} = \sigma_{PhiCal} = \frac{2\sigma}{C_{tot}M}\sqrt{\frac{1}{N_0}}$$

where M is the amplitude of the calibration signal.

Finally, the standard error in the bias is given as

$$\sigma_{bias} = \frac{\sigma}{\sqrt{N_0}}$$

(In the generator, bias is added directly in Volts).

The standard error in the initial misalignment can not be easily compared with the value calculated from the analytical solution to the covariance matrix because in the analytical solution of the covariance matrix, the state was taken to be the average misalignment not the initial misalignment.

These analytical expressions for the standard errors are compared to the test results in the Table 1.

Parameters that were used to calculate values for the analytical expressions were:

A_O=5.0 arcsec,

N_0=13866 points,

Cg0=1.15e-3 Volt/Arcsec,

Cs=4,

Cg=1,

T=12 hours,

M=8 arcsec,

σ = 0.568 mvolts.

The agreement between the analytical value of the standard error and the standard error determined from test results is generally very good.

Parameter	Expected error (Std)	Run 1 (5T01)	Run 2 (5T02)	Run 3 (5T03)
Relative Scale Factor (Cg) (d-less)	6.75E-4	6.41E-4	7.11E-4	8.76E-4
Roll Phase Offset (deg)	3.87E-2	3.67E-2	4.05E-2	5.08E-2
Drift Rate (arcsec/yr)	3.825	3.56	3.6	4.14
Calibration Signal Amplitude (arcsec)	2.10E-3	3.14E-1	3.13E-1	3.63E-1
Calibration Signal Phase (deg)	1.50E-2	2.25	2.25	2.60
Bias (Volts)	4.83E-6	4.7E-6	5.76E-6	6.55E-6

Table 1. Comparison of the standard error calculated analytically using the input parameters with the standard error determined using the post-fit residuals.

Finally the true values of the nine parameteres may be compared with their estimated values. Table 2 shows the true values for each of the parameters, the estimated values, the error in the estimated value (the difference between the estimated value and the true value), and the standard error determined from the test results. Except in exceptional circumstances, the error should be less than several standard errors. For four main paramaters (Drift rates, Scale factor and roll phase offset), this information is also shown graphically in Figures 1 and 2. For comparison, the expected result shown in these figures is included to show the analytical value of the standard error.

Parameters (see Generator Manager GM0 run 5T01)	True Values	Estimated Values	Error	Relative Error	Sigma Experimental
Cg (Scale factor) (d-less)	**1**	1.00052E+00	5.2300E-04	5.23E-04	6.4134E-04
Deltaphideg (Roll phase error) (deg)	**17**	1.70070E+01	7.0000E-03	4.12E-04	3.6707E-02
Rg (NS drift rate) (arcsec/yr)	**9.2**	8.68980E+00	5.1020E-01	5.55E-02	3.5676E+00
Rf (EW drift rate) (marcsec/yr)	**49**	-7.41980E+02	7.9098E+02	1.61E+01	3.5720E+03
Nso (NS init. Misalign.) (arcsec)	**-2**	-1.99530E+00	4.7000E-03	2.35E-03	9.4420E-03
EWo (EW init. Misalign.) (arcsec)	**7.5**	7.50590E+00	5.9000E-03	7.87E-04	9.4059E-03
Bias (volts)	**-1.6**	-1.59999E+00	1.0000E-05	6.25E-06	4.7036E-06
M (Output cal. Sign amp.) (arcsec)	**8**	7.99980E+00	2.0000E-04	2.50E-05	3.1407E-01
Phical (Calibr. Signal phase) (deg)	**19**	1.90128E+01	1.2800E-02	6.74E-04	2.2490E+00

Table 2. Comparison of True and Estimated Values for the Nine Parameters Determined in the Data Analysis.

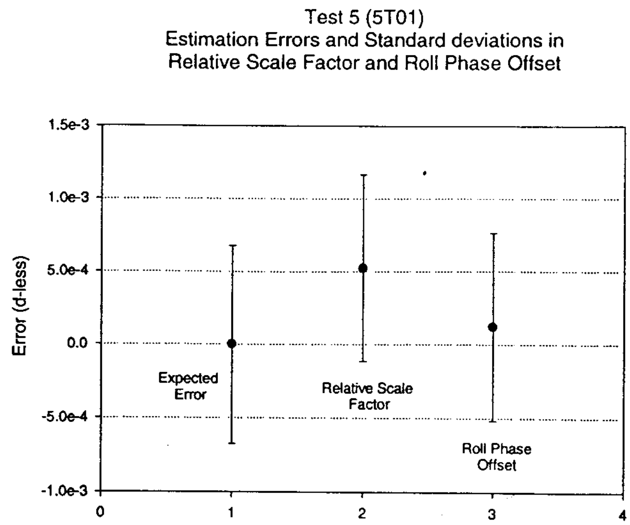

Figure 1. Estimation and Standard deviations in Scale Factor and Roll Phase Offset.

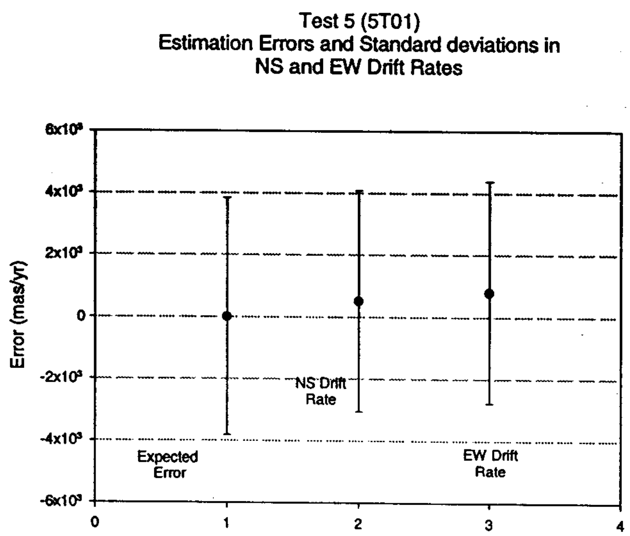

Figure 2. Estimation and Standard deviations in NS and EW Drift Rates

Estimation of the Magnetic Field Attenuation Coefficients

One of main purposes of the End To End Test 5 was to detect how accurately the magnetic field attenuation coefficients can be determined from the SQUID readout signal analysis. It is important to emphasize that all twelve attenuation coefficients (that correspond to the twelve magnetometer signals) can not be detected separately due to their linear dependence. It is only possible to determine the observable L-dim subspace that defines L linear combinations of the magnetometer signals and the corresponding attenuation coefficients.(See eq.(1))

These coefficients ($\alpha_1, \ldots, \alpha_L$) are additional components of the augmented state vector, observable and estimated in the algorithm CV1 (See description above).

Results for Run 2 (5T02)

Originally all twelve attenuation coefficients were assigned to twelve magnetometer signals:

Parameters	att1	att2	att3	att4	att5	att6	att7	att8	att9	att10	att11	att12
Values	**1E-11**	**0**	**0**	**0**	**-1E-11**	**0**	**0**	**0**	**1E-11**	**0**	**0**	**0**

Table 3. True attenuation coeficients (run 5T02)

Analysis of the full observability matrix B_{ob} (performed with the use of the standard Matlab function *rank*) has detected that its rank was equal to six , so we have chosen six largest eigenvalues with the corresponding eigenvectors, presented below in the table 4 .
Based on these eigenvectors we have calculated the corresponding linear combinations of the initial attenuation coefficients att1, ... att12, and determined six equivalent coefficients ($\alpha_1, \ldots, \alpha_6$), presented in the table 5.

Eigenvalue	**Eigenvalue**	**Eigenvalue**	**Eigenvalue**	**Eigenvalue**	**eigenvalue**
-1.2317e-11	**1.0465e-10**	**1.2155e-10**	**2.9385e+03**	**2.9418e+03**	**5.2663e+03**
Eigenvector	**Eigenvector**	**Eigenvector**	**Eigenvector**	**Eigenvector**	**eigenvector**
-5.3928e-04	-1.1742e-01	4.8576e-01	3.8660e-01	3.1708e-01	4.9271e-04
-1.3495e-05	-4.8603e-01	-1.1737e-01	3.1708e-01	-3.8660e-01	-1.9388e-04
4.9491e-01	7.3616e-04	8.0410e-05	-2.5802e-04	-4.6237e-04	5.0000e-01
1.1745e-03	1.1750e-01	-4.8555e-01	3.8660e-01	3.1708e-01	4.9271e-04
4.9692e-03	4.8624e-01	1.1736e-01	3.1708e-01	-3.8660e-01	-1.9388e-04
-4.8087e-01	3.8202e-03	5.8747e-04	-2.5802e-04	-4.6237e-04	5.0000e-01
3.3246e-04	-1.1746e-01	4.8627e-01	3.8660e-01	3.1708e-01	4.9271e-04
-8.6020e-05	-4.8598e-01	-1.1752e-01	3.1708e-01	-3.8660e-01	-1.9388e-04
5.0469e-01	-1.0148e-03	7.1778e-05	-2.5802e-04	-4.6237e-04	5.0000e-01
-9.6769e-04	1.1738e-01	-4.8647e-01	3.8660e-01	3.1708e-01	4.9271e-04
-4.8697e-03	4.8577e-01	1.1752e-01	3.1708e-01	-3.8660e-01	-1.9388e-04
-5.1872e-01	-3.5416e-03	-7.3966e-04	-2.5802e-04	-4.6237e-04	5.0000e-01

Table 4. Eigenvalues and Eigenvectors (run 5T02)

Run 5T02					
Equivalent attenuation Coefficients	Values	Estimated values	Error	Relative error	Sigma
α_1	4.99182E-12	-4.15E-01	4.1500E-01	8.31E+10	8.06E-01
α_2	-6.04675E-12	-2.37E-04	2.3700E-04	3.92E+07	4.51E-04
α_3	3.68472E-12	9.29E-04	9.2900E-04	2.52E+08	1.80E-03
α_4	6.9262E-13	5.50E-12	4.8074E-12	6.94E+00	4.84E-12
α_5	7.03218E-12	1.04E-11	3.3678E-12	4.79E-01	4.84E-12
α_6	5.00687E-12	7.36E-12	2.3531E-12	4.70E-01	3.33E-12

Table 5. Estimation results for Run 2: 3 Independent Magnetometer Signals

Results for Run 3 (5T03)

Originally all twelve attenuation coefficients were assigned to twelve magnetometer signals:

Param.	att1	att2	att3	att4	att5	att6	att7	att8	att9	att10	att11	att12
Values	**2.5E-10**	**1.5E-10**	**2.5E-100**	**-5.E-11**	**5.E-11**	**-5.E-11**	**0**	**0**	**0**	**0**	**0**	**0**

Table 6. True attenuation coeficients (run 5T03)

Analysis of the full observability matrix B_{ob} has detected that its rank =6 , so we have chosen again six largest eigenvalues with corresponding eigenvectors, and those eigenvalues and eigenvectors are presented below in the table 7 .

Eigenvalue	**Eigenvalue**	**Eigenvalue**	**eigenvalue**	**Eigenvalue**	**eigenvalue**
9.02E-02	**1.12E-01**	**1.20E-01**	**6.52E+03**	**6.58E+03**	**1.19E+04**
Eigenvector	**Eigenvector**	**Eigenvector**	**eigenvector**	**Eigenvector**	**eigenvector**
5.99E-03	-5.03E-01	-2.66E-03	2.42E-03	4.97E-01	8.56E-04
5.03E-01	6.00E-03	-6.86E-04	4.97E-01	-2.41E-03	-2.00E-04
7.22E-04	-2.65E-03	5.03E-01	1.91E-04	-8.52E-04	4.97E-01
-5.92E-03	4.97E-01	2.62E-03	2.44E-03	5.03E-01	8.63E-04
-4.97E-01	-5.91E-03	6.83E-04	5.03E-01	-2.45E-03	-2.00E-04
-7.11E-04	2.62E-03	-4.97E-01	2.00E-04	-8.69E-04	5.03E-01
5.99E-03	-5.03E-01	-2.66E-03	2.42E-03	4.97E-01	8.56E-04
5.03E-01	6.00E-03	-6.86E-04	4.97E-01	-2.41E-03	-2.00E-04
7.22E-04	-2.65E-03	5.03E-01	1.91E-04	-8.52E-04	4.97E-01
-5.92E-03	4.97E-01	2.62E-03	2.44E-03	5.03E-01	8.63E-04
-4.97E-01	-5.91E-03	6.83E-04	5.03E-01	-2.45E-03	-2.00E-04
-7.11E-04	2.62E-03	-4.97E-01	2.00E-04	-8.69E-04	5.03E-01

Table 7. Eigenvalues and Eigenvectors (run 5T03)

Based on these eigenvectors we have calculated the corresponding linear combinations of the initial attenuation coefficients **att1, ... att12,** and determined six equivalent coefficients ($\alpha_1,\ldots, \alpha_6$), presented in the table 8.

The estimates of paramaters $\alpha_1,\ldots, \alpha_6$ with their standard deviations are also presented in the table 8.

Run 5T03 Equivalent attenuation Coefficients	**Values**	Estimated Values	Error	Relative error	Sigma
α_1	5.26511E-11	8.04E-10	7.5135E-10	1.43E+01	7.43E-10
α_2	-1.50844E-10	-7.00E-10	5.4916E-10	3.64E+00	6.60E-10
α_3	1.49723E-10	3.90E-10	2.4028E-10	1.60E+00	5.63E-10
α_4	1.00193E-10	1.02E-10	1.8071E-12	1.80E-02	3.47E-12
α_5	9.83342E-11	9.79E-11	4.3423E-13	4.42E-03	3.48E-12
α_6	9.92403E-11	1.01E-10	1.7597E-12	1.77E-02	2.24E-12

Table 8. Estimation results for Run 3: 6 Independent Magnetometer Signal

As we can see from tables 5 and 8, the standard deviations for $\alpha_1, \alpha_2, \alpha_3$ are much larger then standard deviations for α_4, α_5 and α_6. The reason is that the eigenvalues that correspond to $\alpha_1, \alpha_2, \alpha_3$ are several orders of magnitude larger then eigenvalues that correspond to α_4, α_5 and α_6. We can control the way which eigenvalues (eigenvectors) are chosen, so we performed an additional data analysis where only three largest eigenvalues were used.

Eigenvalue	**Eigenvalue**	**eigenvalue**
6.52E+03	**6.58E+03**	**1.19E+04**
Eigenvector	**Eigenvector**	**eigenvector**
2.42E-03	4.97E-01	8.56E-04
4.97E-01	-2.41E-03	-2.00E-04
1.91E-04	-8.52E-04	4.97E-01
2.44E-03	5.03E-01	8.63E-04
5.03E-01	-2.45E-03	-2.00E-04
2.00E-04	-8.69E-04	5.03E-01
2.42E-03	4.97E-01	8.56E-04
4.97E-01	-2.41E-03	-2.00E-04
1.91E-04	-8.52E-04	4.97E-01
2.44E-03	5.03E-01	8.63E-04
5.03E-01	-2.45E-03	-2.00E-04
2.00E-04	-8.69E-04	5.03E-01

Table 9. Eigenvalues and Eigenvectors (run 5T03.1)

Based on these eigenvectors we have calculated the corresponding linear combinations of the initial attenuation coefficients att1, ... att12, and determined three equivalent coefficients ($\alpha_1,\ldots, \alpha_3$), presented in the table 10.

The estimates of paramaters $\alpha_1, \ldots, \alpha_3$ with their standard deviations are also presented in the table 10.

Equivalent attenuation Coefficients	Values	Estimated Values	Error	Relative error	Sigma
α_1	1.00193E-10	1.03E-10	2.8071E-12	2.80E-02	3.45E-12
α_2	9.83342E-11	9.83E-11	3.4230E-14	3.48E-04	3.46E-12
α_3	9.92403E-11	1.01E-10	1.7597E-12	1.77E-02	2.24E-12

Table 10. Estimation results for Run 3.1: 6 Independent Magnetometer Signal

Comparison of the results presented in the tables 8 and 10 shows that the accuracy of the attenuation coefficients estimation is better in the case where only three largest eigenvalues are used.

Results of the performed data analysis demonstrate that

1) The accuracy of the main state vector estimation in the runs 2 and 3 is practically the same as it was obtained when there was no magnetic field in the SQUID signal (run 1);
2) Attenuation coefficients of the order of 10^{-10} are detectable with sufficient accuracy and statistical significance based on 12 hours of data (run3);
3) Attenuation coefficients of the order of 10^{-11} can be hardly detectable based on 12 hours of data. (Estimates are statistically insignificant) (run2). It is very likely that those attenuation coefficients would be detectable with sufficient accuracy given a longer period of observations. (We are going to show that during the "Long-term End-To-End test");
4) The observability and estimation accuracy of the attenuation coefficients can be enhanced by choosing only few (~3) largest eigenvalues of the magnetometers observability matrix.

References

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2. M.I. Heifetz, A.S.Krechetov. Magnetometers Signal Generator (G8). Data Reduction Software Documentation. Stanford University, November, 2000.

3. M.I. Heifetz, A.S.Krechetov. Short-Term Analysis with Magnetometers (CV.1). Data Reduction Software Documentation. Stanford University, January, 2001

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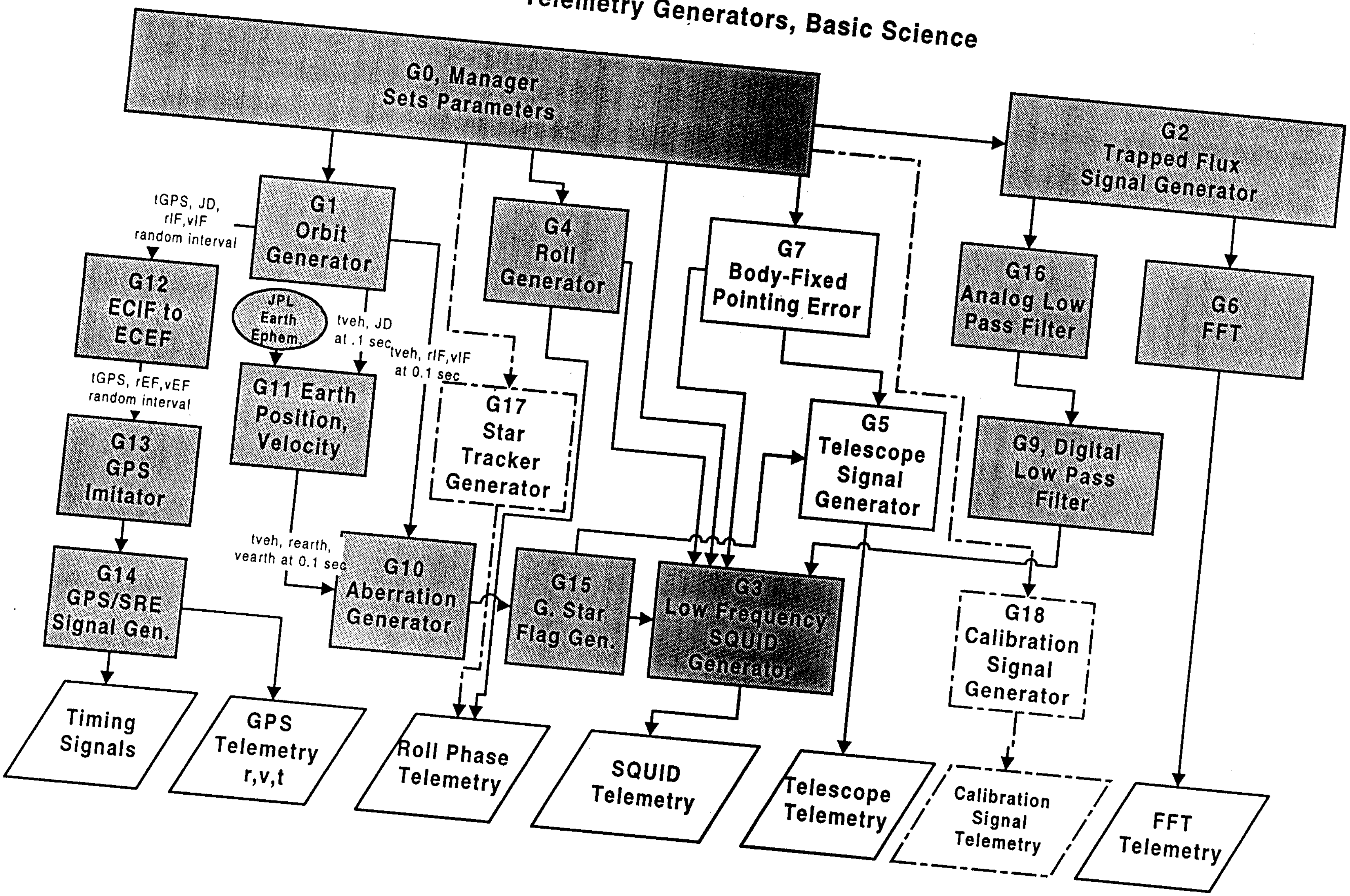
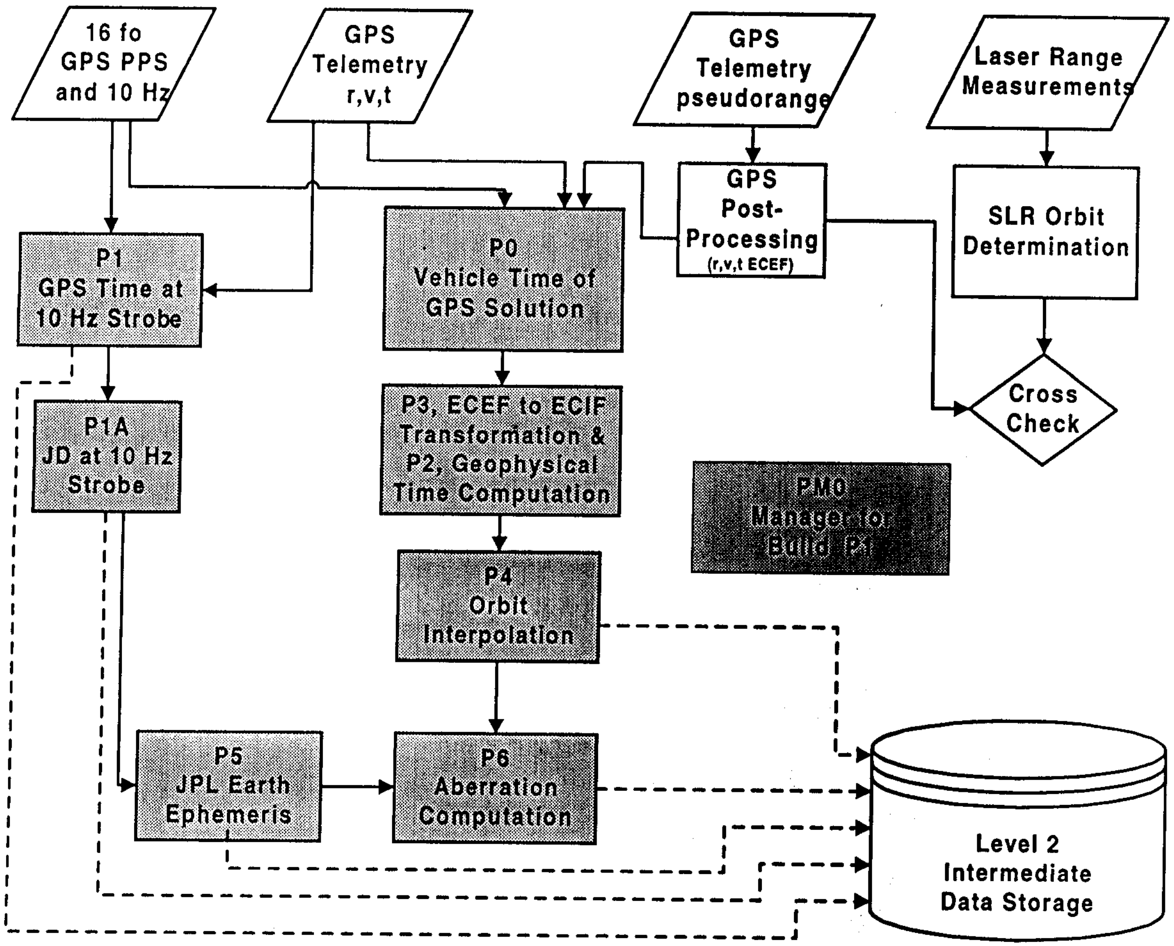
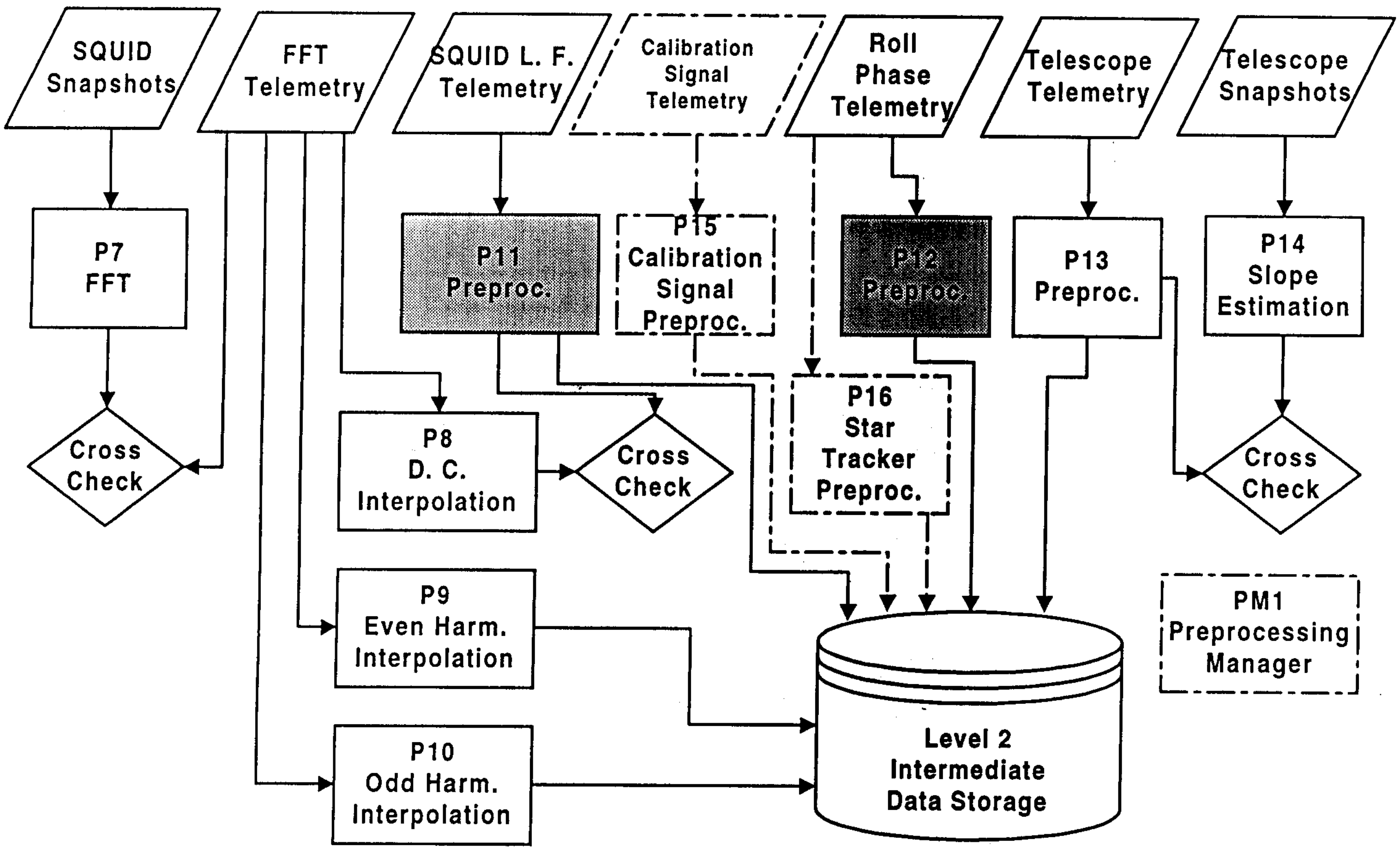
Telemetry Generators, Basic Science



Preprocessing Algorithms - Orbital Information



Preprocessing - SQUID, Roll, Attitude



Analysis - Basic Science

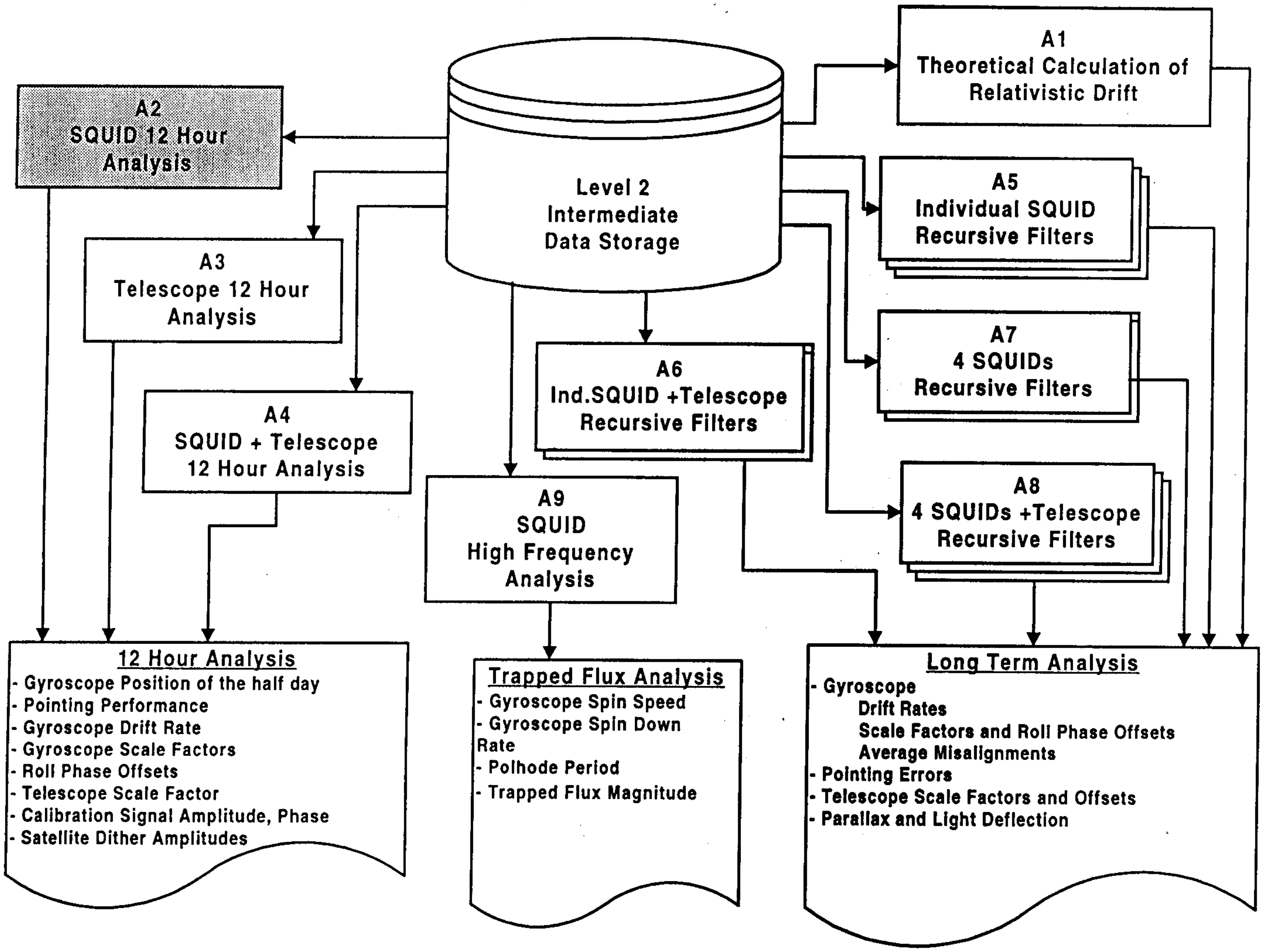


Table 1. Parameters not Estimated in the Analysis

Parameters (see Generator Manager GM0)	Value as set
randTmean (sec)	10
randTrange (sec)	0.2
RandTskip	0
GPSposNoiseSigmaGauss (m)	25
GPSposNoiseSigmaWhite (m)	25
GPSvelNoiseSigmaGauss (m/s)	0.15
GPSvelNoiseSigmaWhite (m/s)	0.15
Cg (Gyro scale factor) (d-less)	1
fspin (Spin freq.) (Hz)	100
Tr (Roll period) (min)	3
wc (Calibr. signal freq.) (rad/sec)	2*pi/62
SQ0 (Ref. Noise) (marcs/sqr(Hz))	100
Cnonlin (nonlinearity coeff.)	0

Table 2. Results for Run 1: no Magnetometer Signals

Parameters (see Generator Manager GM0)	**Values as set**	Estimated values	Error	**Relative error**	Sigma
Cg(Scale factor) (d-less)	**1**	1.00052E+00	5.2300E-04	5.23E-04	6.4134E-04
deltaphideg (Roll phase error) (deg)	**17**	1.70070E+01	7.0000E-03	4.12E-04	3.6707E-02
Rg (NS drift rate) (arcsec/yr)	**9.2**	8.68980E+00	5.1020E-01	5.55E-02	3.5676E+00
Rf (EW drift rate) (marcsec/yr)	**49**	-7.41980E+02	7.9098E+02	1.61E+01	3.5720E+03
NSo (NS init. misalign.) (arcsec)	**-2**	-1.99530E+00	4.7000E-03	2.35E-03	9.4420E-03
EWo (EW init. misalign.) (arcsec)	**7.5**	7.50590E+00	5.9000E-03	7.87E-04	9.4059E-03
Bias (volts)	**-1.6**	-1.59999E+00	1.0000E-05	6.25E-06	4.7036E-06
M (Output cal. Sign amp.) (arcsec)	**8**	7.99980E+00	2.0000E-04	2.50E-05	3.1407E-01
phical (Calibr. Signal phase) (deg)	**19**	1.90128E+01	1.2800E-02	6.74E-04	2.2490E+00

Table 3. Results for Run 2: 3 Independent Magnetometer Signals

Parameters (see Generator Manager GM0)	**Values as set**	Estimated values	Error	Relative error	Sigma
Cg(Scale factor) (d-less)	**1**	1.00062E+00	6.2000E-04	6.20E-04	7.1138E-04
Deltaphideg (Roll phase error) (deg)	**17**	1.70400E+01	4.0000E-02	2.35E-03	4.0528E-02
Rg (NS drift rate) (arcsec/yr)	**9.2**	1.16731E+01	2.4731E+00	2.69E-01	3.6035E+00
Rf (EW drift rate) (marcsec/yr)	**49**	-2.84940E+03	2.8984E+03	5.92E+01	3.5902E+03
NSo (NS init. misalign.) (arcsec)	**-2**	-2.00075E+00	7.5000E-04	3.75E-04	1.4388E-02
EWo (EW init. misalign.) (arcsec)	**7.5**	7.50510E+00	5.1000E-03	6.80E-04	1.4394E-02
Bias (arcsec)	**-1.6**	-1.60002E+00	2.0000E-05	1.25E-05	5.7574E-06
M (Output cal. Sign amp.) (arcsec)	**8**	7.99700E+00	3.0000E-03	3.75E-04	3.1384E-01
Phical (Calibr. Signal phase) (deg)	**19**	1.89920E+01	8.0000E-03	4.21E-04	2.2486E+00
α_1 (equiv.att.coeff, d-less)	**4.99182E-12**	-4.15E-01	4.1500E-01	8.31E+10	8.06E-01
α_2("-")	**-6.04675E-12**	-2.37E-04	2.3700E-04	3.92E+07	4.51E-04
α_3("-")	**3.68472E-12**	9.29E-04	9.2900E-04	2.52E+08	1.80E-03
α_4("-")	**6.9262E-13**	5.50E-12	4.8074E-12	6.94E+00	4.84E-12
α_5("-")	**7.03218E-12**	1.04E-11	3.3678E-12	4.79E-01	4.84E-12
α_6("-")	**5.00687E-12**	7.36E-12	2.3531E-12	4.70E-01	3.33E-12

Table 4. Results for Run 3: 6 Independent Magnetometer Signals

Parameters (see Generator Manager GM0)	**Values as set**	Estimated values	Error	Relative error	Sigma

Cg (Scale factor) (d-less)	**1**	9.99580E-01	4.2000E-04	4.20E-04	8.7463E-04
Deltaphideg (Roll phase error) (deg)	**17**	1.70085E+01	8.5000E-03	5.00E-04	5.0772E-02
Rg (NS drift rate) (arcsec/yr)	**9.2**	1.22575E+01	3.0575E+00	3.32E-01	4.1292E+00
Rf (EW drift rate) (marcsec/yr)	**49**	4.29790E+03	4.2489E+03	8.67E+01	4.1438E+03
NSo (NS init. Misalign.) (arcsec)	**-2**	-2.01870E+00	1.8700E-02	9.35E-03	2.3412E-02
EWo (EW init. Misalign.) (arcsec)	**7.5**	7.47480E+00	2.5200E-02	3.36E-03	2.3503E-02
Bias (arcsec)	**-1.6**	-1.60000E+00	3.0000E-06	1.88E-06	6.5460E-06
M (Output cal. sign amp.) (arcsec)	**8**	8.00027E+00	2.7000E-04	3.38E-05	3.6222E-01
Phical (Calibr. signal phase) (deg)	**19**	1.90124E+01	1.2400E-02	6.53E-04	2.5945E+00
α_1 (equiv.att.coeff, d-less)	**5.26511E-11**	8.04E-10	7.5135E-10	1.43E+01	7.43E-10
α_2("-")	**-1.50844E-10**	-7.00E-10	5.4916E-10	3.64E+00	6.60E-10
α_3("-")	**1.49723E-10**	3.90E-10	2.4028E-10	1.60E+00	5.63E-10
α_4("-")	**1.00193E-10**	1.02E-10	1.8071E-12	1.80E-02	3.47E-12
α_5("-")	**9.83342E-11**	9.79E-11	4.3423E-13	4.42E-03	3.48E-12
α_6("-")	**9.92403E-11**	1.01E-10	1.7597E-12	1.77E-02	2.24E-12