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Gravity Probe B Relativity Mission

IMPACT OF POWER BUS SENSITIVITY ON VERIFICATION OF T002 AND T003 REQUIREMENTS

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**Impact of Power Bus Sensitivity on SRE Bias Variation
and Re-Verification of T003 Requirement 3.5.2
and T002 Requirement 10**

Table of Contents

List of Tables	3
List of Figures	3
I. Introduction	4
I. Data Summary	4
II. Impact on Science Data Analysis	6
A. Variation in Roll Frequency Component of Power Bus Voltage	6
B. Estimated Variations in Roll Frequency Output of SRE	10
C. On-Orbit Calibration of the Sensitivity of the SQUID Readout to Power Bus Variation	10
III. Impact on T003 and T002 Requirements	12
A. System Design and Performance Requirements (T003)	12
B. Fundamental Requirements T002	13
C. Summary	14
IV. Re-Verification of T003 and T002 Requirements	16
A. T003 Requirement 3.5.2	16
B. T002 Requirement 10	17
References:	18

List of Tables

Table 1. Sensitivity of SQUID Output to Variations in Power Bus Voltage for Tests on October 17 and 20, 2003	4
Table 2. Explanation of and Additional Information on Various Cases Given in Table 1.5	
Table 3. Sensitivity of SRE Output to Variations in Power Bus Voltage (mas/volt)	6
Table 4. Space Vehicle Estimated Roll Frequency Bus Ripple Test and Analysis Summary	7
Table 5. Estimated Linear, Annual, and Orbital Variation in Power Bus Voltage at the Satellite Roll Frequency	9
Table 6. Estimates the Linear (mas/yr), Orbital (mas), and Annual (mas) Effective Gyroscope Misalignment with No Calibration	10
Table 7. Estimates the Linear (mas/yr), Orbital (mas), and Annual (mas) Effective Gyroscope Misalignment with Calibration. Baseline Mode.	12
Table 8. Limits on Error in Gyroscope Drift Rate (mas/yr) due to Variation in Roll Frequency Component of Power Bus Voltage.	14
Table 9. Contribution of Power Bus Variations to Estimates of T003 and T002 Requirements with No Calibration. Baseline Configuration	15
Table 10. Contributions of Power Bus Variations to Estimates of T003 and T002 Requirements with No Calibration. Contingency Configuration: ECU Off.	15
Table 11. Contributions of Power Bus Variations to T003 and T002 Requirements Verification with Modest Calibration of Power Bus Sensitivity.....	16
Table 12. Roll Frequency Bias Variations from All Sources	17
Table 13. Errors in Measured Gyroscope Drift Rate due to All Sources of Roll Frequency Bias Variation.....	17

List of Figures

Figure 1. Fraction of Each Guide Star Valid Period When Satellite is in Earth's Shadow 8

I. Introduction

During a simulated test of space vehicle operations (SIM 5) in May, 2003, with the Gravity Probe B space vehicle in Sunnyvale, the output of the SRE electronics was found to depend on the spacecraft bus voltage. The dependence was observable because of the 2 volt drop in the bus voltage during 5 minute intervals every 90 minutes when the spacecraft bus power was supplied by the batteries. The sensitivity varied from one SQUID to another, with SQUID 1 having the largest sensitivity. Subsequent investigation showed that this sensitivity was observable when the ECU ± 15 volt power supply was turned on. In addition, the sensitivity was found to be larger when the ECU was operating with the aft ± 15 volt A-side power supply than with the aft ± 15 volt B-side power supply. One further observation was that with the ECU operating on the aft ± 15 volt B-side power supply, the sensitivity was smallest when the QBS heater was turned off and only the forward ECU A-Side Instrumentation was turned on. A summary of the various tests and the best estimate of the sensitivity of each SQUID to changes in the power bus under different operating conditions is provided in part II of this document.

The purpose of this document is to analyze the impact of this power bus sensitivity on the overall accuracy of the Gravity Probe B experiment and to re-verify the requirements related to bias variations in the SQUID Readout Electronics. Further information on the investigation of the SRE power bus sensitivity may be found in the GP-B Discrepancy Report 446.

I. Data Summary

The results of the analysis by J. Turneure of the test data collected on the spacecraft on October 17, and October 20, 2003, is presented in Table 1 with an

Table 1. Sensitivity of SQUID Output to Variations in Power Bus Voltage for Tests on October 17 and 20, 2003

CASE	SQUID (marcsec/V)							
	1		2		3		4	
	0 deg	90 deg	0 deg	90 deg	0 deg	90 deg	0 deg	90 deg
FF	0.2 ± 19.8	7.1 ± 19.8	-3.2 ± 1.0	-0.6 ± 1.0	6.1 ± 1.6	-1.4 ± 1.6	-1.8 ± 1.3	0.5 ± 1.3
GG	1135.5 ± 49.1	-215.1 ± 71.7	7.3 ± 1.2	-3.2 ± 1.3	94.2 ± 3.7	-18.8 ± 5.7	8.7 ± 1.5	-2.8 ± 1.6
HH	1212.9 ± 53.4	117.4 ± 78.0	7.2 ± 1.1	1.3 ± 1.1	85.5 ± 4.0	7.2 ± 5.6	4.7 ± 0.9	1.0 ± 1.0
JJ	1373.9 ± 62.4	287.7 ± 89.0	5.3 ± 1.2	0.1 ± 1.2	99.6 ± 3.5	13.1 ± 5.8	11.3 ± 1.4	0.4 ± 1.5
KK	-259.8 ± 35.3	39.7 ± 35.5	4.4 ± 0.8	-1.3 ± 0.8	-18.9 ± 1.0	0.6 ± 1.1	6.9 ± 0.9	-1.4 ± 0.9
LL	-293.4 ± 78.2	60.8 ± 79.3	2.8 ± 1.2	0.9 ± 1.3	-25.6 ± 1.9	3.0 ± 2.2	4.1 ± 1.5	1.1 ± 1.5
MM	-479.0 ± 87.9	117.1 ± 90.5	0.7 ± 1.6	0.1 ± 1.6	-23.9 ± 2.2	-3.2 ± 2.4	4.2 ± 1.7	-0.4 ± 1.7
NN	-79.7 ± 64.4	39.0 ± 64.4	1.6 ± 1.0	-1.0 ± 1.0	-0.3 ± 1.5	-2.7 ± 1.5	3.8 ± 1.4	-1.1 ± 1.4
PP	-326.0 ± 57.2	26.9 ± 58.2	-1.2 ± 1.3	0.0 ± 1.3	11.9 ± 1.4	-0.4 ± 1.4	3.4 ± 1.9	4.1 ± 1.9
QQ	No Good	No Good	0.0 ± 0.3	-0.1 ± 0.3	3.5 ± 0.5	-0.3 ± 0.5	1.3 ± 0.7	-1.3 ± 0.7
RR	No Good	No Good	1.4 ± 0.6	-0.5 ± 0.6	6.7 ± 1.5	-0.2 ± 1.5	-15.7 ± 1.8	2.4 ± 1.8

Table 2. Explanation of and Additional Information on Various Cases Given in Table 1.

CASE	# Points	Buss V Correlation	Info	Comments
FF	1200	0.805	SRE Only	
GG	456	0.663	SRE, ECU A/A,B	
HH	450	0.849	SRE, ECU A/A,B, QBS AC CL SP=0 K	
JJ	450	0.791	SRE, ECU A/A,B, QBS DC OL HSP=0	
KK	3600	0.791	SRE, ECU B/A,B	
LL	450	0.780	SRE, ECU B/A,B, QBS AC CL SP=0 K	
MM	450	0.786	SRE, ECU B/A,B, QBS DC OL HSP=0	
NN	855	0.748	SRE, ECU B/A	
PP	900	0.786	SRE, ECU B/ B	
QQ	3780	0.736	SRE + GSS	SQUID 1 Has Large Jumps
RR	3600	0.424	SRE + GSS + ECU B/A,B	SQUID 1 Has Large Jumps Correlation low, but fit is good. Buss V data is probably anomalous Flux jumps in SQUID 4 probably produces the increased sensitivity

explanation of the various cases given in Table 2. The numbers given in Table 1 are derived using the estimated spin speed for each of the four gyroscopes. For each of the four SQUIDs the column labeled "0 deg" gives the sensitivity of the SQUID to power bus variations determined from the variation in the output of the SRE that is in phase with the square wave modulation of the power bus voltage. The column labeled "90 deg" gives the sensitivity of the SQUID to variations in the power bus voltage that are 90 degrees out of phase with the commanded variations. This second column gives a measure of the error in the analyzed data. Under normal conditions, it should be zero to within the 1 sigma error bars 67% of the time.

The notation in the column labeled "Info" gives the operating conditions of the test. In the notation ECU x/y, x denotes which of the two aft ± 15 volt power supplies is turned on (A or B), and y denotes which of the sets of forward electronics is turned on (A,B, or both). The condition of the Quartz Block Support (QBS) heater temperature control is denoted as follows: QBS AC CL SP=0 K means that the QBS heater was turned on the a.c. closed-loop control mode with the set point at 0K, QBS DC OL HSP=0 means that the QBS heater was turned on in the d.c. open loop control mode with the heater set point at 0.

From this analysis, it can be seen that, while the ECU is turned on, the lowest sensitivity is for case NN, where the ECU is operated on the B side power supply and the only the A-side forward electronics is turned on. Because this condition has the lowest sensitivity, it has been chosen as the baseline condition. As a contingency, the ECU may be turned off which corresponds to case FF or case QQ. For case QQ, the data from SQUID 1 was not useable, so case FF is chosen as representative of this condition.

II. Impact on Science Data Analysis

Based on these measurements, the output of the SQUID Readout Electronics (SRE) will vary at the same rate as the variations in the power bus voltage. The magnitude of these variations will depend on the size of the coupling between the output of the SRE and the power bus voltage and the magnitude of the variations in the power bus voltage. For the purpose of this document, it is assumed that the ECU is operating with the aft ± 15 volt B-side power supply and the forward ECU A-side instrumentation. Under these conditions, the best estimate of the sensitivity of each of the SQUIDS to variations in the power bus voltage is summarized in Table 3 below.

Table 3. Sensitivity of SRE Output to Variations in Power Bus Voltage (mas/volt)

ECU Configuration	SQUID 1	SQUID 2	SQUID 3	SQUID 4
Baseline : B-side ± 15 power supply A-Side Electronics	-79.7 ± 64.4	1.6 ± 1.0	-0.3 ± 1.0	3.8 ± 1.4
Contingency: ECU Off	0.2 ± 19.8	-3.2 ± 1.0	6.1 ± 1.6	-1.8 ± 1.3

A. Variation in Roll Frequency Component of Power Bus Voltage

There are three modes of regulating the power bus voltage depending on whether the power is supplied by the batteries, the power is supplied by the solar arrays with the battery charging, or the power is supplied by the solar arrays with the batteries fully charged. While the bus voltage is supplied by the batteries during eclipse, the d.c. power bus voltage will operate up to several volts below the level while the solar arrays are supplying the power. In addition, the components of the variation in the power bus voltage at the roll frequency, twice roll frequency, and four times roll frequency are expected to be different depending on the operating mode for the power bus voltage regulation. The roll frequency variations in the power bus sensitivity have been estimated in reference [1] and are reproduced in Table 4 below.

Table 4. Space Vehicle Estimated Roll Frequency Bus Ripple Test and Analysis Summary

Condition	Method	Ref. EM	Peak Power Tracking (PPT)	Voltage/ Temperature (V/T)	Eclipse
2 Batteries BOL	Test	EPS-054	150 mv p-p at 2*roll (no roll signal)	< 1 mv p-p (amb noise floor)	< 1 mv p-p (amb noise floor)
	Analysis	EPS-255	72 mv p-p	25.1 mv p-p	0.1 mv p-p
2 Batteries EOL	Analysis	EPS-208	60 mv p-p	25.04 mv p-p	0.04 mv p-p
1 Battery BOL	Analysis	EPS-255	144 mv p-p	25.2 mv p-p	0.2 mv p-p
1 Battery BOL	Analysis	EPS-208	120 mv p-p	25.08 mv p-p	0.08 mv p-p

The amount of time the electrical power system operates in each of these different modes will vary as the orientation of the sun changes with respect to the spacecraft over the course of the year. Figure 1 below shows the expected fraction of each guide star valid period where the spacecraft lies in the shadow of the Earth. In this figure, day 0 corresponds to Jan. 1. From day 160 (June 9) to day 325 (Nov.20), the satellite spends a significant fraction of each guide star valid period in the earth's shadow. During this interval the gamma angle (angle between the guide star and the sun) lies between 120° and 160° , and the sun is behind the spacecraft

During all other times of the year the spacecraft spends less than 10% of each guide star valid period in the Earth's shadow, and for the most part the spacecraft electrical power system will be operating in the VT and PPT modes. In the Voltage/Temperature (VT) mode, the roll frequency variation in power bus voltage is expected to be 25 mV, whereas in the peak-power-tracking (PPT) mode it may be as large as 72 mV.

The impact of this coupling between the power bus voltage and the SQUID readout electronics will depend on how the amplitude and phase of the roll frequency component changes with time. A constant roll frequency component will have no impact on the measurement of the gyroscope drift rate. Linear variations in the amplitude or phase of the roll frequency component will cause errors in the gyroscope drift rate. Variations in the amplitude or phase of the roll frequency component which are modulated at the orbital or annual frequency will cause errors in the scale factor or roll phase offset of the gyroscope readout system [2]. These errors, in turn, will cause errors in the gyroscope drift rate.

The linear, annual, and orbital frequency components of the roll frequency power bus variations may be estimated using the information in reference [1]. The largest linear

variation in the roll frequency component is due to a potential 15% degradation of the solar arrays over the course of a year. With a 5% imbalance or the loss of a string in the solar array, the linear variation in the power bus voltage while the electrical power system is in the peak-power-tracking mode amounts to a 12 mv variation in the peak-to-peak amplitude. Although the electrical power system is expected to be in this mode approximately 1/3 of the time, it seems appropriate to take this full amount as the linear variation in the roll frequency component.

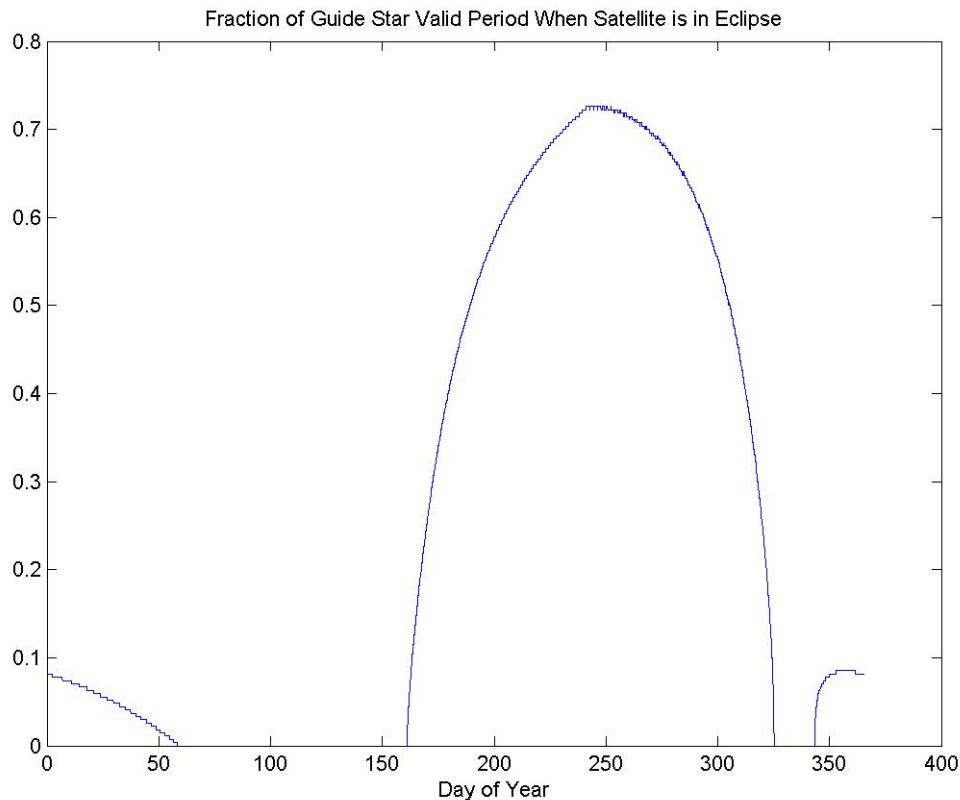


Figure 1. Fraction of Each Guide Star Valid Period When Satellite is in Earth's Shadow

The annual variation of the roll frequency component of the bus voltage may also be estimated in each of the different modes. The phase of the estimated roll frequency components will vary at an annual period as the sun changes its orientation with respect to the satellite roll axis. In the absence of a detailed simulation, the annual variation in the roll frequency component may be estimated by finding the average roll frequency component of the three operating modes since the electrical power system is expected to be in each operating mode one third of the time. This estimate gives a 32 mv annual variation in the roll frequency component.

The orbital variation in the roll frequency component is more difficult to estimate because it will very likely be dominated by the switching between the different modes of

the electrical power system. Except for two intervals of approximately three weeks each, the satellite will spend up to 37 minutes of each orbit in eclipse. As it comes out of eclipse, it will switch to the PPT mode to recharge the batteries. Finally, it will enter the V/T mode before entering into eclipse again. A reasonable estimate is that the amplitude of the orbital variation in the roll frequency component of the power bus voltage will be equal to the amplitude of the roll frequency component of the power bus voltage in the PPT mode, or 36 mv (See Table 4 which gives peak-to-peak amplitude). Over the course of one year the orbital phase of each of these modes will slowly change, so the orbital frequency variation averaged over one year will be significantly smaller.

These expected temporal variations in the roll frequency component of the power bus voltage are summarized in the table below.

Table 5. Estimated Linear, Annual, and Orbital Variation in Power Bus Voltage at the Satellite Roll Frequency

Time History	Amplitude
Linear Variation in Roll Frequency Component	12 mv/yr
Annual Variation in Roll Frequency Component	32 mv
Orbital Variation in Roll Frequency Component	36 mv

B. Estimated Variations in Roll Frequency Output of SRE

The product of the estimated variations in the power bus voltage given in Table 5 and the sensitivity of each of the SQUIDs to variations in the power bus voltage given in Table 3 gives the effective drift in the gyroscope spin axis and the annual and orbital variation in the gyroscope spin direction. These values are given in Table 6 for each of the four gyroscopes with the ECU in the baseline mode and the contingency mode.

Table 6. Estimates the Linear (mas/yr), Orbital (mas), and Annual (mas) Effective Gyroscope Misalignment with No Calibration

ECU Mode	Variation in Power Bus Voltage at Roll Frequency	SQUID 1	SQUID 2	SQUID 3	SQUID 4
Baseline B-Side PS A-Side Elect.	Linear Variation 12 mv/yr	1.0 ± 0.8	0.02 ± 0.01	0.0 ± 0.01	0.05 ± 0.02
	Annual Variation 32 mv	2.6 ± 2.1	0.05 ± 0.03	0.01 ± 0.03	0.12 ± 0.04
	Orbital Variation 36 mv	2.9 ± 2.3	0.06 ± 0.04	0.01 ± 0.04	0.14 ± 0.05
Contingency ECU Off	Linear Variation 12 mv/yr	0.0 ± 0.24	0.04 ± 0.01	0.07 ± 0.02	0.02 ± 0.02
	Annual Variation 32 mv	0.01 ± 0.6	0.10 ± 0.03	0.20 ± 0.05	0.06 ± 0.04
	Orbital Variation 36 mv	0.01 ± 0.7	0.12 ± 0.04	0.22 ± 0.06	0.06 ± 0.05

C. On-Orbit Calibration of the Sensitivity of the SQUID Readout to Power Bus Variation

Several different approaches may be used to reduce the impact of this power bus sensitivity in the ground based data analysis and to check the magnitude of this effect. One method would be to place the data in various bins depending on the operating mode of the electrical power system. Because the roll frequency variations in the electrical power bus are significantly different depending on the operating mode, any error in the gyroscope drift rate due to this effect should be considerably different depending on the operating mode. Although the statistical error in the gyroscope drift rate could be expected to increase because of the decreased amount of data in any one bin, it should be possible to place limits on this systematic error by comparing the gyroscope drift rate determined in each operating mode.

Another promising method of reducing this error in the data analysis would be to perform a regression analysis between the power bus voltage and the output of the SQUID readout electronics. In order to perform such a regression analysis, it would be necessary to have an accurate measure of the power bus voltage. Assuming that the noise in each SQUID meets its required value of $190 \text{ mas}/\sqrt{\text{Hz}}$, the amplitude of a sinusoidal component in the output of the SRE can be determined to an accuracy of 0.65 mas with one day of integration time. If the variation in the power bus at the second harmonic of the satellite roll frequency is 150 mv peak-peak while the electrical power system is operated in the peak power tracking (PPT) mode [1] (See Table 4 in this document), then by monitoring the SRE output at the second harmonic of the satellite roll frequency, the sensitivity of the SRE output to variations in the power bus sensitivity could be measured to an accuracy of 4.3 mas/volt in one day of operating time in the PPT mode. If the electrical power system is in the PPT mode one third of the time, then it would take three days to collect this data. If the data is collected over an interval of one month, the sensitivity could be measured to an accuracy of 1.4 mas/volt.

Additional information of the sensitivity of the output of the SRE to variations in the power bus voltage could be obtained as the satellite makes the transition from sunlight to the Earth's shadow. As the satellite passes into the shadow of the Earth, the power bus voltage is expected to drop approximately 0.6 v and then decrease 100 mv per minute as the batteries discharge. When the satellite returns to the sunlight, the power bus voltage will quickly return to its original value. The maximum time in eclipse will be 37 minutes resulting in an increase in the power bus voltage of 4.3 v when it comes out of eclipse. Comparing the bias in the output of the SRE during a 5 minute interval before the transition with the output during a 5 minute interval after the transition, the difference in the bias may be measured to an accuracy of 17 mas with a SQUID noise of $190 \text{ mas}/\sqrt{\text{Hz}}$. Then, the power bus sensitivity may be measured with an accuracy of 28 mas/volt during the transition into eclipse and 4 mas/volt during the transition out of eclipse. Assuming that two transitions occur on each orbit and that there are 16 orbits per day, then this error could be reduced to 1.0 mas/volt in a one day interval.

Table 7 below gives variation in the gyroscope angular orientation for each of the four SQUIDS assuming that the sensitivity of each SQUID may be calibrated to the larger of 1.4 mas/volt or 10% of the uncalibrated value in the baseline mode. For SQUID 1, the assumed calibration accuracy is 10% of the measured sensitivity in the baseline configuration. For this reason the values given in Table 7 for SQUID 1 are significantly larger than those values for the other SQUIDS.

Table 7. Estimates the Linear (mas/yr), Orbital (mas), and Annual (mas) Effective Gyroscope Misalignment with Calibration. Baseline Mode.

ECU Mode	Variation in Power Bus Voltage at Roll Frequency	SQUID 1	SQUID 2	SQUID 3	SQUID 4
Baseline Mode with Modest Calibration	Linear Variation 12 mv/yr	0.1 ± 0.08 mas/yr	0.017 mas/yr	0.017 mas/yr	0.017 mas/yr
	Annual Variation 32 mv	0.26 ± 0.2 mas	0.045 mas	0.045 mas	0.045 mas
	Orbital Variation 36 mv	0.29 ± 0.23 mas	0.050 mas	0.050 mas	0.050 mas

III. Impact on T003 and T002 Requirements.

A. System Design and Performance Requirements (T003)

The following T003 requirements may be impacted by the sensitivity of the SQUID readout system to variations in the power bus sensitivity.

3.5.2 Bias Variation

The bias variation in the sampled SQUID readout signal due to the SQUID, the top hat filters, the SQUID electronics, and the A/D converter meets the following requirements:

3.5.2.1 Linear Variation

The amplitude of any linear variation in the body-fixed bias signal at roll frequency (linear variation in the inertially fixed bias) and the body fixed calibration signal shall be less than 0.1 marcsec for data taken over the course of one year during the time the guide star is valid.

3.5.2.2 Annual Variation

The amplitude of any variation at annual rate in the body-fixed bias at roll frequency (annual variation in the inertially fixed bias) and the body-fixed calibration signal shall be less than 0.4 marcsec for data taken over the course of one year during the time the guide star is valid.

3.5.2.3. Orbital Variation

The amplitude of any variation at orbital rate in the body-fixed bias signal at roll frequency (orbital variation in the inertially fixed bias), the body-fixed calibration frequencies, or the body-fixed dither frequency shall be less than 0.1 marcsec for data taken over the course of one year during the time the guide star is valid when these components are simultaneously estimated with the components at roll frequency, calibration frequency, and dither frequency.

The expected values of the worst-case and probable-case unmodeled bias variations for each of the four SQUIDs are shown in Table 7 for the baseline configuration and the contingency configuration.

B. Fundamental Requirements T002

The following fundamental requirements may be affected by this sensitivity of the SQUID readout to variations in the power bus voltage.

Req. 2. Detection and Calibration

The overall apparent change in direction of the gyroscope spin axis with respect to distant inertial space, excluding the uncertainty in proper motion of the effective guide point (as defined in requirement 3) and the inertial drift of the quartz block (see reqmt. 9), shall be measured, including instrument calibration, with an error not to exceed 0.34 marcsec/year (1 sigma) over the duration of the experiment. Calibration shall be performed using orbital ephemeris and known physical processes such as the aberration and bending of starlight.

Req. 10. Bias Rejection

The joint effects of all bias drifts, including electronic, magnetic, optical, thermal, and mechanical, on the SG and ST readouts and on the science-data instrumentation and data reduction systems shall be reduced to less than 0.15 marcsec/year when referenced to inertial space.

The impact of the T003 Requirements on the T002 Requirements may be made assuming that these effects are treated as unmodeled errors. The effect of these unmodeled errors on the determination of the gyroscope rate, scale factor, and roll phase offset are calculated in reference [2]. A linear drift in the amplitude or phase of the roll frequency signal leads directly to an error in the gyroscope drift rate measurement. Over intervals of nine months or more, the annual aberration signal is more effective than the orbital aberration signal in determining the scale factor and orientation of the gyroscope readout. However, SRE bias variations in the roll frequency which are modulated at the annual frequency will produce an error in the scale factor or roll phase offset, which in turn will produce an error in the measured drift rate. The worst-case errors in the measured drift rate, assuming the annual modulation is in phase with the annual aberration, is given by [2]

$$\delta R = \frac{b_a}{A_a} R_g$$

where b_a is the magnitude of the annual modulation of the roll frequency bias, A_a is the magnitude of the annual aberration signal, and R_g is the magnitude of the geodetic drift rate (6.6 arc sec/yr).

Table 8 below gives the contribution to the error in the measured drift rate due to the linear and annual variation in the roll frequency bias. Here, the assumption is made that the data is collected over an interval of 12 months and that the linear variation and

the annual modulation of the roll frequency bias are given by the values listed in Table 6 and 7. Over these long intervals, an orbital variation in the roll frequency bias will make a much smaller contribution to the error in the measured drift rate.

Table 8. Limits on Error in Gyroscope Drift Rate (mas/yr) due to Variation in Roll Frequency Component of Power Bus Voltage.

	Variation in Power Bus Voltage at Roll Frequency	SQUID 1	SQUID 2	SQUID 3	SQUID 4
Baseline: B-side P. S. A-side electr.	Linear Drift	1.0 ± 0.8	0.02 ± 0.01	0.0 ± 0.01	0.05 ± 0.02
	Annual Modulation	0.9 ± 0.7	0.02 ± 0.01	0.0 ± 0.01	0.04 ± 0.01
	Root Square Sum	1.3 ± 1.2	0.03 ± 0.01	0.0 ± 0.01	0.06 ± 0.02
Contingency: ECU Off	Linear Drift	0.0 ± 0.24	0.04 ± 0.01	0.07 ± 0.02	0.02 ± 0.02
	Annual Modulation	0.0 ± 0.2	0.03 ± 0.01	0.07 ± 0.02	0.02 ± 0.02
	Root Square Sum	0.0 ± 0.3	0.05 ± 0.02	0.10 ± 0.03	0.03 ± 0.03
Calibration	Linear Drift	0.1 ± 0.08	< 0.017	< 0.017	< 0.017
	Annual Modulation	0.09 ± 0.1	< 0.015	< 0.015	< 0.015
	Root Square Sum	0.13 ± 0.12	< 0.023	< 0.023	0.023

C. Summary

The overall impact of the power bus sensitivity on the gyroscope readout in the baseline configuration without calibration are summarized in Table 9 below. The summary for the contingency configuration with the ECU off is given in Table 10. Finally, Table 11 summarizes the impact on the of the power bus variations assuming that the SQUID power-bus sensitivity may be calibrated to an accuracy of 1.4 mas/volt. For SQUIDS 2, 3, and 4, it is clear that the calibration may be done this accurately since it represents a modest improvement over the uncalibrated value. The calibrated value for SQUID 1 is potentially just as accurate, but here the more conservative assumption is made that the sensitivity may be calibrated to 10% of its value in the baseline configuration.

SQUID 1 does not meet the T003 requirements 3.5.2 or the T002 Requirement 10 in either of the operating modes or with the assumed 10% calibration. It also does not meet T002 Requirement 2 in the contingency configuration with the ECU off, but it does meet the requirement if this effect is calibrated on orbit. The combined measurements from all four of the SQUIDS meet the top level requirement T001 requirement in either the baseline or the contingency configuration without calibration.

Table 9. Contribution of Power Bus Variations to Estimates of T003 and T002 Requirements with No Calibration. Baseline Configuration

Requirement	SQUID 1	SQUID 2	SQUID 3	SQUID 4
T003 Requirement 3.5.2.1 Linear Variation at Roll 0.1 mas/yr	Fail 1.0 ± 0.8	Pass 0.02 ± 0.01	Pass 0.0 ± 0.01	Pass 0.05 ± 0.02
T003 Requirement 3.5.2.2 Annual Variation in Roll Bias 0.4 mas	Fail 2.6 ± 2.1	Pass 0.05 ± 0.03	Pass 0.01 ± 0.03	Pass 0.12 ± 0.04
T003 Requirement 3.5.2.3 Orbital Variation in Roll Bias 0.1 mas	Fail 2.9 ± 2.3	Pass 0.06 ± 0.04	Pass 0.01 ± 0.04	Fail 0.14 ± 0.05
T002, Req 10 Bias Rejection 0.15 mas/yr	Fail 1.3 ± 1.2	Pass 0.03 ± 0.01	Pass 0.0 ± 0.01	Pass 0.06 ± 0.02
T002 Req. 2 Detection and Calibration 0.34 mas/yr	Fail	Pass	Pass	Pass
T001 0.5 mas/yr	Pass			

Table 10. Contributions of Power Bus Variations to Estimates of T003 and T002 Requirements with No Calibration. Contingency Configuration: ECU Off.

Requirement	SQUID 1	SQUID 2	SQUID 3	SQUID 4
T003 Requirement 3.5.2.1 Linear Variation at Roll 0.1 mas/yr	Unverified 0.0 ± 0.24	Pass 0.04 ± 0.01	Pass 0.07 ± 0.02	Pass 0.02 ± 0.02
T003 Requirement 3.5.2.2 Annual Variation in Roll Bias 0.4 mas	Unverified 0.01 ± 0.6	Pass 0.10 ± 0.03	Pass 0.20 ± 0.05	Pass 0.06 ± 0.04
T003 Requirement 3.5.2.3 Orbital Variation in Roll Bias 0.1 mas	Unverified 0.01 ± 0.7	Fail 0.11 ± 0.03	Fail 0.22 ± 0.06	Unverified 0.06 ± 0.05
T002, Req 10 Bias Rejection 0.15 mas/yr	Unverified 0.0 ± 0.3	Pass 0.05 ± 0.02	Pass 0.10 ± 0.03	Pass 0.03 ± 0.03
T002 Req. 2 Detection and Calibration 0.34 mas/yr	Unverified	Pass	Pass	Pass
T001 0.5 mas/yr	Pass			

Table 11. Contributions of Power Bus Variations to T003 and T002 Requirements Verification with Modest Calibration of Power Bus Sensitivity

	SQUID 1	SQUID 2	SQUID 3	SQUID 4
T003 Requirement 3.5.2.1 Linear Variation at Roll 0.1 mas/yr	Unverified 0.1 ± 0.08 mas.yr	Pass 0.017 mas/yr	Pass 0.017 mas/yr	Pass 0.017 mas/yr
T003 Requirement 3.5.2.2 Annual Variation in Roll Bias 0.4 mas	Unverified 0.26 ± 0.2 mas	Pass 0.045 mas	Pass 0.045 mas	Pass 0.045 mas
T003 Requirement 3.5.2.3 Orbital Variation in Roll Bias 0.1 mas	Unverified 0.29 ± 0.23 mas	Pass 0.050 mas	Pass 0.050 mas	Pass 0.050 mas
T002, Req 10 Bias Rejection 0.15 mas/yr	Unverified 0.13 ± 0.12 mas	Pass 0.023 mas/yr	Passs 0.023 mas/yr	Pass 0.023 mas/yr
T002 Req. 2 Detection and Calibration 0.34 mas/yr	Pass	Pass	Pass	Pass
T001 0.5 mas/yr	Pass			

IV. Re-Verification of T003 and T002 Requirements

A. T003 Requirement 3.5.2

The T003 requirements on bias variation at the satellite roll frequency were verified in reference [3], based on the analysis in reference [4]. In this reference the variation in the roll frequency bias is shown to less than 0.042 mas for the supported gyroscopes and less than 0.050 mas for the unsupported gyroscope.

As shown above, the sensitivity to the power bus variation may be calibrated on orbit to an accuracy of 1.4 mas/volt. For gyroscopes 2, 3, and 4 this calibrated sensitivity represents a modest improvement over the measured sensitivity. Combining the results from reference [3] with the values given in Table 11 above, for these gyroscopes the overall linear variation amounts to 0.045 mas/yr for the supported gyroscopes and 0.053 mas/yr for the unsupported gyroscope. The annual variation amounts to 0.062 mas for the supported gyroscopes and 0.067 mas for the unsupported gyroscope. Finally, the orbital variation is 0.065 mas for the supported gyroscopes and 0.071 mas for the unsupported gyroscope. For gyroscope 1, if the sensitivity to the power bus variation may be calibrated to an accuracy of only 10%, then the existing data is not adequate to verify T003 Requirements 3.5.2. These roll frequency bias variations for all four gyroscopes are summarized in Table 12 below.

Table 12. Roll Frequency Bias Variations from All Sources

	SQUID 1	SQUID 2 Unsupported	SQUID 3	SQUID 4
T003 Requirement 3.5.2.1 Linear Variation at Roll 0.1 mas/yr	Unverified 0.1 ± 0.08 mas.yr	Pass 0.053 mas/yr	Pass 0.045 mas/yr	Pass 0.045 mas/yr
T003 Requirement 3.5.2.2 Annual Variation in Roll Bias 0.4 mas	Unverified 0.26 ± 0.2 mas	Pass 0.067 mas	Pass 0.062 mas	Pass 0.062 mas
T003 Requirement 3.5.2.3 Orbital Variation in Roll Bias 0.1 mas	Unverified $0.29 \pm$ 0.23 mas	Pass 0.071 mas	Pass 0.065 mas	Pass 0.065 mas

B. T002 Requirement 10

T002 Requirement 10 was verified in reference [5]. In this reference, the error in the measured drift rate due to roll frequency bias variations was shown to be 0.077 mas/yr for the supported gyroscopes and 0.082 mas/yr for the unsupported gyroscope. With the additional contributions to the systematic error in the measured gyroscope drift rate due to the sensitivity of the SRE to variations in the power bus voltage, the updated errors for each gyroscope are listed below.

Table 13. Errors in Measured Gyroscope Drift Rate due to All Sources of Roll Frequency Bias Variation.

	SQUID 1	SQUID 2 Unsupported	SQUID 3	SQUID 4
T002, Req 10 Bias Rejection 0.15 mas/yr	Unverified 0.13 ± 0.14 mas/yr	Pass 0.085 mas/yr	Pass 0.080 mas/yr	Pass 0.080 mas/yr

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1. Shehatz, S. and M. Anderson, *Space Vehicle EPS Modes Bus Ripple Estimates*, Lockheed Martin, EM EPS 256, October 20, 2003
2. Keiser, G.M., *Analytical Solution for Unmodeled Errors in the Gravity Probe B Experiment*, GP-B, Hansen Laboratories, Stanford University, S0895, Rev. -, June 9, 2003
3. Keiser, G.M. and B. Muhlfelder, *Verification of Gyroscope Readout Bias Requirements*, T003 3.6.2, 3.5.2.1, 3.5.2.2, 3.5.2.3, Hansen Laboratories, Stanford University, S0884, Rev. -, April 15, 2003
4. Keiser, G.M., J. Lockhart, and B. Muhlfelder, *Bias Variations in the Gravity Probe B Readout System*, GP-B, Hansen Laboratories, Stanford University, S0352, Rev. -, May 16, 2003
5. Keiser, G.M., *Verification of T002 Requirement 10: Bias Rejection*, Hansen Laboratories, GP-B, Stanford University, S0920, Rev. -, July, 2003