W. W. Hansen Experimental Physics Laboratory
STANFORD UNIVERSITY
STANFORD, CALIFORNIA 94305 - 4085

Gravity Probe B Relativity Mission

VERIFICATION OF T002 REQUIREMENT 11, Part A: CONDITIONING, SAMPLING, PROCESSING OF SCIENCE GYROSCOPE AND SCIENCE TELESCOPE SIGNALS

S0861, Rev. -
April, 2003

Prepared by:	Approved by:
7/10/03 Date	04/28/03 Date
G. M. Keiser GP-B Chief Scientist	John P. Turneaure
Approved by:	Approved by:
7/11/03 Date	7/11/2003 Date
D.Ross FOR QA	R. Whelan Systems Engineering

ITAR Assessment Performed ______ Tom Langenstein 7.12.03

ITAR Control Req'd? ☐ Yes ☒ No

the time a signal is sampled and when it is time tagged. Since the effective sample times of these two signals are known to an accuracy better than 0.1 msec [6], the worst case bias error at the maximum satellite roll frequency (1 rpm) due to the worst case roll frequency pointing error (3.4mas) is $b < 3.4 \times 10^{-5}$ mas.

V. Conclusion

The science gyroscope and science telescope conditioning, sampling, and on-board processing have been designed so that the phase shift and attenuation of these signals is much less than the accuracy with which these signals must be differenced. The dominant error is due to the variation in the relative attenuation and a roll frequency pointing error, which combine to give a worst-case roll frequency bias error of 3.4×10^{-4} mas. This value is significantly below the required value of 0.1 mas.

References:

1. Everitt, C.W.F., W.M. Fairbank, D.B. DeBra, *et al., Report on a Program to Develop a Gyro Test of General Relativity in a Satellite and Associated Control Technology*, Stanford University, S0018, June 1980
2. Lockheed, *System Design and Documentation, Control System Error Budgets and Capability*, Lockheed Martin Missiles and Space, SCSE-04, Part 5, Rev. C, 18 Oct. 2002
3. Turneaure, J.P., *Verification of T002 Pointing Requirements*, Hansen Laboratories, GP-B, Stanford University, S0850, Rev. -, April 21, 2003
4. Whelan, R. and B. Muhlfelder, *Verification of Trapped Flux Requirements*, GP-B, Hansen Laboratories, Stanford University, S0564 Rev. A, September 20, 2001
5. Keiser, G.M. and A.S. Silbergleit, *Pick-up Loop Symmetry and Centering*, Gravity Probe B, Stanford University, S0243,
6. Li, J., P. Shestople, and G.M. Keiser, *Gravity Probe B Timing System*, Hansen Labs, Stanford University, S0307, Rev. A, June 20, 2003
7. Heifetz, M. and G.M. Keiser, *SQUID Readout (Science) Digital Lowpass Filter: Algorithm*, GP-B, Hanson Laboratories, Stanford University, S0250, Rev. A, April 2002
8. Ehrensberger, P., *Forward Telescope Readout Electronics (TRE) Specification*, Lockheed Martin Missles and Space, LMMS/P480135 Rev B, June 9, 1999
9. Boretsky, P., *Estimation of Slope for Science Telescope*, Stanford University, GP-B, S0252 Rev. A, February 1, 2001

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Frequency Dependence of the Gyroscope and Telescope Transfer Functions

The relative scale factors of the gyroscope and telescope readout systems are measured at the dither frequency. However, it is near the satellite roll frequency where the signals must be accurately differenced. Therefore, any frequency dependence of the gain between these two frequencies has the potential for causing errors in the subtraction.

Requirements on the signal conditioning of the gyroscope readout system are specified in paragraph 3.5.7.2 of the System Design and Performance Requirements (T003):

T003 Requirement 3.5.7.2 Matching
The frequency dependence and phase shift of the SQUID transfer function (volts/SQUID flux quanta) shall be either known and stable or matched to the frequency dependence of the telescope transfer function to an accuracy of 2 % over the frequency range from the SV roll frequency to the dither frequency (Section 19.5).

At frequencies below the maximum dither frequency (1/20 Hz), the frequency response of the conditioned science gyroscope readout is dominated by the 4 Hz analog low pass filter. Using the frequency response given is Section II above, the attenuation of the output at 1/20 Hz is less than 7×10^{-5}. Since this attenuation is less than 1/360, there is no need to account for this attenuation when matching the scale factor ratios at frequencies as high as 1/20 Hz. Also, at a frequency of 1/20 Hz or less, the on-board digital low pass filter causes an attenuation of less than 10^{-4}.

Similarly, the conditioning, sampling, and processing of the science telescope signals has a negligible effect on the frequency dependence of the transfer function of the telescope readout system below 1/20 Hz. The 500 Hz double-pole low-pass filter attenuates the frequencies below 1/20 Hz less than 10^{-8}. The sampling and processing of the telescope signals with the Kalman filter every 0.1 seconds attenuates the signals above the 5 Hz Nyquist frequency but has a negligible effect below the Nyquist frequency. To the accuracy required, there is no significant attenuation of the gyroscope or telescope signals below 1/20 Hz. Therefore, there will be no significant difference between the scale factor ratios at the pointing control dither frequency compared to the satellite roll frequency.

If the difference between the relative attenuation at the satellite dither frequency and the satellite roll frequency is less than 10^{-4}, then the residual roll frequency pointing error is the product of the amplitude of the roll frequency pointing error (3.4 mas worst-case) multiplied by the error in the relative attenuation, 10^{-4}. The product of factors produce a bias error of 3.4×10^{-4} mas.

Frequency Dependence of the Phase Shifts of the Gyroscope and Telescope Signals

To first order, the ratios of the telescope and gyroscope scale factors defined above are insensitive to a phase shift at the satellite dither frequency since this ratio depends only on the amplitude of the signals. However, when the telescope signals are subtracted from the gyroscope signals, an error in the relative effective sample time of these two signals will leave a residual pointing error signal at the satellite roll frequency. This roll frequency bias error is given by

$$b = \theta_p \sin \omega_r \Delta t$$

where θ_p is the amplitude of the roll frequency pointing error, ω_r is the angular roll frequency, and Δt is the error in the relative effective sample time of the gyroscope and telescope signals. This effective sample time includes any time delay due to a linear phase shift in the low pass filters as well as any data latency between

LIST OF ACRONYMS, ABBREVIATIONS, AND SYMBOLS

δ_X	Phase shift of gyroscope readout relative to telescope readout at x-axis dither frequency
δ_Y	Phase shift of gyroscope readout relative to telescope readout at y-axis dither frequency
Δ	Angle between normal to gyroscope pickup loop and telescope x-axis
Δt	error in data latency
θ_p	Roll frequency pointing error
ω_X	angular frequency of x-axis pointing dither
ω_Y	angular frequency of y-axis pointing dither
ω_r	Angular roll frequency
μas	micro-arc-seconds
A/D	Analog-to-Digital
b	bias
cm	centimeter
C_G	Gyroscope scale factor
C_{TX}	x-axis telescope scale factor
C_{TY}	y-axis telescope scale factor
d_X	angular amplitude of x-axis pointing dither
d_Y	angular amplitude of y-axis pointing dither
FLL	Flux Locked Loop
FIR	Finite Impulse Response

Hz	Hertz
marcsec	milli-arc-second
mas	milli-arc-second
MHz	Mega-Hertz
nsec	nanosecond
rms	root mean square
rpm	Revolutions per minute
R_X	ratio used to combine x-axis telescope signal with gyroscope signal
R_Y	ratio used to combine y-axis telescope signal with gyroscope signal
s	Complex Variable used for Laplace Transforms
SG	Science Gyroscope
SQUID	Superconducting Quantum Interference Device
SRE	SQUID Readout Electronics
ST	Science Telescope
SV	Space Vehicle
t	time
z	combined gyroscope and telescope signals
z_G	gyroscope readout signal
z_{TX}	x-axis telescope pointing signal
z_{TY}	y-axis telescope pointing signal

$$R_X = \frac{|z_G(\omega_x)|}{|z_{TX}(\omega_x)|} = \frac{C_G \cos\Delta}{C_{TX}}$$

$$R_Y = \frac{|z_G(\omega_y)|}{|z_{TY}(\omega_y)|} = \frac{C_G \sin\Delta}{C_{TY}}$$

These ratios will be different for each of the four gyroscopes. To remove the residual pointing error signal from the gyroscope readout, the telescope pointing signal on the x-axis and the y-axis is multiplied respectively by R_X and R_Y, and result is subtracted from the gyroscope readout signal (ref. telescope document). The resulting quantity,

$$z = z_G - R_X z_{TX} - R_Y z_{TY},$$

contains no pointing error signals, only the roll modulated orientation of the gyroscope spin axis relative to the guide star. This quantity, z, will be used for subsequent analysis. The phase shift at the satellite dither frequencies may also be determined from these analyses:

$$\delta_X = \angle z_G(\omega_X) - \angle z_{TX}(\omega_X)$$

$$\delta_Y = \angle z_G(\omega_Y) - \angle z_{TY}(\omega_Y)$$

Although these phase shifts are expected to be zero, they provide a useful check on any data latency or phase shift in the signal processing chain.

The accuracy with which the relative scale factor of the telescope and gyroscope signals must be matched depends on the amplitude of the roll frequency pointing error. This amplitude is specified by paragraph 19.3.1 of the System Design and Performance Requirements (T003):

T003 Requirement 19.3.1 Science Pointing, Guide Star Valid
During science operation and Guide Star Valid, the satellite telescope axis shall be pointed to within 50 marcsec rms of the guide star optical direction for each axis. The rms is computed over the time period of the guide star valid. The roll frequency component of the observable body-fixed pointing error (d.c. component of inertially-fixed bias) during guide star valid shall be less than 5 marcsec averaged over any period of four orbits.

The expected worst case amplitude is 3.4 mas [2]. To remove any residual pointing error from the gyroscope readout to an accuracy of 0.01 mas, the relative scale factor must be known to an accuracy of 1 part in 360. The time required to match the scale factors to this accuracy is largely determined by the noise in the gyroscope readout system.

Error Sources due to Signal Conditioning, Sampling, and Processing

Potential sources of error due to the signal conditioning, sampling, and processing of the science telescope and gyroscope signals are

(1) frequency dependence of the gyroscope and telescope readout systems that causes the relative scale factors at the dither frequency to differ from the relative scale factor at the satellite roll rate, and
(2) phase shifts in the telescope signals relative to the gyroscope signals at the satellite roll frequency.

Each of these potential sources of error is discussed below.

I. Introduction

The purpose of this document is to verify Part A of Requirement 11 of the Gravity Probe B Twelve Fundamental Requirements (T002), which is

11. Telemetry and Data Processing

The SG and ST signals shall be conditioned, sampled and processed so that they can be differenced on the ground with an added error due to telemetry and data processing of less than 0.1 marcsec over a range which covers aberration, dither and nominal drift.

The science gyroscope (SG) readout measures the orientation of the gyroscope spin axis relative to the quartz block reference frame, while the science telescope (ST) readout measures the orientation of the quartz block reference frame with respect to the apparent position of the guide star. To determine the orientation of the gyroscope spin axis relative to the apparent position of the guide star, the relative scale factor of these signals must be determined and the signals must be subtracted. There are a number of different ways that this subtraction could be done. In early designs of the Gravity Probe B satellite [1], this scale factor matching and subtraction of these signals were performed on orbit, and only the difference between these two signals was included in the telemetry. However, to reduce the risk of an error in this subtraction process, both signals are now included in the telemetry, and the scale factor matching and subtraction are done with ground-based data analysis. The purpose of this requirement is to insure that the on-board conditioning, sampling, and processing of these signals is adequate so that the scale factor matching and subtraction may be done using the ground-based data analysis.

The attitude and translation control system on the Gravity Probe B satellite is designed to operate so that the science telescope is always pointing toward the apparent position of the guide star as long as the guide star is in view. However, there will be some residual pointing error at the satellite roll frequency. Requirement 19.3 of the System Design and Performance Requirements (T003) specifies that this roll frequency pointing error shall be less than 5 mas. Based on simulations, the worst-case amplitude of the roll frequency pointing error is expected to be less than 3.4 mas [2]. With this expected worst-case performance, the residual error due to the pointing error will be less than 0.1 mas as long as the scale factor ratios are know to an accuracy of 3 %. The dominant source of error in determining these ratios is the noise in the gyroscope readout system as described in "Verification of T002 Pointing Requirements" [3]. This document describes the small additional contribution to the residual pointing error, after the science gyroscope and science telescope signals have been differenced, due to the conditioning, sampling, and data processing.

The conditioning, sampling, and processing of the science gyroscope and science telescope signals are each described in the two following sections. Then, the fourth part of this document describes potential errors that may be introduced when these signals are differenced. Section V concludes that the worst-case added error due to telemetry and data processing is less than 0.34 μas (0.00034 mas). Thus T002 requirement 11A is met since 0.34 μas is negligible compared to the required value of 0.1 mas.

On-Board Processing

Of the 220 sampled data points, a Kalman filter uses every other data point to compute the slope of the TRE output every 0.1 seconds. This algorithm is described in the document, "Estimation of Slope for Science Telescope" [9].

Ground Based Processing

The two photocurrents from a pair of photodiodes are combined in ground-based processing to determine the normalized pointing signal along that axis. The normalized pointing signal is the difference in the photocurrents of the photodiodes divided by their sum. The result is a signal proportional to the pointing error within the linear region of the telescope readout.

IV. Matching of Telescope Scale Factor to Gyroscope Scale Factor and Differencing the Signals

The procedure for matching the telescope signals to gyroscope signals using the dither signal is as follows. A deliberate pointing dither is injected into both axes of the pointing control system with an amplitude of approximately 30 mas (milli-arc-seconds), which is well within the linear range of the telescope output. This dither signal is specified in the System Design and Performance Requirements (T003) Paragraph 19.5:

T003 Requirement 19.5 Dither
The SV shall have the capability to sinusoidally slew (dither) the pitch or yaw angles with a magnitude up to 30 marcsec ± 10% and frequencies in the range from twice orbital frequency to 0.05 Hz. The specific amplitude, phase, and frequency of both the pitch and yaw angles shall be independently commandable from the ground.

The components of the dither signals in the telescope readouts are given by

$$z_{TX} = C_{TX} d_X \cos \omega_X t$$
$$z_{TY} = C_{TY} d_Y \cos \omega_Y t$$

where C_{TX} and C_{TY} are the scale factors of the telescope readout on the x- and y-axes. The signals at these dither frequencies in readouts of each of the four gyroscopes are

$$z_G = C_g \left[d_X \cos(\omega_X t + \delta_X) \cos \Delta + d_Y \cos(\omega_Y t + \delta_Y) \sin \Delta \right]$$

where C_g is the gyroscope scale factor at the dither frequencies, d_X and d_Y are the amplitudes of the pointing dither on the telescope x and y axes, ω_X and ω_Y are the dither frequencies on the corresponding axes, and Δ is the angle between the telescope x-axis and the normal to the pickup loop. This angle Δ is known to an accuracy of 1^0 for each of the four gyroscopes and is approximately 0^0 or 90^0 depending on which gyroscope is used. The phase shifts δ_X and δ_Y have been included to account for any potential phase shift in the gyroscope readout relative to the telescope readout. Nominally these phase shifts are zero.

By measuring the amplitude and phase of the gyroscope and telescope signals at each of the dither frequencies, the following ratios may be calculated:

II. Science Gyroscope Readout

The design of the conditioning, sampling, and processing of the science gyroscope readout signals is driven by characteristics of the output signal and the requirements on the stability and frequency response of the filters.

The output signal consists of contributions from both the London Moment and the trapped magnetic flux in the gyroscope rotor. For a gyroscope spinning at approximately 150 Hz, the London moment equivalent magnetic field is 1.1×10^{-4} gauss. Since the area of the pickup loop is approximately 10 cm^2 and it has four turns, the magnetic flux through the pickup loop is 4×10^{-3} gauss cm^2 multiplied by the sine of the angle between the gyroscope spin axis and the plane of the pickup loop. For an angle between the gyroscope spin axis and the satellite roll axis of 20 arc seconds, the variation of the magnetic flux through the pickup loop at the satellite roll frequency is 4×10^{-7} gauss cm^2.

Although the equivalent dipole field of the trapped magnetic flux is more than a factor of ten smaller than the London Moment field, the variation through the pickup loop at harmonics of the rotor spin speed is significantly larger. The required value of the dipole equivalent trapped magnetic flux is less than 9 microgauss.This requirement has been verified in reference [4]. Based on this requirement the maximum possible flux through the four-turn pick up loop is then 3.6×10^{-4} gauss cm^2. The orientation of the trapped magnetic flux relative to the gyroscope spin axis is unknown and will vary at the polhode period, but the amplitude of magnetic flux through the pickup loop at the odd harmonics [5] of the gyroscope spin speed could be as large as 3.6×10^{-4} gauss cm^2, which is a factor of 1000 larger than the London Moment signal at the satellite roll rate. These high frequency signals provide valuable information on the gyroscope spin speed, but to take advantage of the London Moment signals for the gyroscope readout, the high frequency trapped flux signals must be adequately filtered.

Since the frequency separation between these signals is large, a significant part of the high frequency signal may be filtered out with an analog low pass filter. Because of tight requirements on the stability of the amplitude, phase shift, and frequency response of this low pass filter, a combined analog and digital filtering approach is used. The analog low pass filter has a relatively high cut off frequency of 4 Hz, the signal is sampled at 2200 Hz, and an additional digital low pass filter with a cut-off frequency of 2.5 Hz removes the residual signals from the trapped magnetic flux at the gyroscope spin speed. The filtered signals are time tagged and included in the telemetry at a rate of 5 Hz. The relatively high cut off frequency of the analog low pass filter reduces the temperature sensitivity of the phase shift and gain at the satellite roll rate. It also improves the flat frequency response over the region of the satellite roll frequency (1/600 to 1/60 Hz), the SRE calibration signal frequency (1/62 or 1/124 Hz), and satellite dither frequency (1/2700 Hz to 1/20 Hz).

Conditioning

Figure 1 is a schematic diagram of the conditioning of the science gyroscope signals.

Sampling

After the conditioning described above, the signals in the low frequency processing chain are sampled at a rate of 2200 Hz by the 16-bit multiplexed A/D converter. The time at which they are sampled is determined by the 16 MHz Oven Controlled Crystal Oscillator in the Aft SRE, and the sampling time is controlled to an accuracy better than 15 nsec.

Processing

The on-board digital filter is a non-causal finite impulse response (FIR) filter that collects 2200 samples over a 1 second interval and multiplies each of these 2200 samples by a predetermined filter coefficient. The 2200 input data points and the filter output are updated every 0.2 seconds. Because the update interval is shorter than the interval over which the data is collected, there is necessarily some overlap between adjacent input data sets. The data is sent over the 1553 bus from the aft SRE processor to the flight computer (CCCA) every 0.1 seconds, but included in the telemetry every 0.2 seconds. The time tagging of the telemetry monitors from this science low pass filter is discussed in the Gravity Probe B Timing Document (S0307) [6].

There are four sets of filter coefficients available in the on-board database. All four sets of filter coefficients are designed to act as a low pass filter with a cutoff frequency at the 2.5 Hz Nyquist frequency of the update rate. All of these sets of filter coefficients are designed to provide a nearly flat frequency response below the fastest dither frequency, which is flat to better than 1 part in 10^4, but provides strong attenuation above the 2.5 Hz cutoff frequency. The characteristics of these filters are described in the S-document on the Science Low Pass Filter [7]. Since these filters are non-causal, and the data latency in the data collection, processing, and time-tagging is taken into account [6], these filters introduce no phase shift.

III. Science Telescope Readout

Conditioning

The output of the telescope charge locked loop is proportional to the charge on the feedback capacitor. Over each 0.1 second interval, the charge on this feedback capacitor increases with a slope that is proportional to the photocurrent. At the end of each 0.1 second interval, the charge on the feedback capacitor is reset. This signal is amplified and filtered with a second order Butterworth filter with a 500 Hz corner frequency (Telescope Readout Electronics Specification [8], Paragraph 3.2.3.5.4). The photocurrent from the diode is proportional to the rate of change of the output.

Sampling

The multiplexed A/D converter samples the filtered signals from each of the photodiodes at a rate of 2200 Hz. The timing of each of the sampled signals is known to an accuracy better than 15 nsec.

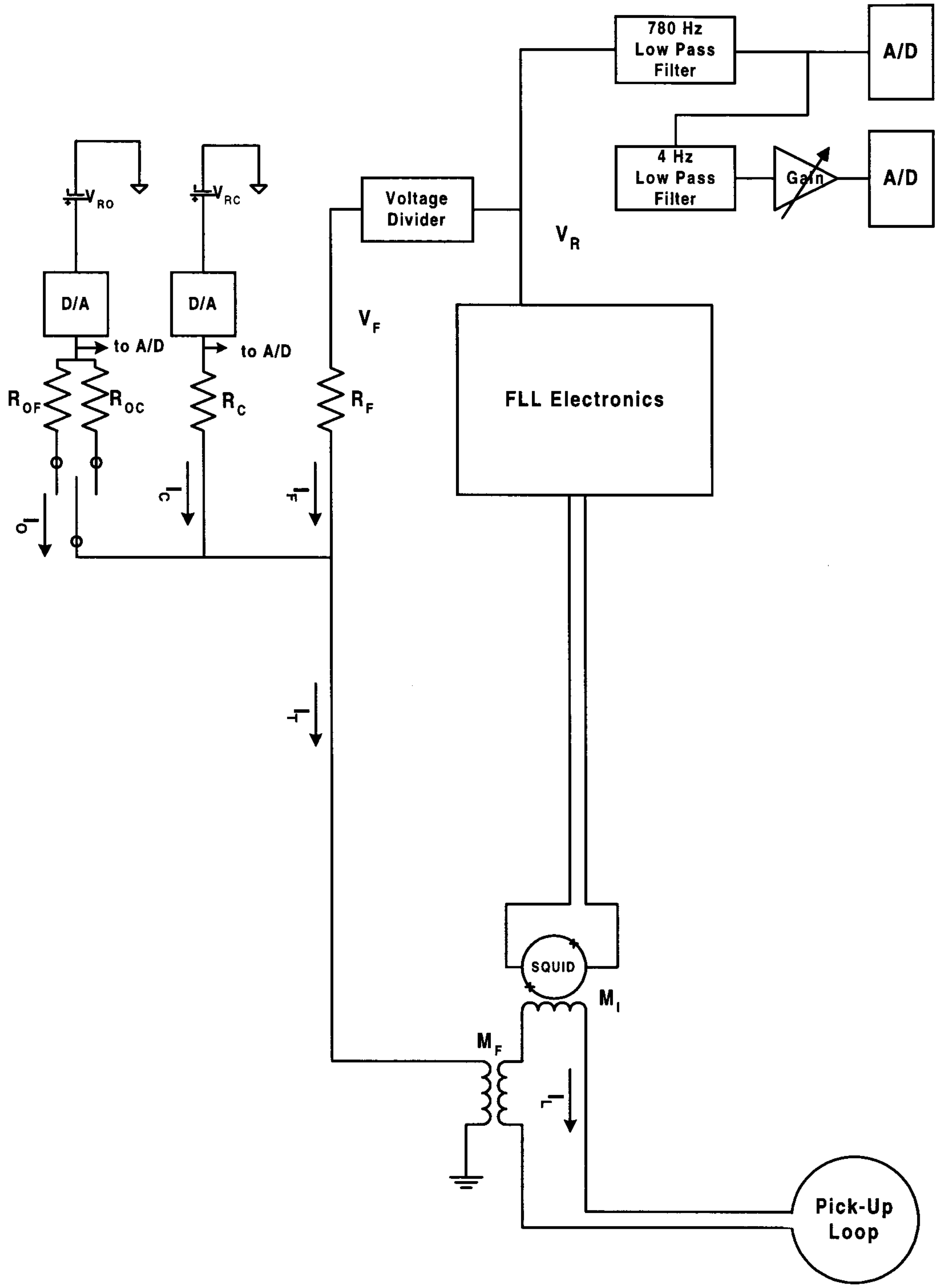

Figure 1. Flux Locked Loop and Signal Conditioning in SQUID Readout Electronics

The feedback voltage for the flux locked loop (FLL) passes through a 780 Hz analog low pass filter and then a 4 Hz analog low pass filter. At that point the signal is amplified before being sampled by the multiplexed analog-to-digital converter. The 780 Hz analog low pass filter is a three pole filter and the 4 Hz analog filter is a two pole filter. The combined transfer function of these two filters is [6]:

$$\left(\frac{1}{10^{-4}s+1}\right)\left(\frac{5.92\times10^{7}}{s^{2}+13333s+5.92\times10^{7}}\right)\left(\frac{4132}{s^{2}+181.8s+4132}\right)$$