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Gravity Probe B Relativity Mission

NEW ERROR TREE

S0292, Rev. D

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Introduction 1.

This is a completely "live" version of the GP-B New Error Tree (NET) Excel book , which we call New Live Error Tree (NLET), with the input parameters as they are known for gyro #3 before the flight. It consists of two main parts describing non-relativistic (classical) drifts and measurement errors, respectively; it is to accompany doc S0529 providing verification of T002 Req. 1 on the classical drift rate, and also doc. S0** providing verification of T002 Req. 5 on the measurement error.

The second part, which occupies the sheets of the NLET from *"Modeled Errors Theory"* to *"Total Measurument.Errors"*, contains the sequence of analitical calculations to determine covariance matrices for "modeled" and "unmodeled" cases for the general GP-B data reduction problem. These calculations are based on the analytical approach developed in "Analytical Solution for the Gravity Probe B Covariance Matrix (S0351), and "Gravity Probe B Data Reduction: Analysis of Unmodeled Errors"(S0354). The major simplifying assumptions used in it are the absence of the state vector process noise and the telescope part of the signal (the latter basicly can also be included). Analytical formalism is described in sheets "Modeled Errors Theory" and "Unodeled Errors Theory". You can input and change the relevant parameters (listed in sheet *"Measur. Errors Input Par."*) and see how they influence the measurement errors (standard deviations for the state vector estimates) shown in the sheet *"Measur.Errors"*; the relativistic drift errors are picked up from there to the sheet *"Main"* of the NLET to contribute to the overall error of the experiment.

The first part of the NLET, dealing with the classical torques acting on the GP-B gyros, **is, of course, also a "live" one,** and thus can serve as a useful tool for various pertinent purposes. This means that one only has to specify (type in) the numbers into the sheets *"Constants", Parameters", "Coefficients", "Preloads"* and a few places in *"Accelerations"* and *Sheet 10,* and all the remaining numbers, including the values of classical drift rates from all kind of torques, are immediately calculated by the spreadsheet according to the formulas which are derived and explained in other accompanying documents S0263, S0264 and S0269. In particular, accelerations acting on the gyros are analyzed in S0269 (and a list of most significant of them is also included in S0263), suitable expressions for support dependent torques are derived in S0263, and an account of support independent torques is given in S0264.

Let us explain in more detail how this first part is organized and how it works.

In the table *"Constants"* the quantities are collected which are real fundamental constants, such as the speed of light, or have a pretty well specified number in the GP-B project, such as the rotor's radius; their numerical values can be changed, in principle, causing automatically respective changes in the NLET as a whole, but such changes are neither typically needed nor expected. Still, any variation of any constant affects the values given in all the dependent cells, that is, in all the cells of the NLET spreadsheet where this constant is used.

All the rest quantities listed in *"Parameters", "Coefficients",* etc., **have two numerical values each, which correspond to what is called the `worst' and `plausible' cases,** respectively. The worst case values are either taken directly from the requirements, or derived from them, or are the result of our estimate of what the worst case may be in reality for this particular parameter; these estimates are based on the collective experience of the GP-B team. The plausible case values are either those which have already been experimentally achieved, or are expected to be achieved with the "99% certainty". Some of these plausible values just differ several times from the corresponding worst case ones: for instance, if we are speaking about transient accelerations, we expect them to be several times smaller in reality (plausible case) than in the worst case we can imagine.

We need to specially explain the numbers in the table *"Coefficients"*, where the values of fifteen major torque coefficients are given. The torque coefficients determine, together with the voltages (i. e., accelerations and preloads), all the support dependent torques on the gyro; the definitions and expressions of these coefficients in terms of the spherical harmonics of the rotor shape are found in expressions of these coefficients in terms of the spherical harmonics of the rotor shape are found in S0263. Evidently, their values differ for different rotors, because they are rotor shape dependent. The values of the torque coefficients used in the NLET for the flight rotors in the plausible case are calculated as follows. First, the range of every torque coefficient for an uncoated rotor is established by averaging the ranges of the two quartz rotors from doc. S0289, tables 3.1.a and 3.1.b. Second, the contribution of the mass unbalance (see Table 3 of doc S0529) is added to the maxima of the eight torque coefficients depending on the odd harmonics of the rotor shape, according to formula (4.7) from S0289. Third, the contribution of the centrifugal asphericity (with the proper value of the spin speed, Table 3, S0529) is added to the maxima of the remaining seven torque coefficients depending on the even harmonics of the rotor shape, according to formula (4.9) from S0289. In the latter case the addition was done to either lower or upper bound of the range so that the resulting coefficient has the maximum absolute value.

The numbers for the torque coefficients obtained as described above are really the largest possible for a given rotor, up to its shape and mass unbalance measurement error. Moreover, these values are extremely improbable to be achieved in reality, because they occur only when the spin axis stays all the time at one particular direction in the rotor's. In addition, for the eight odd harmonics coefficients the direction of the mass unbalance must coincide with the spin axis as well. This means that the used plausible values are, in fact, some upper bounds of the torque coefficients expected in reality. For this reason, allowing for some measurement error, we calculate the worst case torque coefficients for a given rotor by a 15% increase of the plausible case ones.

All in all, the plausible case shows what we expect to face in the real experiment with the "99% certainty", while the worst case represents the upper bound of errors: they definitely are not going to be worse if everything is all right.

Naturally, the things in the NLET book are arranged so that **a change of a worst case value automatically implies variations in the dependent worst case values (of classical drift rates, in particular) ONLY, not influencing the plausible case at all, and vice versa.**

The *"Main"* sheet shows the top portion of the error tree with the contributions from classical torques, measurement errors and the proper motion of the Guide Star. From there on, **four values of the drift rates** (in *marcsec/year*) **are given in every box, namely, the worst and plausible case values for both EW (motional) and NS (geodetic) drifts.**

The part of the NLET we are discussing continues in the *"Sheet 0"* where our classification of all classical torques is depicted, along with the contributions of each group of torques into gyro's drift; the contributions are represented graphically in the next *"Sh.0 Chart"*. **The two major groups are Support Dependent and Support Independent torques.**

Inside the **support dependent** group, we recognize **acceleration dependent** (sheets 1 - 3) and **preload dependent** (sheets 4 - 6) torques, which are subsequently subdivided regarding the rotor's position in which they are produced - **nominal** (no misalignment and miscentering), **misaligned** (proportional to misalignment only), and **miscentered** (proportional to miscentering only), see S0263 for details. Therefore there are exactly 6 =2*3 sheets with support dependent torques, from *"Sheet 1"* (acceleration dependent, nominal position torques) to *"Sheet 6"* (preload dependent, miscentered position torques). In each of the sheets torques are finally sorted out with respect to their physical source.

Support independent torques are primarily classified as **housing fixed** (sheets 7 - 9) and **inertially fixed** (*"Sheet 10"*), with the former again sorted relative to the **nominal, misaligned,** and **miscentered** rotor's position, which accounts for the number of sheets (3) which this group of torques occupies. So far, no housing fixed, miscentered position torques have been discovered, therefore *"Sheet 9"* for the housing fixed, miscentered position torques remains blank.

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All those ten sheets with the torques are filled in in the following way. There are **the lowest level boxes** showing drift rates produced by a torque belonging to a particular group, specified `all the way down from the top', which is caused by a particular physical source. **The drift rates in each of the four respective cells (EW worst, EW plaus., NS worst, NS plaus.) are calculated by the proper formulas from either S0263 or S0264 in which only either the worst or plausible case values from** *"Parameters", "Coefficients",* etc. **are used**.

Example: *Sheet 1, Box 1.1.1.* , Centrifugal Force. The drift rates shown there are generated by a support dependent, acceleration dependent torque due to the centrifugal acceleration perpendicular to the roll axis, which torque appears because of the variation of the roll phase at roll and is produced by D.C. voltages. The last two characterizations are found in *Box 1.1*. uniting a group of three lowest level boxes with the torques which differ only by a physical source of acceleration; the perpendicularity of acceleration to the roll axis is a signature of a larger group of torques shown in Box 1. uniting them. The general formula for such torques is in p.22 of S0263, under Sheet 1., 1.; the expression for the roll averaged voltages is in p.30 of the same document, group a). Finally, the (worst or plausible case) values of centrifugal accelerations are taken from the appropriate cells of the sheet "Accelerations" in which, in their turn, they are computed according to the expressions given in S0269 and S0263, p.32, group 3.

Clearly, the separate sheets for the accelerations and effective accelerations squared (labeled "Squared") are expedient because they are used a large number of times throughout the sheets with

Introduction 2

As soon as the lowest level boxes are filled which, to say that again, occurs automatically after all the data in "Constants", Parameters", etc. are in place, all the upper level boxes in all the sheets, all the way up to those in "Main", also get the numbers in them automatically, by means of the built-in procedure borrowed from the old Error Tree. The assumption is that different torques act independently, so the squares of corresponding drift rates, say, EW, worst case, from all the boxes of a group at any given level are summed up, and the square root of this sum is inserted as a value EW, worst case, into the next level box to which the group is attached.

As an **example,** let us again consider *Sheet 1, Boxes 1.1.1. - 1.1.3.*, which are attached to the upper level Box 1.1. (note that the box level is reflected in its number, to which a new digit is added when one goes a level down). The number in a cell in *Box 1.1.* is the square root of the sum of squares of numbers standing in the proper cells of *Boxes 1.1.1., 1.1.2., 1.1.3.* And so on: the numbers in *Box 1.* are the square roots of the sum of squares of the corresponding numbers from *Boxes 1.1., 1.2., 1.3.,* and finally the contributions of all the support dependent, acceleration dependent, nominal position torques in the uppermost box of *Sheet 1* are the respective square roots of sums of squared drift rates shown in *Boxes 1.* and *2.* This uppermost box of *Sheet 1*, in its turn, is nothing else as the lowest level *Box 1.1.1.1.1.* of *Sheet 0,* etc.

After the sheets with torques, the dominant torque drift rates are listed in *"TopTorques"* and plotted in *"TopEW"*, *"TopNS"* sheets. The values there change automatically with any changes in the data which affect the torques listed, but no new large torques are automatically added if appear as a result of such changes (the selection is fixed).

To conclude, we note that a user of the NLET can avoid reading and understanding all the above. Just type your own numbers in *"Parameters", "Coefficients",* etc., and look at the results! The only advice is to use a separate copy of the file, keeping this original one as an official standard.

CONSTANTS

Docu	Req. #	Parameter Name	Value	Units	Variable Name
1	2	3	4	5	6
		Fundamental Constants			
		Speed of light, c	3.00E+08	m/sec	c_light
		Gravitational constant, G	6.67E-11	Nm^2/kg^2	G
		Boltzman constant, K_B	1.38E-23	joule/K	Kb
		Dielectr. permitivity of vacuum,ε_0	8.85E-12	$(Coulomb)^2/(N\ m^2)$	epsi_0
		Magnetic permeability of vacuum, μ	1.26E-06	Henry/m	mu_0
		Magnetic flux quantum, Φ_{0}.	2.00E-15	Wb	phi_0
		London moment per ω, $\mu_L=M_L/\omega$	3.97E-10	$A\ m^2$ sec	mu_l
		London field per ω, $b_L=B_L/\omega$	1.14E-11	Tesla sec	b_l
		Earth's Gravitational Constant, μ_e	3.99E+14	N*m/kg	mu_e
		Earth Radius, R_e	6.37E+06	m	R_e
		Earth's oblateness J_2	1.08E-03	D less	J_2
		Constants in GPB Design			
T003	1.1	Rotor mass, m	6.30E-02	kg	m
T003	1.1	Rotor moment of inertia, I	9.19E-06	$kg\ m^2$	I
SIA	3.7.1.6.2.2.1	Rotor radius, r	1.91E-02	m	rot_rad
T003	1.6.1	Electrode/rotor gap spacing, d_0	3.20E-05	m	d0
T003	3.4	Rotor temperature, T_R	2.80E+00	K	Tr
T003	1a.3.1	Electrode thickness, d_e	3.20E-06	m	de
T003	2.5	Eletrode to rotor capacitance, C	7.00E-11	farad	cap
		Electrode to ground resistor, Rp	3.00E+04	ohm	Rp
		Electrode conductivity, σ_e	6.50E+10	$(ohm * m)^{-1}$	sigma_e
		Caging rod conductivity, σ_c	2.40E+07	$(ohm * m)^{-1}$	sigma_c
SIA	3.7.1.7.2.1.6	Inductance, L_s+L_R	3.60E-06	Henry	Ls_Lr
		Gain of magnetic feedback system,	1.00E+04	D less	K
SIA	3.7.1.5.2.2.2	Effective number of tuns in pickup lo	3.50E+00	D less	Nt
SIA	3.7.1.4.9	Cylindrical shield radius, R_S	2.79E-02	m	Rs
		Resid. gas decay rate per pressure	1.05E-01	$(sec\ Torr)^{-1}$	lambda
T003	24.2	Orbital Radius, R	7.01E+06	m	R_
		Conversion Factors			
		Picofarads x Radians/sec to farads x marcsec/yr, CAT	6.30E+03	(f marcsec/yr)/ /(pf rad/sec)	CAT
		Acceleration to voltage const., C	5.09E+04	$volts^2/(m/sec^2)$	acc_volt
		Radians/Degree, RD	1.75E-02	deg^{-1}	RD
		Number π	3.1416E+00	Dless	pi
		Conversion factor, marcs to radians	4.8481E-09	marcsec/rad	u

PARAMETERS

Docume	Req. #	Parameter Name	Worst Case Value	Units	Variable Name	Plausible Case Value	Variable Name
1	2	3	4	5	6	7	8
		Rotor and Housing, Mechanical					
T003	1.3.1	Fract. dif. in rotor's mom. of inertia, $\Delta I/I$	2.90E-06	D less	difr	2.90E-06	difc
T003	1.7.1	Distance of gyro from satellite roll axis, d	1.00E-04	m	d_rr	4.80E-05	d_rc
Design		*) Dist. of gyro from unsupport. gyro, d_{pg}	1.65E-01	m	d_pgr	1.65E-01	d_pgc
T003	5.2	Average Misalignment, EW_0	1.00E+03	marcsec	ew0r	1.00E+03	ew0c
T003	5.2	Average Misalignment, NS_0	1.00E+03	marcsec	ns0r	1.00E+03	ns0c
T003	4.1	Gyroscope spin frequency, f	143	Hz	fspinr	143	fspinc
T003	4.1	Spin angular velocity, ω	8.98E+02	rad/sec	spinr	8.98E+02	spinc
T003	6	Residual gas pressure, P	2.00E-10	torr	pressr	2.00E-10	pressc
T003	6 (deriv.)	Spindown time, particles in housing, T_P	1.00E+13	sec	Tpr	1.00E+14	Tpc
Green	B-k, p.576	Mass of particles hitting the rotor, M_p	1.00E-14	kg	mpr	1.00E-15	mpc
SIA	3.7.1.3.4	Number of particles hitting the rotor year	1.00E+08	D less	nyr	1.00E+07	nyc
Green	B-k, p.566	Number of pairs of crystals on surface, n	1.00E+03	D less	ncr	1.00E+06	ncc
T003	2.3.3.1	Fractional miscentering at roll, a1	1.00E-05	D less	Pr	1.00E-05	Pc
T003	2.3.3.1	Random fract. misc. at roll, ar	7.00E-06	D less	arr	7.00E-06	arc
T003	2.3.2	Fractional miscentering, D.C., a0	2.00E-02	D less	delta_r	2.00E-02	delta_c
T003	2.3.3.3.2	Fract. misc. perp to roll axis at roll +/- 2 orbit, a2	1.00E-03	D less	a2r	1.00E-04	a2c
T003	2.3.3.3.1	Difference in perp. fractional miscentering at roll +2 orb. and roll - 2 orb., Δa_2	1.00E-04	D less	da2r	1.00E-05	da2c
T003	2.3.3.3.3	Difference in miscent. phase shifts at roll + 2 orb. and roll - 2 orb., $\Delta\vartheta$	1.00E-02	rad	dnur	1.00E-03	dnuc
T003	2.3.3.3.3	Fractional miscent. parallel to roll axis at 2 orbital, a_3	1.00E-03	D less	a3r	1.00E-04	a3c
GSS	3.4.3.3.3.2	Miscent. phase shift at 2 orbital, ϑ	1.00E-02	rad	nur	1.00E-03	nuc
		Rotor and Housing, Electrical and Magnetic					
S0577		Asymmetry coeff. (patch eff., res. gas) α	1.00E-01	D less	alphpr	1.00E-02	alphpc
T003	1a.3.3	Asymmetry coefficient for electrodes, α_e	1.00E-01	D less	alpher	1.00E-02	alphec
Design		Asymmetry coefficient for caging rod, α_{cr}	1.00E-01	Dless	alphcrr	1.00E-02	alphcrc
Design		Asymmetry in R/Gr. plane capacitance, Δ	50	pf	delta_cr	10	delta_cc
Ph.D.Th. Darling		Patch potential, V	1.00E-01	volt	patchr	1.00E-01	patchc
Ph.D.Th. Darling		Contact potential difference, V_C	2.00E-01	volt	vcr	1.00E-01	vcc
T003	2.2.1	Preload voltage, V_P	2.00E-01	volt	vpr	2.00E-01	vpc
T003	2.5.2.	Rotor charge voltage, V_q	1.50E-02	volt	vqr	1.50E-02	vqc
Rev Sci	Ins.,66/1	Variation in V_q at roll, δV_q	5.00E-06	volt	delta_vqr	5.00E-06	delta_vqc
T003	1a.2	Capacitance change with spin axis					

		orientation, $C_1 = C_2$	1.00E-01	pf/rad	c_12r	5.00E-02	c_12c
T003	1a.2(der.)	Difference in the above, $\Delta C_1 = C_1 - C_2$	1.00E-03	pf/rad	delta_c1r	5.00E-04	delta_c1c
T003	1a.2(der.)	Second deriv. of capacitance in angle,ΔC	5.00E-04	pf/rad^2	delta_c2r	1.00E-04	delta_c2c
GSS	3.4.10.3.6	Bridge sensing voltage, V_S	2.00E-02	volt	vsr	1.35E-02	vsc
GSS	3.4.14.3.9	Variation in V_s at roll, δV_s	5.00E-06	volt	delta_vsr	5.00E-06	delta_vsc
Design		Diff. in V_s between a and b axes, ΔV_S	2.00E-04	volt	bdelta_vsr	1.00E-04	bdelta_vsc
T003	1.5	Trapped field perpendicular to the rotor,	9.00E-06	gauss	hsr	9.00E-07	hsc
T003	8.2	Residual field, B_r	9.00E-10	tesla	brr	9.00E-10	brc
T003	8.3	Variation in B_r at roll, δB_r	1.00E-14	tesla	delta_brr	1.00E-15	delta_brc
T003	1.5	Number of Fluxon-Antifluxon pairs, N_f	1.00E+02	D less	nfr	5.00E+01	nfc
		Satellite, Orbit, Guide Star					
T003	21.2	Satellite roll period, T_{roll}	3	minutes	T_rollr	3	T_rollc
T003	21.2	Roll angular velocity, ω_r	3.49E-02	rad/sec	rollr	3.49E-02	rollc
T003	21.4.1	Amplitude of roll phase variation at roll frequency, $\Delta\Phi$	1.00E-05	radians	delta_phir	5.00E-06	delta_phic
T003	19.3.1	Roll frequency pointing error, $\Delta\Phi_P$	5	marcsec	delta_phipr	5	delta_phipc
T003**)	21.4.2	Fractional roll rate variation, $\delta\omega/\omega_r$	1.00E-05	D less	rrvr	5.00E-06	rrvc
T003	17.1	Total satellite mass, M_S	3.34E+03	kg	M_sr	3.00E+03	M_sc
T003	17.1, 17.4.1	Effective satellite radius, r_S	1.00E+00	m	r_sr	1.50E+00	r_sc
T003	17.4	Satellite gravity gradient, g_G	2.50E-07	sec^{-2}	ggr	5.00E-08	ggc
Estimate		Gravity gradient due to liquid He, g_H	2.50E-10	sec^{-2}	ghr	5.00E-12	ghc
T003	20.4	Vibration frequency, ω_V	1.00E-01	rad/sec	w_vr	1.00E-02	w_vc
T003	20.4(der.)	Vibration amplitude, a_V	5.00E-07	m	a_vr	5.00E-07	a_vc
T003	19.3.1	Pointing system oscillation freq., ω_P	1.00E+01	rad/sec	w_pr	1.00E+00	w_pc
T003	19.3.1	Pointing system angular amplitude, Θ	4.50E-09	rad	thetar	1.00E-09	thetac
T003	24.4	Av. coinclination, I' (eta-av.=0.025 deg)	1.50E-03	rad	inclr	1.50E-03	inclc
T003	24.4	Av. Asc. node, Ω (eta-av.=0.025 deg)	0	rad	omegar	0	omegac
T003	25.1	Declination of guide star, δ	2.9391E-01 (HR 8703)	rad	dlt_r	2.9391E-01 (HR 8703)	dlt_c
T003	25.1	Right ascention of Guide Star, α	5.9910E+00	rad	ra_r	5.9910E+00	ra_c
T003	24.2.	Satellite orbital velocity	8.10E+05	cm/sec	Vorb	8.10E+05	Vorb

*) Gyro #2 unsupported
**) Also SCSE-04, Part 5, Rev. C, p. 23

COEFFICIENTS

Coefficient Name	Value Worst Case	Units	Variable Name	Value Plausible Case	Variable Name
1	2	3	4	5	6
Torque coefficients for nominal and misaligned position: K_1	0.0315	pf/rad	krcoeff1	0.0274	kccoeff1
K_2	-0.1141	pf/rad	krcoeff2	-0.0992	kccoeff2
K_3	-0.1003	pf/rad	krcoeff3	-0.0872	kccoeff3
K_4	0.0258	pf/rad	krcoeff4	0.0224	kccoeff4
K_5	0.0628	pf/rad	krcoeff5	0.0546	kccoeff5
K_6	-0.0315	pf/rad	krcoeff6	-0.0274	kccoeff6
k (due to electrode imperfection)	0.0024	pf/rad	ksmallr	0.0001	ksmallc
Torque coefficient for miscentered position: K_7 (D_1)	0.0344	pf/rad	drcoeff1	0.0299	dccoeff1
K_8 (D_2)	0.0032	pf/rad	drcoeff2	0.0028	dccoeff2
K_9 (D_3)	0.0377	pf/rad	drcoeff3	0.0328	dccoeff3
K_{10} (D_4)	-0.0044	pf/rad	drcoeff4	-0.0038	dccoeff4
K_{11} (D_5)	-0.0117	pf/rad	drcoeff5	-0.0102	dccoeff5
K_{12} (D_6)	0.0197	pf/rad	drcoeff6	0.0171	dccoeff6
K_{13} (D_7)	0.0835	pf/rad	drcoeff7	0.0726	dccoeff7
K_{14} (D_8)	0.0228	pf/rad	drcoeff8	0.0198	dccoeff8
K_{15} (D_9)	-0.0835	pf/rad	drcoeff9	-0.0726	dccoeff9
d (due to electrode imperfection)	0.0001	pf/rad	dsmallr	0.0001	dsmallc

Torque coefficients for the worst case ar(case are calculated basing on requirements from T003, 1.4

PRELOADS

Document	Req. #	Name	Worst Case	Variable name	Plausible Case	Variable name
		PRELOADS				
			Preloads, m/sec^2			
		I. D.C.				
T003	2.2.1	$h_C^{(0)}$	3.14E-06	hc0r	2.10E-06	hc0c
T003	2.2.1, 2.2.2	$h_a^{(0)}+h_b^{(0)}$	6.28E-06	hab0r	4.20E-06	hab0c
S0714		$h_a^{(0)}-h_b^{(0)}$	2.00E-08	habm0r	1.80E-08	habm0c
		h	3.14E-06	h0r	2.10E-06	h0c
		II. Amplitudes at roll				
T003	2.2.4	$h_C^{(1)}$	6.70E-10	hc1r	4.00E-11	hc1c
T003	2.2.4	$h_a^{(1)}+h_b^{(1)}$	1.34E-09	hab1r	8.00E-11	hab1c
T003	2.2.3.1	$h_a^{(1)}-h_b^{(1)}$	6.70E-11	habm1r	2.00E-11	habm1c
		III. Amplitudes at twice roll				
T003	2.2.5	$h_C^{(2)}$	1.96E-06	hc2r	2.36E-08	hc2c
T003	2.2.5	$h_a^{(2)}+h_b^{(2)}$	3.92E-06	hab2r	4.72E-08	hab2c
S0714		$h_a^{(2)}-h_b^{(2)}$	2.00E-10	habm2r	1.57E-10	habm2c
		IV. Transients	**in $(m/sec^2)sec$**			
		$h\Delta t$ (preload)	5.00E-10	hdt_r	1.00E-10	hdt_c
		$\Delta h\Delta t$ (preload difference)	5.00E-12	dhdt_r	1.00E-12	dhdt_c

ACCELERATIONS

ACCELERATIONS (housing-fixed components)				
NOTATIONS: D.C. accelerations - g_X, g_Y, g_Z amplitudes at cosine and sine of roll in x-direction - g_1 g_2 amplitudes at cosine and sine of roll in y-direction - g_3, g_4 SUPERSCRIPT at asymbol denotes the PHYSICAL SOURCE (**EXAMPLES:** $g^E{}_Z$ -- D.C. acceleration due to Earth gravity gradient; $g^S{}_1$, $g^S{}_2$ -- accelerations at roll in x-direction due to satellite gravity gradient, etc.)				
	ACCELERATIONS, in m/sec^2			
	Worst Case	Variable name	Plausible Case	Variable name
1. Gravity gradient, Earth				
$g^E{}_Z$	-9.56E-08	ezr	-9.56E-08	ezc
$g^E{}_1$	1.77E-11	e1r	1.77E-11	e1c
$g^E{}_2$	1.23E-10	e2r	1.23E-10	e2c
$g^E{}_3 (= - g^E{}_2)$	-1.23E-10	e3r	-1.23E-10	e3c
$g^E{}_4 (= g^E{}_1)$	1.77E-11	e4r	1.77E-11	e4c
2. Gravity gradient, Satellite				
$g^S{}_Z$	3.68E-08	szr	9.79E-09	szc
$g^S{}_X$	3.68E-09	sxr	9.79E-10	sxc
$g^S{}_Y$	3.68E-09	syr	9.79E-10	syc
3. Centrifugal acceleration				
$g^C{}_X$	1.22E-07	cxr	5.85E-08	cxc
$g^C{}_Y$	1.22E-07	cyr	5.85E-08	cyc
4. Roll frequency variation and centrifugal force				
$g^{RC}{}_1$	-2.44E-12	rc_1r	-5.85E-13	rc_1c
$g^{RC}{}_2$	-2.44E-12	rc_2r	-5.85E-13	rc_2c
$g^{RC}{}_3$	-2.44E-12	rc_3r	-5.85E-13	rc_3c
$g^{RC}{}_4$	-2.44E-12	rc_4r	-5.85E-13	rc_4c
5. Roll frequency variation and inertial force				
$g^{RI}{}_1$	-1.22E-12	ri1r	-2.92E-13	ri1c
$g^{RI}{}_2$	1.22E-12	ri2r	2.92E-13	ri2c
$g^{RI}{}_3$	1.22E-12	ri3r	2.92E-13	ri3c
$g^{RI}{}_4$	-1.22E-12	ri4r	-2.92E-13	ri4c
6. Acceleration due to nonzero rotor charge				
$g^N{}_Z$	2.79E-09	nzr	5.58E-10	nzc
$g^N{}_X$	2.79E-09	nxr	5.58E-10	nxc
$g^N{}_Y$	2.79E-09	nyr	5.58E-10	nyc
$g^N{}_1$	5.58E-12	n1r	1.12E-12	n1c
$g^N{}_2$	5.58E-12	n2r	1.12E-12	n2c
$g^N{}_3 (= -g^N{}_2)$	-5.58E-12	n3r	-1.12E-12	n3c
$g^N{}_4 (= g^N{}_1)$	5.58E-12	n4r	1.12E-12	n4c
7. Transients, in (m/sec^2) x sec integrated over a year				
$g\ \Delta t$	1.00E-03	gdtr	1.00E-04	gdtc
8. Random, in m/sec^2 (Hz)$^{1/2}$				
g_r	2.00E-08	gr_r	2.00E-09	gr_c

ACCELERATIONS SQUARED

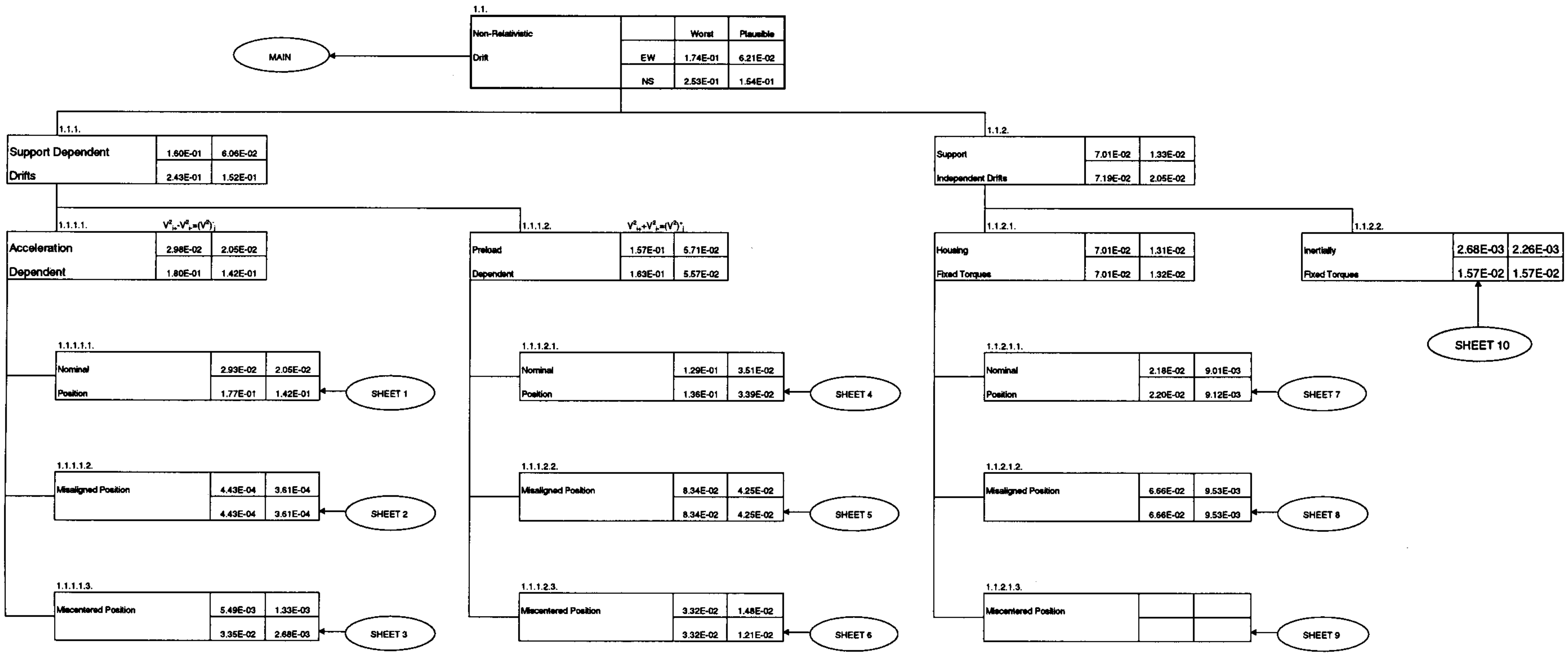
EFFECTIVE ACCELERATIONS SQUARED				
(housing - fixed components, same notations as in 'ACCELERATIONS'; angle brackets mean averaging over a proper period)				
	Accelerations squared, in m^2/sec^4			
	Worst Case	**Variable name**	**Plausible Case**	**Variable name**
1. Gravity gradient, Earth				
$\langle (g^E{}_Z)^2 \rangle$ (D.C.)	2.66E-14	ez2r	2.66E-14	ez2c
$\langle (g^E{}_X)^2 \rangle$ (D.C.)	2.06E-14	ex2r	2.06E-14	ex2c
$\langle (g^E{}_Y)^2 \rangle$ (D.C.)	2.06E-14	ey2r	2.06E-14	ey2c
$\langle g^E{}_Z g^E{}_3 \rangle$ (at roll)	4.97E-18	eze3r	-4.96E-18	eze3c
$\langle g^E{}_Z g^E{}_4 \rangle$ (at roll)	-9.33E-18	eze4r	-9.33E-18	eze4c
$\langle (g^E{}_1)^2 \rangle$ (at twice roll)	2.06E-14	e12r	2.06E-14	e12c
$\langle (g^E{}_4)^2 \rangle$ (at twice roll)	-2.06E-14	e42r	2.06E-14	e42c
2. Due to pointing error				
$\langle (g^P{}_X)^2 \rangle$ (D.C.)	1.50E-16	px2r	7.43E-22	px2c
$\langle (g^P{}_Y)^2 \rangle$ (D.C.)	1.50E-16	py2r	7.43E-22	py2c
3. Due to frequences other than roll				
$\langle (g^O{}_X)^2 \rangle$ (D.C.)	6.25E-18	ox2r	6.25E-22	ox2c
$\langle (g^O{}_Y)^2 \rangle$ (D.C.)	6.25E-18	oy2r	6.25E-22	oy2c

Gravity Probe B Error Tree , gyro#4

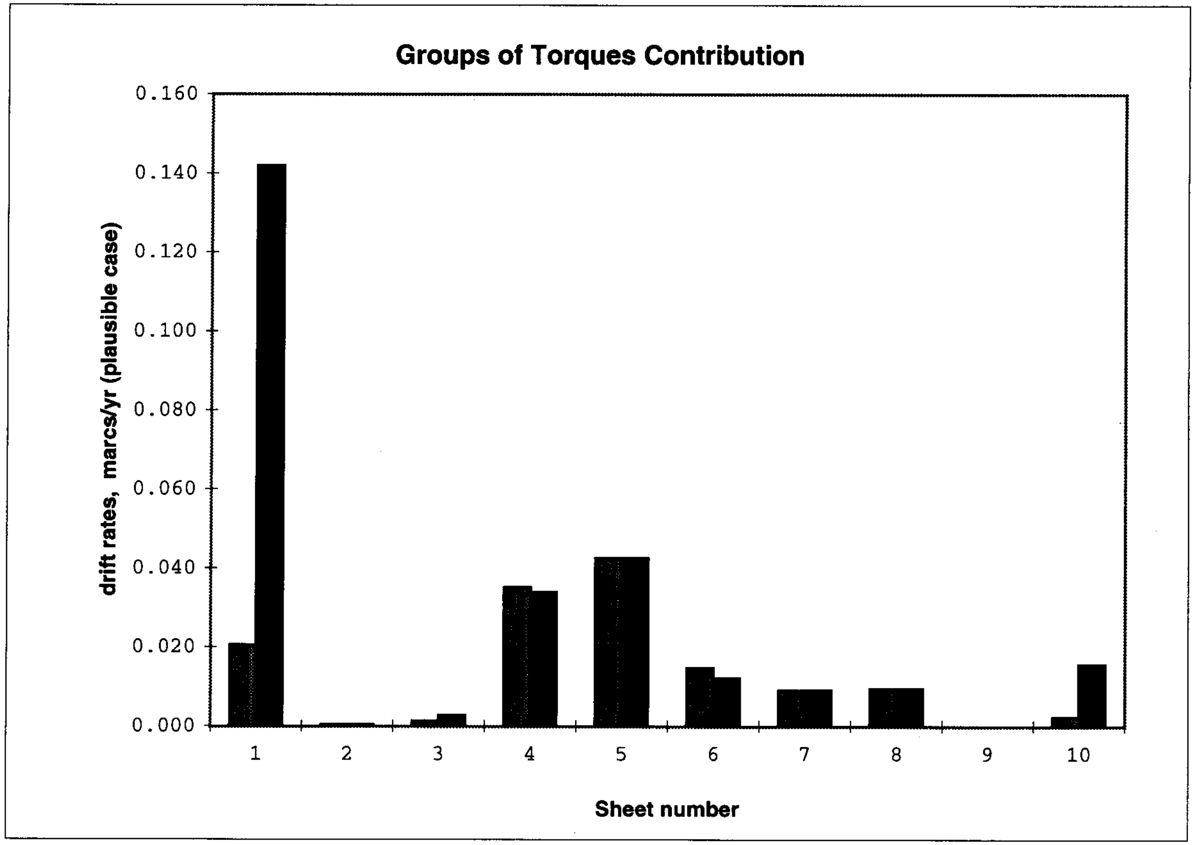
Relativistic Drift Rate marcsec/yr		Worst	Plausible
	Motional (EW)	2.93E-01	2.09E-01
	Geodetic (NS)	3.42E-01	2.46E-01

- 1. Drift Rate Relative to Guide Star: 2.52E-01 | 1.78E-01 / 3.07E-01 | 2.21E-01
  - 1.1. Non-Relativistic Drifts: 1.74E-01 | 6.21E-02 / 2.53E-01 | 1.54E-01 ← SHEET 0
  - 1.2. Measurement Error: 1.82E-01 | 1.66E-01 / 1.74E-01 | 1.58E-01 ← Sheet "Total Measur.Error"
- 2. Proper Motion of Guide Star: 1.50E-01 | 1.10E-01 / 1.50E-01 | 1.10E-01

Non - Relativistic Drift, gyro #4



Sh.0 Chart



Groups of Torques Contribution
0.160
0.140
0.120
0.100
0.080
0.060
0.040
0.020
0.000
drift rates, marcs/yr (plausible case)
1
2
3
4
5
6
7
8
9
10
Sheet number

SHEET 1

Acceleration Dependent, Nominal Position

SHEET 0 ← Nominal Position

Nominal Position		Worst	Plausible
	EW	2.93E-02	2.05E-02
	NS	1.77E-01	1.42E-01

1.

Accel. Perp. to Roll Axis		
	2.93E-02	2.05E-02
	1.77E-01	1.42E-01

1.1. K_2 , k

D.C.Voltages and Variation in Roll Phase at Roll		
	8.73E-04	1.68E-04
	8.73E-04	1.68E-04

1.1.1.

Centrifugal Force		
	8.72E-04	1.68E-04
	-8.72E-04	-1.68E-04

1.1.2.

Gravity Gradient, Satellite		
	2.63E-05	2.82E-06
	-2.63E-05	-2.82E-06

1.1.3.

Rotor Charge		
	2.00E-05	1.61E-06
	-2.00E-05	-1.61E-06

1.2. K_1 , K_4 , k

Voltages at Roll		
	2.69E-02	2.05E-02
	1.77E-01	1.42E-01

1.2.1.

Gravity Gradient, Earth		
	2.54E-02	2.04E-02
	-1.76E-01	-1.42E-01

1.2.2.

Roll Rate Variation and Centrifugal Forc(		
	-3.49E-03	-6.73E-04
	-2.80E-04	-4.90E-05

1.2.3.

Roll Rate Variation and Inertial Force		
	-1.74E-03	-3.37E-04
	1.40E-04	2.45E-05

1.2.4.

Variation in Rotor Charge at Roll		
	7.99E-03	1.29E-03
	-7.99E-03	-1.29E-03

1.3. K1, K4, k

Transient and Random Voltages		
	1.16E-02	5.93E-04
	1.16E-02	5.93E-04

1.3.1.

Transient		
	1.16E-02	5.93E-04
	-1.16E-02	-5.93E-04

1.3.2.

Random		
	-9.40E-10	-8.17E-11
	-9.40E-10	-8.17E-11

SHEET 1

2.

Accel. Parall to	7.33E-04	1.66E-06
Roll Axis	7.33E-04	1.66E-06

2.1. k

D.C.Voltages and Variation in	3.38E-05	6.60E-07
Roll Phase at Roll	3.38E-05	6.60E-07

2.1.1.

Gravity Gradient,	3.15E-05	6.56E-07
Earth	-3.15E-05	-6.56E-07

2.1.2.

Gravity Gradient,	-1.21E-05	-6.72E-08
Satellite	1.21E-05	6.72E-08

2.1.3.

Rotor Charge	-9.19E-07	-3.83E-09
	9.19E-07	3.83E-09

2.2. k

Transient	-7.32E-04	-1.53E-06
Voltages	-7.32E-04	-1.53E-06

SHEET 2

Acceleration Dependent, Misaligned Position

SHEET 0

Misalignment		Worst	Plausible
$(V^2)^-_j\theta$	EW	4.43E-04	3.61E-04
	NS	4.43E-04	3.61E-04

1.

Accel. Parallel to Roll Axis		
	4.43E-04	3.61E-04
	4.43E-04	3.61E-04

1.1. K_6

D.C.Voltages		
	4.43E-04	3.61E-04
	4.43E-04	3.61E-04

1.1.1.

Gravity Gradient,	4.14E-04	3.60E-04
Earth	-4.14E-04	-3.60E-04

1.1.2.

Gravity Gradient,	-1.59E-04	-3.68E-05
Satellite	1.59E-04	3.68E-05

1.1.3.

Rotor Charge	-1.21E-05	-2.10E-06
	1.21E-05	2.10E-06

Acceleration Dependent, Miscentered Position

SHEET 0 ← Miscentering

Miscentering		Worst	Plausible
	EW	5.49E-03	1.33E-03
	NS	3.35E-02	2.68E-03

1. Accel. Perp. to Roll Axis

Accel. Perp. to	4.14E-03	1.08E-03
Roll Axis	4.14E-03	7.29E-04

1.1. D_3, D_9

D.C.Voltages and Miscentering	2.25E-03	9.39E-04
at Roll	1.16E-03	4.84E-04

1.1.1.

Centrifugal	2.25E-03	9.39E-04
Force	1.16E-03	4.84E-04

1.1.1.1.

Misc. at	-1.84E-03	-7.69E-04
Roll	-9.50E-04	-3.97E-04

1.1.1.2.

Random	-1.29E-03	-5.38E-04
Misc at Ro	-6.65E-04	-2.78E-04

1.1.2.

Gravity Gradient,	-5.56E-05	-1.29E-05
Satellite	-2.87E-05	-6.64E-06

1.1.3.

Rotor Charge	-4.22E-05	-7.34E-06
	-2.18E-05	-3.78E-06

1.2. D_3, D_9, d

Voltages at Roll	1.13E-03	5.41E-04
and D.C. Miscentering	3.84E-03	5.46E-04

1.2.1.

Gravity Gradient,	1.07E-03	-5.40E-04
Earth	-3.83E-03	-5.45E-04

1.2.2.

Roll Rate Variation	-1.48E-04	1.77E-05
and Centrif. Force	-1.48E-04	9.20E-06

1.2.3.

Roll Rate Variation	-7.35E-05	8.92E-06
and Inert. Force	7.35E-05	4.56E-06

1.2.4.

Variation in Rotor	3.37E-04	-3.38E-05
Charge at Roll	-1.73E-04	-1.76E-05

1.3. D_3, D_9

Voltages at roll + - twice orbital,	3.29E-03	2.86E-05
Miscenetering at twice orbital	1.05E-03	9.13E-06

Item	Description	D_5, D_7	
2.	Accel. Parall to	3.61E-03	7.69E-04
	Roll Axis	3.32E-02	2.57E-03
2.1.	D.C.Voltages and	1.42E-03	7.69E-04
	Miscentering at Roll	1.42E-03	7.69E-04
2.1.1.	Gravity Gradient,	1.35E-03	7.66E-04
	Earth	1.35E-03	7.66E-04
2.1.1.1.	Misc at	-1.11E-03	6.27E-04
	Roll	1.11E-03	-6.27E-04
2.1.1.2.	Random	-7.75E-04	4.39E-04
	Misc at Ro	-7.75E-04	4.39E-04
2.1.2.	Gravity Gradient,	4.26E-04	-6.42E-05
	Satellite	4.26E-04	-6.42E-05
2.1.3.	Rotor Charge	3.23E-05	-3.66E-06
		3.23E-05	-3.66E-06
2.2.	Voltages at twice orbital	3.32E-03	2.46E-05
		3.32E-02	2.46E-03
2.2.1.	Misc. at roll + - twice	3.32E-03	-2.46E-05
	orbital,diff. phase shift	0.00E+00	0.00E+00
2.2.2.	Diff. in misc. at roll +	0.00E+00	0.00E+00
	2 orb. and roll - 2 orb.	-3.32E-02	-2.46E-03

SHEET 0

Node	Description		Worst	Plausible
	Nominal Position	EW	1.29E-01	3.51E-02
		NS	1.36E-01	3.39E-02
1.	Preload Differences		1.28E-01	3.51E-02
			1.27E-01	3.39E-02
1.1. (K_3, k)	D.C.Voltages and Variation in Roll Phase at Roll		2.04E-04	7.94E-05
			2.04E-04	7.94E-05
1.1.1.	h_a-h_b, D.C.		1.90E-04	7.61E-05
			-1.90E-04	-7.61E-05
1.1.2	g^2_a-g^2_b, D.C.		7.56E-05	2.27E-05
			7.56E-05	2.27E-05
1.1.2.1.	Gravity Gradient Earth-Centrifugal		7.05E-05	2.25E-05
			-7.05E-05	-2.25E-05
1.1.2.2.	Gravity Gradient Satellite - Centrifugal		-2.71E-05	-2.65E-06
			2.71E-05	2.65E-06
1.1.2.3.	Rotor Charge Centrifugal		-2.06E-06	-1.31E-07
			2.06E-06	1.31E-07
1.1.2.4.	Gravity Gradient, Earth Gravity Gradient, Satellite		2.13E-06	3.77E-07
			-2.13E-06	-3.77E-07
1.1.2.5.	Rotor Charge		-4.71E-08	-1.25E-09
			4.71E-08	1.25E-09
1.2. (K_3, k)	Voltages at Roll		1.27E-01	3.51E-02
			1.27E-01	3.38E-02
1.2.1.	h_a-h_b at Roll		-1.27E-01	-3.38E-02
			-1.27E-01	-3.38E-02
1.2.2.	g^2_a-g^2_b at Roll		7.14E-03	9.49E-03
			1.09E-03	1.37E-03
1.2.2.1.	Gravity Gradient, Earth - Roll Rate Variation and Centr.		1.41E-04	4.50E-05
			1.41E-04	4.50E-05
1.2.2.2.	Gravity Gradient, Satellite - Roll Rate Var. and Centr.		-5.42E-05	-4.61E-06
			-5.42E-05	-4.61E-06
1.2.2.3.	Gravity Gradient, Earth		7.13E-03	9.49E-03
			-1.03E-03	-1.37E-03
1.2.2.4.	Grav. Grad, Earth - Variation in Rotor Charge at Roll		3.23E-04	8.59E-05
			-3.23E-04	-8.59E-05
1.2.2.5.	Grav. Grad, Satellite - Var. in Rotor Charge at Roll		-1.24E-04	-8.80E-06
			1.24E-04	8.80E-06
1.2.2.6.	Rotor Charge - Variation in Rotor Charge at Roll		-9.42E-06	-5.01E-07
			9.42E-06	5.01E-07
1.3. (K_3, k)	Transient Voltages		-5.28E-03	-9.39E-04
			-5.28E-03	-9.39E-04

2.	Preloads	1.58E-02	1.39E-04
		4.68E-02	1.74E-04
2.1. (k)	D.C. Voltage and Variation in Roll Phase at Roll	2.20E-03	3.06E-05
		7.31E-04	1.02E-05
2.1.1.	h_a+h_b, h_c, D.C.	-2.20E-03	-3.06E-05
		7.31E-04	1.02E-05
2.1.2.	$g^2_a+g^2_b$, g^2_c, D.C.	5.49E-06	2.05E-07
		4.58E-07	6.15E-08
2.1.2.1.	Centrifugal	-2.20E-06	-1.58E-08
		0.00E+00	0.00E+00
2.1.2.2.	Gravity Gradient, Earth	-5.03E-06	-2.04E-07
		-4.47E-07	-6.15E-08
2.1.2.3.	Gravity Gradient, Satellite	-1.02E-07	-2.26E-10
		-1.00E-07	-2.21E-10
2.1.2.4.	Pointing Error	-2.23E-08	-3.43E-15
		0.00E+00	0.00E+00
2.1.2.5.	Frequences other than Roll	-9.27E-10	-2.89E-15
		0.00E+00	0.00E+00
2.1.2.6.	Rotor Charge	-1.73E-09	-2.16E-12
		-5.78E-10	-7.20E-13
2.2. (k)	Voltages at Roll	1.56E-02	4.10E-05
		4.68E-02	1.16E-04
2.2.1.	h_a+h_b, h_c at Roll	1.56E-02	3.88E-05
		4.68E-02	1.16E-04
2.2.2.	$g^2_a+g^2_b$, g^2_c at Roll	4.46E-04	1.33E-05
		6.79E-05	1.92E-06
2.2.2.1.	Centrifugal - Roll Freq Var and Centrifugal Force	0.00E+00	0.00E+00
		-8.81E-06	-6.32E-08
2.2.2.2.	Centrifugal Gravity Gradient, Earth	-4.46E-04	-1.33E-05
		6.41E-05	1.92E-06
2.2.2.3.	Centrifugal - Roll Freq Var and Inertial Force	0.00E+00	0.00E+00
		-4.41E-06	-3.16E-08
2.2.2.4.	Centrifugal - Variation in Rotor Charge at Roll	-2.02E-05	-1.21E-07
		2.02E-05	1.21E-07
2.2.2.5.	Rotor Charge - Variation in Rotor Charge at Roll	-4.62E-07	-1.15E-09
		4.62E-07	1.15E-09
2.3. (k)	Transient Voltages	3.88E-04	1.29E-04
		3.88E-04	1.29E-04

Preload Dependent, Misaligned Position

SHEET 0

Misalignment $(V^2)^+_j\theta$		Worst	Plausible
	EW	8.34E-02	4.25E-02
	NS	8.34E-02	4.25E-02

1.

Preload		
	8.34E-02	4.25E-02
	8.34E-02	4.25E-02

1.1. K_2, K_5

D.C. Voltages		
	7.30E-02	4.25E-02
	7.30E-02	4.25E-02

1.1.1.

h_a+h_b, h_c, D.C.		
	-7.30E-02	-4.25E-02
	7.30E-02	4.25E-02

1.1.2.

$g^2_a+g^2_b, g^2_c$, D.C.		
	4.41E-05	7.88E-06
	4.41E-05	7.88E-06

1.1.2.1.

Centrifugal		
	2.35E-05	7.05E-06
	-2.35E-05	-7.05E-06

1.1.2.2.

Gravity Gradient, Earth		
	3.72E-05	-3.51E-06
	-3.72E-05	3.51E-06

1.1.2.3.

Gravity Gradient, Satellite		
	-2.60E-06	-2.40E-07
	2.60E-06	2.40E-07

2.1.2.4.

Pointing Error		
	9.91E-09	1.29E-12
	-9.91E-09	-1.29E-12

1.2. K_3, K_4, K_5

Voltages at Twice Roll		
	4.03E-02	4.23E-04
	4.03E-02	4.23E-04

1.2.1.

h_a+h_b, h_c at Twice Roll		
	4.03E-02	4.22E-04
	4.03E-02	4.22E-04

1.2.2.

$g^2_a+g^2_b, g^2_c$ at Twice Roll		
	3.00E-05	2.84E-05
	3.00E-05	2.84E-05

1.2.2.1.

Gravity Gradient, Earth		
	3.00E-05	-2.84E-05
	3.00E-05	2.84E-05

1.2.2.2.

Roll Freq. Var. and Centrifugal Force		
	-1.73E-14	-1.30E-15
	1.73E-14	1.30E-15

1.2.2.3.

Roll Freq. Var. and Inertial Force		
	4.33E-15	3.24E-16
	-4.33E-15	-3.24E-16

1.2.2.4.

Variation in Rotor Charge		
	-6.61E-14	-3.44E-15
	-6.61E-14	-3.44E-15

Preload Dependent, Miscentered Position

SHEET 0 ← Miscentering

Miscentering		Worst	Plausible
	EW	3.32E-02	1.48E-02
	NS	3.32E-02	1.21E-02

- 1. Preload Differences: 3.03E-04, 8.75E-05 / 3.05E-04, 9.69E-05
  - 1.1. (D_8) D.C.Voltages and Miscentering at Roll: 4.76E-05, 4.07E-05 / 4.76E-05, 4.07E-05
    - 1.1.1. h_a-h_b, D.C.: 4.42E-05, 3.46E-05 / 4.42E-05, 3.46E-05
    - 1.1.2. g^2_a-g^2_b, D.C.: 1.76E-05, 2.14E-05 / 1.76E-05, 2.14E-05
      - 1.1.2.1. Gravity Gradient, Earth -- Centrifugal: 1.64E-05, 2.13E-05 / 1.64E-05, 2.13E-05
      - 1.1.2.2. Gravity Gradient, Satellite -- Centrifugal: -6.31E-06, -2.18E-06 / -6.31E-06, -2.18E-06
      - 1.1.2.3. Rotor Charge -- Centrifugal: -4.79E-07, -1.24E-07 / -4.79E-07, -1.24E-07
  - 1.2. (D_8, d) Voltages at Roll and D.C. Miscentering: 2.99E-04, 7.75E-05 / 3.01E-04, 8.79E-05
    - 1.2.1. h_a-h_b, at Roll: 2.99E-04, 7.73E-05 / 2.99E-04, -7.65E-05
    - 1.2.2. g^2_a-g^2_b, at Roll: 4.93E-06, 6.04E-06 / 3.34E-05, 4.34E-05
      - 1.2.2.1. Gravity Grad., Earth -- Roll Freq. Var. and Centr. Force: 6.63E-07, -2.07E-07 / 6.63E-07, -2.05E-07
      - 1.2.2.2. Gravity Gradient, Earth: -4.63E-06, 6.02E-06 / 3.33E-05, -4.34E-05
      - 1.2.2.3. Gravity Grad., Earth -- Roll Freq. Var. and Inert. Force: 3.28E-07, -1.02E-07 / -3.28E-07, 1.03E-07
      - 1.2.2.4. Gravity Grad., Earth -- Variation in Rotor Charge: -1.50E-06, 3.91E-07 / 1.50E-06, -3.95E-07
      - 1.2.2.5. Grav. Grad., Satellite -- Roll Freq. Var. and Centr. Force: -2.55E-07, 2.12E-08 / -2.55E-07, 2.10E-08
      - 1.2.2.6. Grav. Grad., Satellite -- Roll Freq. Var. and Inert. Force: -1.26E-07, 1.05E-08 / 1.26E-07, -1.06E-08
  - 1.3. (D_8) Voltages at Twice Roll, Miscentering at Roll: 4.42E-07, 3.02E-07 / 4.42E-07, 3.02E-07
    - 1.3.1. h_a-h_b, at Twice Roll: 4.42E-07, 3.02E-07 / 4.42E-07, 3.02E-07

SHEET 6

No.	Item	D_1, D_2, D_4, D_6			
2.	Preloads	3.32E-02	1.48E-02	3.32E-02	1.21E-02
2.1.	D.C. Voltages, Miscentering at Roll	2.78E-02	1.48E-02	2.78E-02	1.21E-02
2.1.1.	h_a+h_b, h_C D.C.	2.78E-02	1.48E-02	2.78E-02	1.21E-02
2.1.1.1.	Miscentering at Roll	2.28E-02	1.21E-02	2.28E-02	-1.21E-02
2.1.1.2.	Random Miscentering at Roll	1.60E-02	8.46E-03	1.60E-02	8.70E-04
2.1.2.	$g^2_a+g^2_b$, g^2_c D.C.	5.71E-05	6.05E-05	5.71E-05	6.05E-05
2.1.2.1.	Centrifugal	2.51E-05	7.27E-06	2.51E-05	7.27E-06
2.1.2.2.	Gravity Gradient, Earth	5.13E-05	6.01E-05	5.13E-05	6.01E-05
2.1.2.3.	Gravity Gradient Satellite	8.65E-07	6.09E-08	8.65E-07	6.09E-08
2.1.2.4.	Pointing Error	2.54E-07	1.58E-12	2.54E-07	1.58E-12
2.1.2.5.	Frequences other than Roll	1.06E-08	1.33E-12	1.06E-08	1.33E-12
2.2.	Voltages at Roll and D.C. Miscentering	1.12E-02	4.83E-05	1.12E-02	4.62E-04
2.2.1.	h_a+h_b, h_C at Roll	1.12E-02	4.81E-05	1.12E-02	-4.62E-04
2.2.2.	$g^2_a+g^2_b$, g^2_c at Roll	1.36E-05	4.00E-06	1.36E-05	4.00E-06
2.2.2.1.	Centrifugal -- Roll Freq. Var. and Centr. Force	-2.28E-06	-1.23E-07	-2.28E-06	-1.23E-07
2.2.2.2.	Centrifugal -- Grav. Grad., Earth	1.26E-05	1.12E-06	1.26E-05	1.12E-06
2.2.2.3.	Centrifugal -- Roll Freq. Var. and Inert. Force	-8.63E-07	-1.96E-08	-8.63E-07	-1.96E-08
2.2.2.4.	Centrifugal -- Var. in Rotor Charge at Roll	4.50E-06	3.84E-06	4.50E-06	3.84E-06
2.3.	Voltages at Twice Roll, Miscentering at Roll	1.42E-02	1.47E-05	1.42E-02	1.47E-05
2.3.1.	h_a+h_b, h_C at Twice Roll	0.014234	1.4E-05	0.014234	1.4E-05
2.3.2.	$g^2_a+g^2_b$, g^2_c at Twice Roll	8.69E-06	4.61E-06	8.69E-06	4.61E-06
2.3.2.2.	Gravity Gradient, Earth	8.69E-06	4.61E-06	8.69E-06	4.61E-06
2.3.2.1.	Roll Freq. Var. and Centr. Force	5.02E-15	2.05E-15	5.02E-15	2.05E-15

Support Independent, Housing Fixed, Nominal Position

SHEET 0

Nominal Position		Worst	Plausible
	EW	2.18E-02	9.01E-03
	NS	2.20E-02	9.12E-03

1\.

Electrostatic Torques		
	1.53E-02	8.98E-03
	1.56E-02	9.09E-03

1\.1.

Var. in V_S At Roll		
	-1.53E-02	-8.98E-03
	-1.53E-02	-8.98E-03

1\.2.

Var. in Roll Phase at Roll and V_S		
	1.224E-05	1.796E-06
	-1.22E-05	-1.8E-06

1\.3.

Var. in V_q At Roll		
	2.86E-05	1.43E-05
	2.86E-03	1.43E-03

1\.4.

Var. in Roll Phase at Roll and V_q		
	-4.29E-05	-1.07E-05
	4.292E-07	1.073E-07

2\.

Magnetic Torques		
	2.98E-03	6.70E-04
	2.98E-03	6.70E-04

2\.1.

Var. in Residual Field at Roll		
	2.72E-03	2.72E-04
	2.72E-03	2.72E-04

2\.2.

Var. in Roll Phase at Roll and Residual Fi		
	-1.22E-03	-6.12E-04
	1.22E-03	6.12E-04

3\.

Direct Gravity Gradient		
	1.53E-02	3.05E-04
	1.53E-02	3.05E-04

3\.1.

Satellite Roll Phase Var. at Roll		
	-7.63E-05	-7.63E-06
	7.63E-05	7.63E-06

3\.2.

Liquid He		
	1.53E-02	3.05E-04
	1.53E-02	3.05E-04

Support Independent, Housing Fixed, Misaligned Position

SHEET 0 ← Misalignment

Misalignment		Worst	Plausible
	EW	6.66E-02	9.53E-03
	NS	6.66E-02	9.53E-03

No.	Item	Worst	Plausible
1..	Electrostatic	2.59E-03	7.93E-05
	Torques	2.59E-03	7.93E-05
1.1.	Rotor Charging	8.80E-05	7.66E-05
	Torque	8.80E-05	7.66E-05
1.1.1.	Rotor to Electrode	8.80E-05	7.66E-05
	Charging	-8.80E-05	-7.66E-05
1.1.2.	Rotor to Ground Plane	1.07E-07	2.15E-08
	Charging	1.07E-07	2.15E-08
1.2.	Electric Dipole	2.59E-03	2.05E-05
	Moment	2.59E-03	2.05E-05
1.3.	Quadrapole	0.00E+00	0.00E+00
	Moment	0.00E+00	0.00E+00
2..	Magnetic	6.85E-03	6.85E-03
	Torques	6.85E-03	6.85E-03
2.1.	Coupling with	6.74E-03	6.74E-03
	Local Shield	6.74E-03	6.74E-03
2.2.	Field from	-2.16E-04	-2.16E-04
	Rolling Electrodes	2.16E-04	2.16E-04
2.3.	Residual Field	1.22E-03	1.22E-03
		1.22E-03	1.22E-03
2.4.	Gyro Readout	8.75E-08	-8.75E-08
	Torque	-8.75E-08	-8.75E-08
3..	Differ. Damping	6.62E-02	6.62E-03
	Torques	6.62E-02	6.62E-03
3.1.	Eddy Currents	1.81E-03	9.03E-05
		1.81E-03	9.03E-05
3.1.1.	Electrodes	1.81E-03	9.03E-05
		1.81E-03	9.03E-05
3.1.2.	Caging Rod	5.95E-06	2.98E-07
		5.95E-06	2.98E-07
3.2.	Patch Effect	5.04E-04	5.04E-05
		5.04E-04	5.04E-05
3.3.	Residual Gas	6.62E-02	6.62E-03
	Pressure	6.62E-02	6.62E-03
3.4.	Particles in	3.15E-04	3.15E-06
	Housing	3.15E-04	3.15E-06
4..	Direct Gravity	7.63E-05	1.53E-05
	Gradient	7.63E-05	1.53E-05
4.1.	Satellite	7.63E-05	1.53E-05
		7.63E-05	1.53E-05
4.2.	Liquid He	7.63E-08	1.53E-09
		7.63E-08	1.53E-09

Sheet 9

	Worst	Plausible
EW	0.002677	0.000641
NS	0.001702	0.000642

Total drift rate from roll phase variation at roll

Support Independent, Inertially Fixed

SHEET 0

Inertially Fixed Torques		Worst	Plausible
	EW	2.68E-03	2.26E-03
	NS	1.57E-02	1.57E-02

No.	Item	Worst	Plausible
1.	Direct Gravity Gradient	2.25E-03	2.25E-03
		1.57E-02	1.57E-02
1.1.	Earth	2.25E-03	2.25E-03
		-1.57E-02	-1.57E-02
1.2.	Moon	1.00E-07	1.00E-08
		1.00E-07	1.00E-08
1.3.	Sun	1.00E-06	1.00E-06
		1.00E-06	1.00E-06
2.	Momentum Exchange Torques	1.46E-03	1.12E-04
		1.46E-03	1.12E-04
2.1.	Cosmic Rays	1.46E-03	1.12E-04
		1.46E-03	1.12E-04
2.1.1.	Primary Rays	1.30E-03	1.00E-04
		1.30E-03	1.00E-04
2.1.2.	Sun Showers	6.60E-04	5.00E-05
		6.60E-04	5.00E-05
3.	Stochastic torques	2.35E-05	2.35E-05
		2.35E-05	2.35E-05
3.1.	Random Walk Mass Exchange	1.05E-09	4.69E-11
		1.05E-09	4.69E-11
3.2.	Brownian Motion	2.35E-05	2.35E-05
		2.35E-05	2.35E-05

TopTorques

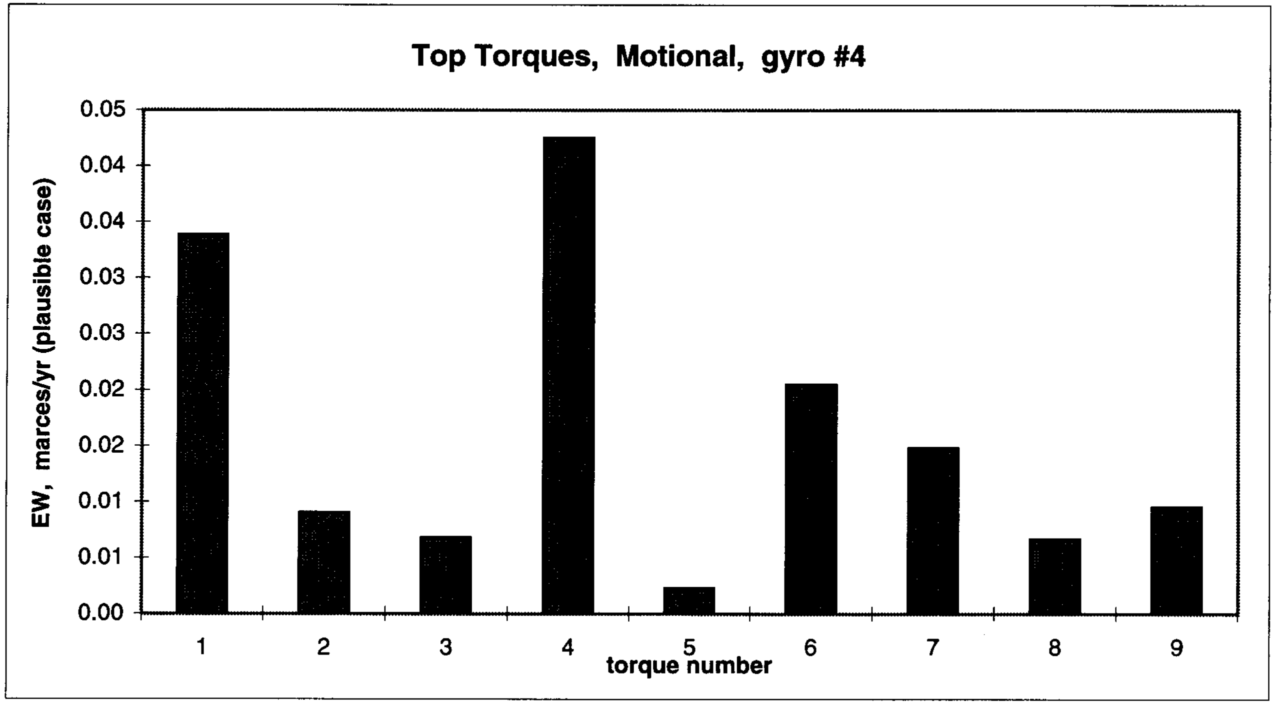
Top Torques (plausible case, g.#4)						
	Category	Sheet	Box	Name / Position	Drift Rate, marcs/yr	
				I. EW		
1	Support Dependent	4	1.2.1.	h_a-h_b at roll	3.38E-02	
2	Support Independent	7	1.1	Variation of sensing bridge voltage at roll	8.98E-03	
3	Support Independent	8	2.1.	Coupling with local shield	6.74E-03	
4	Support Dependent	5	1.1.1.	h_a+h_b , h_c D.C.	4.25E-02	
5	Support Independent	10	1.1	Direct grav.ity gradient, Earth	2.25E-03	
6	Support Dependent	1	1.2.1.	Gravity gradient, Earth, perp. to roll axis	2.04E-02	
7	Support Dependent	6	2.1.1.	h_a+h_b , h_c D.C. + Miscentering at roll	1.48E-02	
8	Support Independent	8	3.3	Residual gas pressure	6.62E-03	
9	Support Dependent	4	1.2.2.3	g^2_a-g^2_b at roll, due to Earth gravity gradient	9.49E-03	
				All top EW torques:	6.20E-02	99.90%
				II. NS		
1	Support Dependent	4	1.2.1.	h_a-h_b at roll	3.38E-02	
2	Support Independent	7	1.1	Variation of sensing bridge voltage at roll	8.98E-03	
3	Support Independent	8	2.1.	Coupling with local shield	6.74E-03	
4	Support Dependent	5	1.1.1.	h_a+h_b , h_c D.C.	4.25E-02	
5	Support Independent	10	1.1	Direct gravity gradient, Earth	1.57E-02	
6	Support Dependent	1	1.2.1	Gravity gradient, Earth, perp. to roll axis	1.42E-01	
7	Support Dependent	6	2.1.1.	h_a+h_b , h_c D.C. + Miscentering at roll	1.21E-02	
8	Support Independent	8	3.3	Residual gas pressure	6.62E-03	
9	Support Dependent	3	2.2.2.	Difference in Miscenter. Amplitudes at ω_r +/- $2\omega_0$	2.46E-03	
				All top NS torques:	1.54E-01	99.98%

(% of total EW dr. rate see Sheet 0)

(% of total NS dr. rate see Sheet 0)

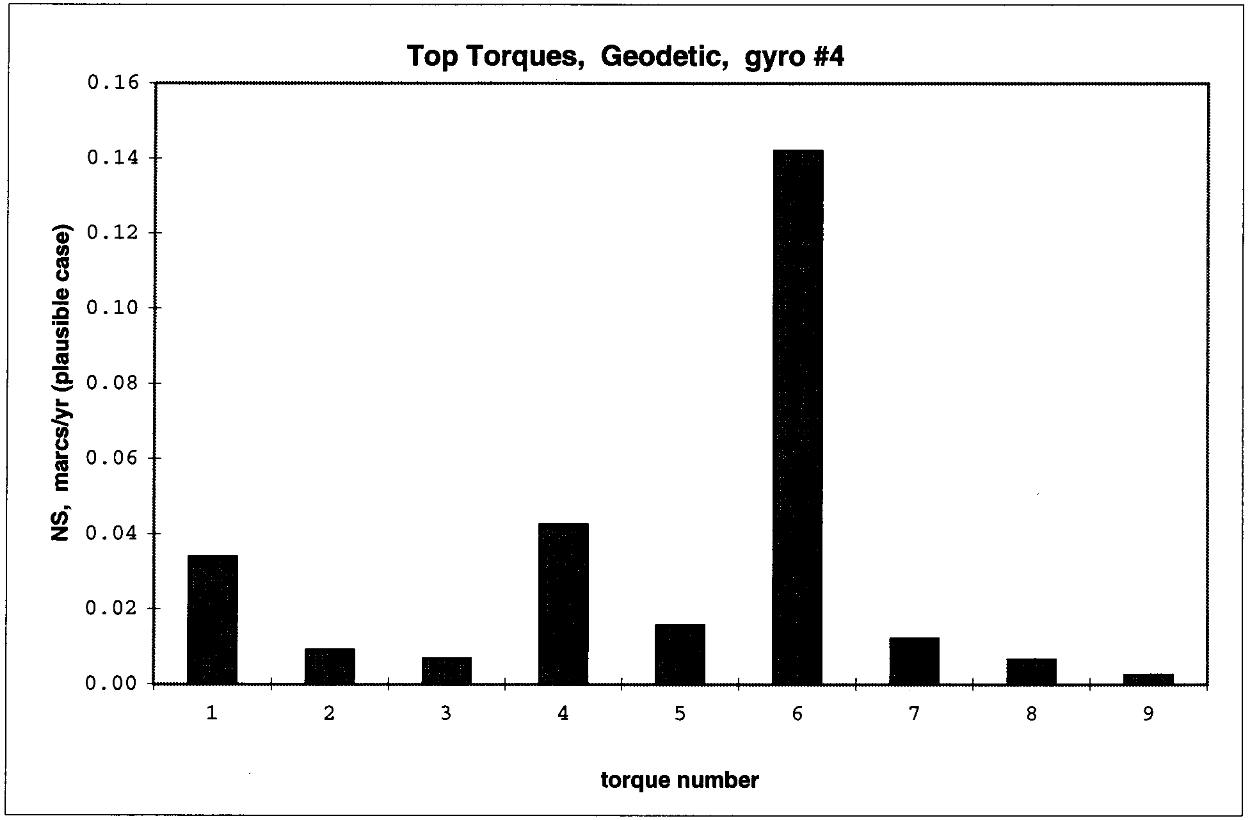
The two lists differ by the last torques only. The order of the first eight torques is the same in both lists, the magnitudes are generally different.

TopEW



Top Torques, Motional, gyro #4
EW, marces/yr (plausible case)
0.05
0.04
0.04
0.03
0.03
0.02
0.02
0.01
0.01
0.00
1
2
3
4
5
6
7
8
9
torque number

TopNS



Top Torques, Geodetic, gyro #4
NS, marcs/yr (plausible case)
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
1
2
3
4
5
6
7
8
9
torque number

MODELED ERRORS (Theory)

MEASUREMENT EQUATION

$$Z(t) = C_g \{[NS + R_g t + I_1 \sin(\omega_a t) + I_2 \sin(\omega_a t) + A_0 \cos(w_o t)]\cos(\omega_r t + \delta\phi) + $$
$$+ [EW + R_m t + I_3 \cos(\omega_a t) + I_4 \sin(\omega_a t)] \sin(\omega_r t + \delta\phi)\} + b + n(t)$$

State Vector $X = [\ C_g \ \ \delta\phi \ \ NS \ \ EW \ \ R_g T_a \ \ R_m T_a \ \ b\ ]$

Two-Step Estimator : Covariance Matrix Calculation

Step 1 **Change of variables: y = f(x)**

$y_1 = C_g\,[NS\cos(\delta\phi) + EW\sin(\delta\phi)]$
$y_2 = C_g[R_g T_a\cos(\delta\phi) + R_m T_a\sin(\delta\phi)]$
$y_3 = C_g[I_1\cos(\delta\phi) + I_3\sin(\delta\phi)]$
$y_4 = C_g[I_2\cos(\delta\phi) + I_4\sin(\delta\phi)]$
$y_5 = C_g A_0\cos(\delta\phi)$
$y_6 = C_g[-NS\sin(\delta\phi) + EW\cos(\delta\phi)]$
$y_7 = C_g[-R_g T_a\sin(\delta\phi) + R_m T_a\cos(\delta\phi)]$
$y_8 = C_g[-I_1\sin(\delta\phi) + I_3\cos(\delta\phi)]$
$y_9 = C_g[-I_2\sin(\delta\phi) + I_4\cos(\delta\phi)]$
$y_{10} = -C_g A_0\sin(\delta\phi)$

Linear Measurement Equation: $Z(t) = H(t)\,y + \nu(t)$

$$H(t) = [\ \cos(\omega_r t) \quad (t/Ta)\cos(\omega_r t) \quad \cos(\omega_r t)\cos(\omega_a t) \quad \cos(\omega_r t)\sin(\omega_a t) \quad \cos(\omega_r t)\cos(\omega_o t)$$
$$\sin(\omega_r t) \quad (t/Ta)\sin(\omega_r t) \quad \sin(\omega_r t)\cos(\omega_a t) \quad \sin(\omega_r t)\sin(\omega_a t) \quad \sin(\omega_r t)\cos(\omega_o t)\ ]$$

1) Calculate Information Matrix I_1 :

$$I_1 = (N_0/2\sigma^2) \quad \boxed{\sum_{i=1}^{N} h_k(t_i)h_l(t_i) = \int_{-T/2}^{T/2} h_k(t)h_l(t)g(t)\,dt} \quad k=1\text{-}10,\ l=1\text{-}10$$

g(t) ={1, if GS valid;
0, if GS invalid}

2) Calculate Covariance Matrix P_1:

$$P_1 = I_1^{-1}$$

Step 2

3) Calculate Information Matrix I_2 :

$$I_2 = (dy/dx)^T\, I_1 (dy/dx)$$

4) Calculate Covariance Matrix P_2 :

$$P_2 = I_2^{-1}$$

MEASUREMENT ERRORS

INPUT PARAMETERS

Parameter Name	Worst Case Value	Units	Variable Name	Plausible Case Value	Variable Name
3	4	5	6	7	8
Hardware parameters					
SQUID noise at roll, σ_0 (S0575, p.11)*)	1.1273E+02	marcsec/sqrt(Hz)	s0r	1.1273E+02	s0c
Gyro noise, σ_g (S0352, p.18)	5.0000E+00	marcsec/sqrt(Hz)	sgr	5.0000E+00	sgc
Telescope noise, σ_t (S0626, T.3-2)**)	1.6100E+01	marcsec/sqrt(Hz)	str	1.6100E+01	stc
SQUID scale factor, C_g	1.0000E+00	Dless	cgr	1.0000E+00	cgc
Operational Parameters					
Start date (Jan 1 =1)	202.	days	sdr	202.	sdc
Experiment duration, T	12	month	tmr	13	tmc
Measurement time step, Δt	2.0000E+00	sec	dt	2.0000E+00	dt
Gyroscope spin frequency, f	143	Hz	fspinr	143	fspinc
Satellite roll period, T_{roll}	3	minutes	T_rollr	3	T_rollc
Declination of Guide Star (HR 8703), δ	2.9391E-01	rad	dlt_r	2.9391E-01	dlt_c
Right ascention of Guide Star, α	5.9910E+00	rad	ra_r	5.9910E+00	ra_c
Fraction of orbit GS valid, f_0	5.0000E-01	Dless	f0r	5.0000E-01	f0c
Averaged misalignment, NS_0	1.0000E+03	marcsec	ns0r	1.0000E+03	ns0c
Averaged misalignment, EW_0	1.0000E+03	marcsec	ew0r	1.0000E+03	ew0c
Magnitude of orbital aberration, A_0	5.1841E+03	marcsec	orb_r	5.1841E+03	orb_c
Roll phase offset. $\delta\phi$	2.9089E-03	rad	rphasr	4.8481E-05	rphasc
Relativistic drift in NS direction, R_g	6.6000E+03	marcsec/yr	rg_r	6.6000E+03	rg_c
Relativistic drift in EW direction, R_m	4.0000E+01	marcsec/yr	rm_r	4.0000E+01	rm_c
Inclination of earth's rot. axis, i	4.1015E-01	rad	iii	4.1015E-01	iii
Radius of earth's orbit, R_s	1.5000E+11	m	rorb	1.5000E+11	rorb

*) Measured noise for SQUID 4 at 130 Hz spin and 3 min roll = 124 mas/sqrt(Hz) (T003, Req. 3.2.2 verification).

**) Mean value over the telescope channels in proper units (T003, Req. 7.5 verification)

DERIVED PARAMETERS

#	Parameter Name	Worst Case Value	Units	Variable Name	Plausible Case Value	Variable Name
1	Readout noise at roll, (σ)	5.699E+01	marcsec	sigmar	5.699E+01	sigmac
2	Number of observation points, N_0	7.884E+06	Dless	N0r	8.541E+06	N0c
3	$N_0/(2\sigma^2)$	1.214E+03	Hz/(marcsec)^2	nsr	1.315E+03	nsc
4	sin ($\pi T/T_a$)	-4.102E-10	Dless	sin1r	-2.588E-01	sin1c
5	sin π f_0	1.000E+00	Dless	sin2r	1.000E+00	sin2c
6	cos ($\pi T/T_a$)	-1.000E+00	Dless	cos1r	-9.659E-01	cos1c
7	sin ($2\pi T/T_a$)	8.204E-10	Dless	sin3r	5.000E-01	sin3c
8	sin($2\pi f_0$)	-4.102E-10	Dless	sin4r	-4.102E-10	sin4c
9	phase of annual motion at midpoint	5.242E+00	rad	fir	5.504E+00	fic
10	NS cosine aberration amplitude, I_1	1.6649E+02	marcsec	I1r	-2.689E+03	I1c
11	NS sine aberration amplitude, I_2	-1.1012E+04	marcsec	I2r	-1.068E+04	I2c
12	EW cosine aberration amplitude, I_3	-1.4230E+04	marcsec	I3r	-1.701E+04	I3c
13	EW sine aberration amplitude, I_4	-1.2596E+04	marcsec	I4r	-8.483E+03	I4c
14	Determinant of step 1 info matrix, D1	8.467E+07	(marcsec)^(-3)	D1r	1.143E+08	D1c
15	Determinant of step 2 info matrix, D2	2.364E+16	(marcsec)^(-4)	D2r	4.623E+16	D2c

Nominal First-Step variables $Y_{nom} = f(X_{nom})$

	Nominal Values				
$y_1=C_g(NS\cos(\delta\phi) + EW\sin(\delta\phi))$	1.00E+03	marcsec	ynom1r	1.00E+03	ynom1c
$y_2=C_g(R_gT_a\cos(\delta\phi) + R_mT_a\sin(\delta\phi))$	6.60E+03	"-"	ynom2r	6.60E+03	ynom2c
$y_3=C_g(I_1\cos(\delta\phi) + I_3\sin(\delta\phi))$	1.34E+02	"-"	ynom3r	-2.69E+03	ynom3c
$y_4=C_g(I_2\cos(\delta\phi) + I_4\sin(\delta\phi))$	-1.10E+04	"-"	ynom4r	-1.07E+04	ynom4c
$y_5=C_gA_0\cos(\delta\phi)$	5.18E+03	"-"	ynom5r	5.18E+03	ynom5c
$y_6=C_g(-NS\sin(\delta\phi) + EW\cos(\delta\phi))$	9.97E+02	"-"	ynom6r	1.00E+03	ynom6c
$y_7=C_g(-R_gT_a\sin(\delta\phi) + R_mT_a\cos(\delta\phi))$	2.08E+01	"-"	ynom7r	3.97E+01	ynom7c
$y_8=C_g(-I_1\sin(\delta\phi) + I_3\cos(\delta\phi))$	-1.42E+04	"-"	ynom8r	-1.70E+04	ynom8c
$y_9=C_g(-I_2\sin(\delta\phi) + I_4\cos(\delta\phi))$	-1.256E+04	"-"	ynom9r	-8.48E+03	ynom9c
$y_{10}= -C_gA_0\sin(\delta\phi)$	-1.508E+01	"-"	ynom10r	-2.51E-01	ynom10c

Jacobian (G=dy/dx) (10x6)

	Cg	$\delta\phi$	NS	EW	RgTa	RmTa
y_1	1.00E+03	9.97E+02	1.00E+00	2.91E-03	0	0
y_2	6.60E+03	2.08E+01	0	0	1.00E+00	2.91E-03
y_3	1.25E+02	-1.42E+04	0	0	0	0
y_4	-1.10E+04	-1.26E+04	0	0	0	0
y_5	5.18E+03	-1.51E+01	0	0	0	0
y_6	9.97E+02	-1.00E+03	-2.91E-03	1.00E+00	0	0
y_7	2.08E+01	-6.60E+03	0	0	-2.91E-03	1.00E+00
y_8	-1.42E+04	-1.25E+02	0	0	0	0
y_9	-1.26E+04	1.10E+04	0	0	0	0
y_{10}	-1.51E+01	-5.18E+03	0	0	0	0

Modeled Error Step 1

Information matrix I_1, worst case

1213.70	0	0.00	0	772.67	0	0	0	0	0
0	101.14	0	193.17	0	0	0	0	0	0
0.00	0	606.85	0	0.00	0	0	0	0	0
0	193.17	0	606.85	0	0	0	0	0	0
772.67	0	0.00	0	606.85	0	0	0	0	0
0	0	0	0	0	1213.70	0	0.00	0	772.67
0	0	0	0	0	0	101.14	0	193.17	0
0	0	0	0	0	0.00	0	606.85	0	0.00
0	0	0	0	0	0	193.17	0	606.85	0
0	0	0	0	0	772.67	0	0.00	0	606.85

Covariance matrix $P_1=I_1^{-1}$,worst case

4.35E-03	0	2.15E-13	0	-5.54E-03	0	0	0	0	0
0	2.52E-02	0	-8.03E-03	0	0	0	0	0	0
2.15E-13	0	1.65E-03	0	3.20E-28	0	0	0	0	0
0	-8.03E-03	0	4.20E-03	0	0	0	0	0	0
-5.54E-03	0	-3.20E-28	0	8.70E-03	0	0	0	0	0
0	0	0	0	0	4.35E-03	0	2.15E-13	0	-0.0055379
0	0	0	0	0	0	0.025218	0	-0.008027	0
0	0	0	0	0	2.15E-13	0	0.001648	0	3.201E-28
0	0	0	0	0	0	-0.008027	0	0.004203	0
0	0	0	0	0	-0.00554	0	3.2E-28	0	0.008699

Information matrix I_1, plausible case

1314.84	0	-99.99	0	837.06	0	0	0	0	0
0	128.59	0	186.22	0	0	0	0	0	0
-99.99	0	705.71	0	-63.66	0	0	0	0	0
0	186.22	0	609.13	0	0	0	0	0	0
837.06	0	-63.66	0	657.42	0	0	0	0	0
0	0	0	0	0	1314.84	0	-99.99	0	837.06
0	0	0	0	0	0	128.59	0	186.22	0
0	0	0	0	0	-99.99	0	705.71	0	-63.66
0	0	0	0	0	0	186.22	0	609.13	0
0	0	0	0	0	837.06	0	-63.66	0	657.42

Covariance matrix $P_1=I^{-1}$, plausible case

4.02E-03	0	1.09E-04	0	-5.11E-03	0	0	0	0	0
0	1.40E-02	0	-4.27E-03	0	0	0	0	0	0
1.09E-04	0	1.43E-03	0	-1.27E-19	0	0	0	0	0
0	-4.27E-03	0	4.20E-03	0	0	0	0	0	0
-5.11E-03	0	1.27E-19	0	8.70E-03	0	0	0	0	0
0	0	0	0	0	4.02E-03	0	1.09E-04	0	-5.11E-03
0	0	0	0	0	0	1.40E-02	0	-4.27E-03	0
0	0	0	0	0	1.09E-04	0	1.43E-03	0	-1.27E-19
0	0	0	0	0	0	-4.27E-03	0	4.20E-03	0
0	0	0	0	0	-5.11E-03	0	1.27E-19	0	8.70E-03

Modeled Errors Step 2

Information matrix I_2, worst case

2.96E+11	0	5.22E+06	1.21E+06	-1.46E+06	-2.43E+06
0	2.96E+11	1.21E+06	-5.22E+06	-2.43E+06	1.46E+06
5.22E+06	1.21E+06	1.21E+03	0	0	0
1.21E+06	-5.22E+06	0	1.21E+03	0	0
-1.46E+06	-2.43E+06	0	0	1.01E+02	0
-2.43E+06	1.46E+06	0	0	0	1.01E+02

Covariance matrix $P_2=I_2^{-1}$, worst case

	C_g	$\delta\phi$	NS	EW	R_gT_a	R_mT_a
C_g	5.19E-12	0	-2.23E-08	-5.19E-09	7.49E-08	1.25E-07
$\delta\phi$	0	5.19E-12	-5.19E-09	2.23E-08	1.25E-07	-7.49E-08
NS	-2.23E-08	-5.19E-09	9.25E-04	0.00E+00	-4.47E-04	-4.61E-04
EW	-5.19E-09	2.23E-08	0.00E+00	9.25E-04	4.61E-04	-4.47E-04
R_gT_a	7.49E-08	1.25E-07	-4.47E-04	4.61E-04	1.40E-02	0
R_mT_a	1.25E-07	-7.49E-08	-4.61E-04	-4.47E-04	0	1.40E-02

To sheet
"Total Measurement Errors

T_a = 1yr = 12 months = 3.15×10^7 sec

Information matrix I_2, plausible case

3.36E+11	0	5.92E+06	3.02E+06	-1.14E+06	-1.57E+06
0	3.36E+11	3.02E+06	-5.92E+06	-1.57E+06	1.14E+06
5.92E+06	3.02E+06	1.31E+03	0	0	0
3.02E+06	-5.92E+06	0	1.31E+03	0	0
-1.14E+06	-1.57E+06	0	0	1.29E+02	0
-1.57E+06	1.14E+06	0	0	0	1.29E+02

Covariance matrix $P_2=I_2^{-1}$, plausible case

	C_g	$\delta\phi$	NS	EW	R_gT_a	R_mT_a
C_g	3.66E-12	0	-1.65E-08	-8.39E-09	3.24E-08	4.48E-08
$\delta\phi$	0	3.66E-12	-8.39E-09	1.65E-08	4.48E-08	-3.24E-08
NS	-1.65E-08	-8.39E-09	8.54E-04	0.00E+00	-2.49E-04	-1.27E-04
EW	-8.39E-09	1.65E-08	0.00E+00	8.54E-04	1.27E-04	-2.49E-04
R_gT_a	3.24E-08	4.48E-08	-2.49E-04	1.27E-04	8.61E-03	0
R_mT_a	4.48E-08	-3.24E-08	-1.27E-04	-2.49E-04	0	8.61E-03

To sheet
"Total Measurement Errors

Unmodeled Errors (Theory)

Unmodeled Errors (Theory)

$$Z(t) = C_g \{[NS + R_g\, t + I_1 \sin(\omega_a t) + I_2 \sin(\omega_a t) + A_0 \cos(w_o t)] \cos(\omega_r t + \delta\phi) \cdot [EW + R_m\, t + I_3 \cos(\omega_a t) + I_4 \sin(\omega_a t)] \sin(\omega_r t + \delta\phi)\} + b + \nu(t)$$

$$Z(t) = H(t)\, y + \nu(t)$$

Unmodeled Bias Variations $b_{unm}(t)$

If we collect all measurements

$$\mathcal{Z} = \begin{vmatrix} z_1 \\ z_2 \\ \cdots \\ z_N \end{vmatrix} \quad L = \begin{vmatrix} H_1 \\ H_2 \\ \cdots \\ H_N \end{vmatrix} \quad Z_{unm} = \begin{vmatrix} b_{unm}(t_1) \\ b_{unm}(t_2) \\ \cdots \\ b_{unm}(t_N) \end{vmatrix} \quad V = \begin{vmatrix} \nu(t_1) \\ \nu(t_2) \\ \cdots \\ \nu(t_N) \end{vmatrix}$$

Measurement equation:

$$Z = L\, y + Zunm + V,$$

To compute Covariance Matrix P_x:

1) Calculate y_{unm}:

$$y_{unm} = P_1 (L^T Z_{unm})/\sigma_v^2$$

$$L^T Z_{unm} = \boxed{\int_{-T/2}^{T/2} h_k(t)\, g(t)\, b(t)\, dt} \qquad k=1\text{-}10$$

2) Calculate P_{1unm}:

$$P_{1unm} = E\{ y_{unm}\, y_{unm}^T \}$$

3) Calculate P_Y:

$$P_Y = P_1 + P_{1unm}$$

4) Calculate I_{1unm}:

$$I_{1unm} = I_1 P_{1unm} I_1$$

5) Calculate I_{2unm}:

$$I_{2unm} = (df/dx)_{nom} P_{1unm} (df/dx)^T_{nom}$$

6) Calculate I_x:

$$I_x = I_2 - I_{2unm}$$

7) Calculate P_{2unm}:

$$P_{2unm} = P_2 I_{2unm} P_2$$

8) Calculate P_x:

$$P_x = P_2 + P_{2unm}$$

Unmodeled Inputs

UNMODELED BIAS VARIATIONS (b_{unm})

	Frequency and phase: Zunmb	Magnitude (fspin. depend.)	Units	Name
1	Cos Roll	9.11E-02	marcsec	Acrollb
2	Linear in time at Cos Roll	9.11E-02	marcsec/yr	Atcrollb
3	(Cos Roll)(Cos Ann)	4.56E-02	"-"	Acrollcannb
4	(Cos Roll)(Sin Ann)	4.56E-02	"-"	Acrollsannb
5	(Cos Roll)(Cos Orb)	9.11E-02	"-"	Acrollcorbb
6	Sin Roll	9.11E-02	"-"	Asrollb
7	Linear in time at Sin Roll	9.11E-02	marcsec/yr	Atsrollb
8	(Sin Roll)(Cos Ann)	4.56E-02	"-"	Asrollcannb
9	(Sin Roll)(Sin Ann)	4.56E-02	"-"	Asrollsannb
10	(Sin Roll)(Cos Orb)	9.11E-02	"-"	Asrollcorbb

Contributions to unmodeled output: $Z_{unm\delta c} = (1/C_g'') H y_{nom} \delta C_{unm}$

	Zunmδc	Linear in Time	Cos Ann	Sin Ann	Cos Orb	Sin (2roll)	Cos (2roll)	Total
1	Cos Roll	0	3.36E-04	-2.76E-02	0.00E+00	0	0.00E+00	2.76E-02
2	Linear in time at Cos Roll	5.01E-03	0	0	0	0	0.00E+00	5.01E-03
3	(Cos Roll)(Cos Ann)	0	0	0	0	0	0.00E+00	0.00E+00
4	(Cos Roll)(Sin Ann)	0	0	0	0	0	0.00E+00	0.00E+00
5	(Cos Roll)(Cos Orb)	0	0	0	0	0	0.00E+00	0.00E+00
6	Sin Roll	0	-3.56E-02	-3.14E-02	0.00E+00	2.49E-03	0	4.75E-02
7	Linear in time at Sin Roll	4.99E-03	0	0	0	5.20E-05	0	4.99E-03
8	(Sin Roll)(Cos Ann)	0	0	0	0	-3.56E-02	0	3.56E-02
9	(Sin Roll)(Sin Ann)	0	0	0	0	-3.14E-02	0	3.14E-02
10	(Sin Roll)(Cos Orb)	0	0	0	0	-1.88E-05	0	1.88E-05

UNMODELED SCALE FACTOR VARIATIONS (δC_{unm})

	Frequency and phase: δC_{unm}	Magnitude	Units	Name
1	**Linear in Time**	5.00E-06	yr^{-1}	Alins
2	**Cos Ann**	5.00E-06	Dless	Acanns
3	**Sin Ann**	5.00E-06	"-"	Asanns
4	**Cos Orb**	5.00E-06	"-"	Acorbs
5	**Sin (2roll)**	5.00E-06	"-"	As2rolls
6	**Cos (2roll)**	5.00E-06	"-"	Ac2rolls

Contributions to unmodeled output: $Zunm\delta\phi = H (d\, y/d\delta\phi)_{nom} \delta\phi_{unm}$

	Zunmδφ	Linear in Time	Cos Ann	Sin Ann	Cos Orb	Sin (2roll)	Cos (2roll)	Total
1	(cos roll)	0	-1.42E-01	-1.26E-01	-1.51E-04	0	0.00E+00	1.90E-01
2	(t/T) * (cos roll)	1.99E-02	0	0	0	0	0.00E+00	1.99E-02
3	(cos ann)(cos roll)	0	0	0	0	0	0.00E+00	0.00E+00
4	(sin ann)(cos roll)	0	0	0	0	0	0.00E+00	0.00E+00
5	(cos orb)(cos roll)	0	0	0	0	0	0.00E+00	0.00E+00
6	(sin roll)	0	-1.25E-03	1.10E-01	-5.18E-02	0.00E+00	0	1.22E-01
7	(t/T) * (sin roll)	-2.01E-02	0	0	0	0.00E+00	0	2.01E-02
8	(cos ann)(sin roll)	0	0	0	0	0.00E+00	0	0.00E+00
9	(sin ann)(sin roll)	0	0	0	0	0.00E+00	0	0.00E+00
10	(cos orb)(sin roll)	0	0	0	0	0.00E+00	0	0.00E+00

UNMODELED ROLL PHASE OFFSET VARIATIONS ($\delta\phi_{unm}$)

	Frequency and phase: $\delta\phi_{unm}$	Magnitude	Units	Name
1	Linear in Time	2.00E-05	yr^{-1}	Alinp
2	Cos Ann	2.00E-05	Dless	Acannp
3	Sin Ann	2.00E-05	"-"	Asannp
4	Cos Orb	2.00E-05	"-"	Acorbp
5	Sin (2roll)	0.00E+00	"-"	As2rollp
6	Cos (2roll)	0.00E+00	"-"	Ac2rollp

Orbital Velocity Error

$$\delta Z_{unmo} = (\delta V_{orb}/V_{orb}) A_o C_g [\cos(\omega_o t)\cos(\omega_r t)]$$

Earth Velocity Error

$$\delta Z_{unma} = \delta l_1 Cg [\cos(\omega_a t)\cos(\omega_r t)] + \delta l_2 Cg [\sin(\omega_a t)\cos(\omega_r t)] + \delta l3 Cg [\cos(\omega_a t)\sin(\omega_r t)] + \delta l4 Cg [\sin(\omega_a t)\sin(\omega_r t)]$$

Unmodeled Aberration (Errors in S/C Velocity)

		Magnitude	Name	Units
1	δV_{orb}	1.00E+01	dvo	cm/sec
2	$\delta l_1/l_1$	0.00E+00	d1an	Dless
3	$\delta l_2/l_2$	0.00E+00	d2an	Dless
4	$\delta l_3/l_3$	0.00E+00	d3an	Dless
5	$\delta l_4/l_4$	0.00E+00	d4an	Dless

	δZunmab		Name	Units
1	Cos Roll	0	Acrollv	marcsec
2	Linear in time at Cos Roll	0	Atcrollv	marcsec/yr
3	(Cos Roll)(Cos Ann)	0.00E+00	Acrollcannv	marcsec
4	(Cos Roll)(Sin Ann)	0.00E+00	Acrollsannv	"-"
5	(Cos Roll)(Cos Orb)	6.40E-02	Acrollcorbv	"-"
6	Sin Roll	0	Asrollv	"-"
7	Linear in time at Sin Roll	0	Atsrollv	marcsec/yr
8	(Sin Roll)(Cos Ann)	0.00E+00	Asrollcannv	marcsec
9	(Sin Roll)(Sin Ann)	0.00E+00	Asrollsannv	"-"
10	(Sin Roll)(Cos Orb)	-1.86E-04	Asrollcorbv	"-"

TOTAL UNMODELED VARIATIONS

	Frequency and phase:	Magnitude	Units	Name
1	Cos Roll	0.212	marcsec	Acroll
2	Linear in time at Cos Roll	0.093	marcsec/yr	Atcroll
3	(Cos Roll)(Cos Ann)	0.046	marcsec	Acrollcann
4	(Cos Roll)(Sin Ann)	0.046	"-"	Acrollsann
5	(Cos Roll)(Cos Orb)	0.111	"-"	Acrollcorb
6	Sin Roll	0.160	"-"	Asroll
7	Linear in time at Sin Roll	0.093	marcsec/yr	Atsroll
8	(Sin Roll)(Cos Ann)	0.058	marcsec	Asrollcann
9	(Sin Roll)(Sin Ann)	0.055	"-"	Asrollsann
10	(Sin Roll)(Cos Orb)	0.091	"-"	Asrollcorb

Step1(UnmInf)

STEP1: $Y_{unm} = \sigma^{-2} P_1 (H^T Z_{unm})$

$\sigma^{-2}H^T Zunm$ (10 x1)	Yunm (10x1)
3.44E+02	0.212
1.82E+01	9.34E-02
2.76E+01	4.56E-02
4.57E+01	4.56E-02
2.32E+02	1.11E-01
2.64E+02	1.60E-01
2.01E+01	9.34E-02
3.51E+01	5.78E-02
5.16E+01	5.53E-02
1.79E+02	9.11E-02

STEP1: $P1_{unm} = E[\, Y_{unm} Y^T_{unm}]$ **(10x10)**

4.51E-02	0	0	0	0	0	0	0	0	0
0	8.72E-03	0	0	0	0	0	0	0	0
0	0	2.07E-03	0	0	0	0	0	0	0
0	0	0	2.07E-03	0	0	0	0	0	0
0	0	0	0	1.24E-02	0	0	0	0	0
0	0	0	0	0	2.55E-02	0	0	0	0
0	0	0	0	0	0	8.73E-03	0	0	0
0	0	0	0	0	0	0	3.34E-03	0	0
0	0	0	0	0	0	0	0	3.06E-03	0
0	0	0	0	0	0	0	0	0	8.30E-03

Combined Information Matrix $I_y = (P_1 + P_{1unm})^{-1} = I_1 - I_{1unm}$

$I_{1unm} = I_1 * P_{1unm} * I_1$

7.38E+04	0	-9.84E-06	0	4.81E+04	0	0	0	0	0
0	1.67E+02	0	4.14E+02	0	0	0	0	0	0
-9.84E-06	0	7.64E+02	0	-6.41E-06	0	0	0	0	0
0	4.14E+02	0	1.09E+03	0	0	0	0	0	0
4.81E+04	0	-6.41E-06	0	3.15E+04	0	0	0	0	0
0	0	0	0	0	4.25E+04	0	-5.86E-06	0	2.78E+04
0	0	0	0	0	0	2.04E+02	0	5.29E+02	0
0	0	0	0	0	-5.86E-06	0	1.23E+03	0	-3.83E-06
0	0	0	0	0	0	5.29E+02	0	1.45E+03	0
0	0	0	0	0	2.78E+04	0	-3.83E-06	0	1.83E+04

STEP 2 Information Matrix: $I_x = (dy/dx)^T I_y(dy/dx) = I_2 - I_2unm$

$I_2unm = (dy/dx)^T I_1unm\ (dy/dx)$

Intermediate step

$W_{unm} = (dy/dx)^T I_1unm$ (6x10)

3.23E+08	-3.47E+06	9.56E+04	-9.31E+06	2.11E+08	4.19E+07	-6.65E+06	-1.75E+07	-1.82E+07	2.74E+07
7.29E+07	-5.19E+06	-1.09E+07	-1.37E+07	4.75E+07	-1.87E+08	4.51E+06	-1.54E+05	1.26E+07	-1.22E+08
7.38E+04	0	-9.84E-06	0	4.81E+04	-123.4877	0	1.706E-08	0	-80.7592
2.15E+02	0	-2.862E-08	0	1.40E+02	4.25E+04	0.00E+00	-5.86E-06	0	2.78E+04
0	1.67E+02	0	4.14E+02	0	0	-0.591983	0	-1.539901	0
0	0.4847757	0	1.2032601	0	0	2.04E+02	0	5.29E+02	0

Unmodeled Information Matrix $I_2unm = W_{unm}\ (dy/dx)$ (6x6)

2.02E+12	9.52E+10	3.23E+08	4.29E+07	-3.45E+06	-6.66E+06
9.52E+10	1.33E+12	7.34E+07	-1.86E+08	-5.21E+06	4.49E+06
3.23E+08	7.34E+07	7.38E+04	9.13E+01	0.00E+00	0.00E+00
4.29E+07	-1.86E+08	9.13E+01	4.25E+04	0.00E+00	0.00E+00
-3.45E+06	-5.21E+06	0.00E+00	0.00E+00	1.67E+02	-1.07E-01
0.00E+00	4.49E+06	0.00E+00	0.00E+00	-1.07E-01	2.04E+02

STEP 2 Covariance Matrix: $Px = P_2 + P_2unm$

$$P_2unm = P_2{*}I_2unm{*}P2$$

Intermediate step

$Wum = P_2{*}I_2unm$ (6x6)

1.96E+00	-8.40E-03	2.96E-05	2.85E-08	-5.44E-06	-9.19E-06
-1.59E-01	1.38E+00	-2.85E-08	-1.98E-05	-6.24E-06	8.05E-06
2.58E+05	5.92E+04	6.07E+01	9.49E-02	2.97E-02	3.15E-02
3.27E+04	-1.48E+05	4.55E-02	3.49E+01	-2.14E-02	4.38E-02
-9.66E+03	-1.85E+04	4.27E-01	-4.76E-01	1.42E+00	5.97E-02
-1.64E+04	2.43E+04	7.12E-01	2.87E-01	-4.17E-02	1.68E+00

Unmodeled Covariance Matrix: $P_2um = Wum{*}P_2$ (6x6)

7.96E-12	-1.87E-13	-9.62E-09	-8.74E-09	5.66E-08	1.03E-07
-1.87E-13	5.32E-12	-4.54E-09	6.72E-09	6.34E-08	-1.66E-09
-9.62E-09	-4.54E-09	5.01E-02	6.83E-05	4.52E-05	1.46E-04
-8.74E-09	6.72E-09	6.83E-05	2.88E-02	-1.76E-04	1.26E-04
5.66E-08	6.34E-08	4.52E-05	-1.76E-04	1.64E-02	1.03E-03
1.03E-07	-1.66E-09	1.46E-04	1.26E-04	1.03E-03	1.91E-02

MEASUREMENT ERRORS :

	Modeled Error worst	Modeled Error plausible	Unmodeled Error	Total Error worst	Total Error plausible	
C_g	2.28E-06	1.91E-06	2.82E-06	3.63E-06	3.41E-06	dless
$\delta\phi$	2.28E-06	1.91E-06	2.31E-06	3.24E-06	3.00E-06	rad
NS	0.030	0.029	0.224	0.226	0.226	marcsec
EW	0.030	0.029	0.170	0.172	0.172	marcsec
R_gT_a	0.118	0.093	0.128	0.174	0.158	marcsec
R_mT_a	0.118	0.093	0.138	0.182	0.166	marcsec

Main Parameters Affecting Measurement Errors

Input from	Parameter Name	Worst Case Value	Units	Plausible Case Value
Parameters	Gyroscope spin frequency	143	Hz	143
"-"	Satellite roll period	3	minutes	3
ME Inp Par	Experiment duration	12	month	13
"-"	SQUID noise intensity	112.7272727	marcsec/sqrt(Hz)	112.7272727
"-"	Relativistic drift in NS direction, R_g	6600	marcsec/yr	6600
"-"	Relativistic drift in EW direction, R_m	40	marcsec/yr	40
"-"	Fraction of orbit GS valid, f_0	0.5	Dless	0.5
Parameters	Averaged misalignment, NS	1	arcsec	1
"-"	Averaged misalignment, EW	1	arcsec	1
"-"	Declination of Guide Star (HR 8703), δ	2.9391E-01	rad	2.9391E-01
"-"	Right ascention of Guide Star (HR 8703), α	5.9910E+00	rad	5.991017191
ME Inp Par	Start date (Jan 1 =1)	202.	days	202