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Gravity Probe B Relativity Mission

VERIFICATION OF T002 REQUIREMENT 2: DETECTION AND CALIBRATION

S0956, Rev. -
November 10, 2003

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ITAR Assessment Performed ____________ Tom Langenstein   11/10/03 ITAR Control Req'd? ☐ Yes ☒ No

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Abbreviations and Acronyms

$\Delta\phi$	Roll Phase Offset
Δt	interval between measuremets
Δv	Uncertainty in velocity
δC_G	Linear Drift in ScaleFactor
$\delta\phi_c$	Variation in roll phase offset at cosine of annual frequency
$\delta\phi_s$	Variation in roll phase offset at sine of annual frequency
μgauss	microgauss
ϕ_m	Measured Roll Phase
σ	standard deviations
A_o	Amplitude of Orbital Aberration Signal
A/D	Analog-to-Digital
C_G	Scale Factor of Gyroscope Readout System
DMA	Detector Module Assembly
EW_0	Misalignment at time $t=t_0$ in East-West Direction
FLL	Flux Locked Loop
GP-B	Gravity Probe B
GPS	Global Positioning System
G_r	Single-sided autospectral density
km	kilometer
l_1, l_2, l_3, l_4	Components of Annual Aberration Signal

marcsec	milli-arc-second
mas	milli-arc-second
ms	milli-second
msec	milli-second
ppm	part per million
NS_0	Misalignment at time $t=t_0$ in North-South Direction
R_{EW}	Drift Rate in East-West Direction
R_{NS}	Drift Rate in North-South Direction
sqrt	square root
SQUID	Superconducting Quantum Interference Device
SG	Science Gyroscope
SRE	SQUID Readout Electronics
ST	Science Telescope
SV	Space Vehicle
TRE	Telescope Readout Electronics
v	velocity
VLOA	Verification Letter of Acceptance
yr	year
z	Combined gyroscope and telescope readout signals

I. Introduction

The purpose of this document is to verify Requirement 2 of the Gravity Probe B Twelve Fundamental Requirements (T002). This requirement specifies the maximum experimental error due to the measurement of the gyroscope drift rate relative to the effective guide point of the reference star.

T002, Requirement 2, Detection and Calibration
The overall apparent change in direction of the gyroscope spin axis with respect to distant inertial space, excluding the uncertainty in proper motion of the effective guide point (as defined in requirement 3) and the inertial drift of the quartz block (see reqmt. 9), shall be measured, including instrument calibration, with an error not to exceed 0.34 marcsec/year (1 sigma) over the duration of the experiment. Calibration shall be performed using orbital ephemeris and known physical processes such as the aberration and bending of starlight.

The measurement errors are classified as either statistical errors or systematic errors. Statistical errors are any errors that arise from the random noise in the measurement process. These errors decrease as the number of measurements increase. The dominant contribution to the statistical error in the Gravity Probe B measurement system is the statistical noise in the gyroscope readout system, but there is also a smaller contribution from the telescope readout system. Systematic errors, on the other hand, may or may not be indistinguishable from the effects being measured depending of the time signature of the error source. Their magnitude does not decrease as larger amounts of data are used in the analysis. The systematic errors have been further classified as contributions from bias variations, scale factor variations, phase shift variations, and those contributions from the nonlinearity of the gyroscope and telescope readout systems. The contributions of each of these errors are described in the sections below and are shown schematically in Figure 1.

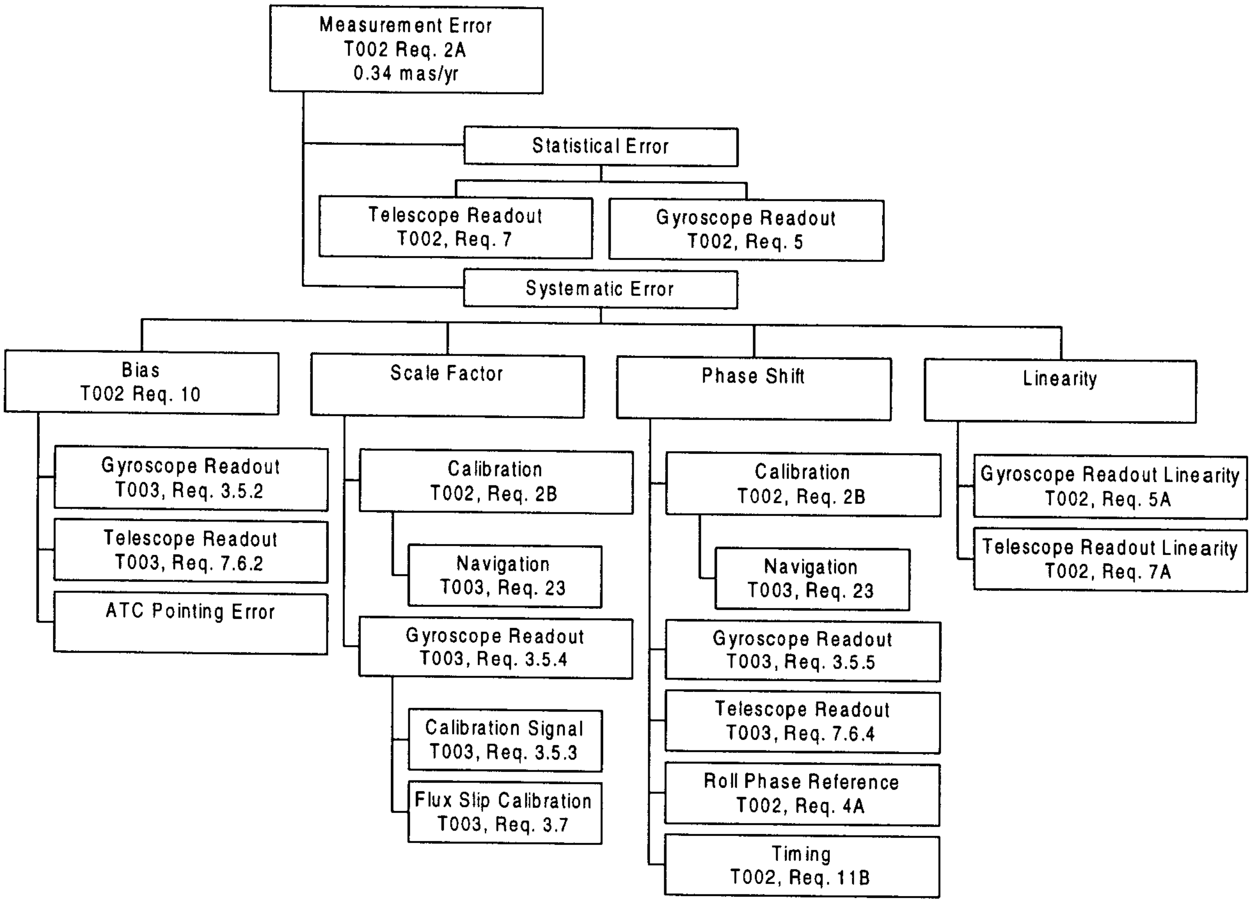

Figure 1. Contributions to Overall Measurement Error

II. Statistical Measurement Errors

The standard errors associated with the parameters (or states) in the measurement model are calculated using a covariance analysis. Any covariance analysis requires a measurement model of the parameters to be determined from the data and the estimated noise in the measurements. The Gravity Probe B measurement model and a covariance analyses based on this model are described below

A. Basic Gravity Probe B Measurement Model

The gyroscope readout system and the telescope readout system are combined as described in reference [1]. This combined signal is then used to determine the orientation and drift rate of each gyroscope relative to the true direction to the guide star. The orbital and annual aberration signals are assumed to be known, and the minimum set of parameters to be determined from the data analysis are

- the gyroscope drift rate relative to the guide star in the East-West and North-South directions, R_{EW} and R_{NS},

- the misalignment of the gyroscope spin axis at time $t = t_0$ in the East-West and North-South directions, EW_0 and NS_0,
- the scale factor of the gyroscope readout, C_G, and the
- roll phase offset, $\delta\phi$.

With these parameters, the model for the combined gyroscope and telescope signals becomes,

$$z = C_G \begin{bmatrix} (NS_0 + R_{NS}(t-t_0) + A_o \cos\phi_o + l_1 \cos\phi_a + l_2 \sin\phi_a)\cos(\phi_m + \delta\phi) + \\ + (EW_0 + R_{EW}(t-t_0) + l_3 \cos\phi_a + l_4 \sin\phi_a)\sin(\phi_m + \delta\phi) + b \end{bmatrix} \quad (1)$$

where A_0 and l_1, l_2, l_3, l_4 are respectively the amplitude of the orbital aberration signal and the components of the annual aberration signal. Explicit expressions for these constants, l_1, l_2, l_3, and l_4, are given in reference [2]. These constants depend on the location of the reference star, the tilt of the Earth's rotation axis with respect to the ecliptic, and the phase of Earth's orbital motion at the midpoint of the data acquisition period. Theses aberration signals are determined from the components of the velocity of the spacecraft perpendicular to the direction to the Guide Star. Here, as an approximation, the orbits of the satellite about the Earth and the Earth about the Sun are taken to be circular with the orbital phase, ϕ_o, and the annual phase, ϕ_a, increasing linearly with time. The orbital phase, ϕ_o, is defined to be zero when the satellite is cloGsest to the guide star. The measured roll phase, ϕ_m, is taken to be zero when the satellite's x-axis [3] passes through the plane which contains the Earth's rotation axis and the direction to the guide star. The time, t_0, is taken to be the time at the midpoint of the data acquisition period.

As discussed in the sections below on the systematic errors due to the scale factor of the gyroscope readout system and the measurement of the satellite's roll phase, the scale factor and the roll phase offset may vary slowly with time. The scale factor will decrease linearly with time since London moment, which is used for the gyroscope readout, is proportional to the gyroscope spin speed, and the spin speed is slowly decreasing due to the residual gas pressure. These changes in the scale factor due to the changing London moment may be accounted for by measuring the spin speed. However, since there will be a small but unknown contribution to the scale factor from the residual trapped flux, it will not be possible to compensate for the changes in the scale factor to the required accuracy if the residual gas pressure is as high as the required value of 2×10^{-10} Torr. In this case, the data analysis model may be augmented to include an additional term due to the slow linear variation of the scale factor:

$$C_G = C_{G0} + \delta C_G (t - t_0). \quad (2)$$

Here, C_{G0} is the value of the scale factor at time $t = t_0$, and δC_G is the contribution from the linear variation in the scale factor.

As discussed in the section below on the systematic errors due to the measured roll phase, the roll phase of the attitude reference platform relative to the quartz block is expected to vary at an annual period by several arc seconds. This motion of the roll phase reference may also be accounted for in the data analysis model by including two additional terms in the data analysis:

$$\delta\phi = \delta\phi_0 + \delta\phi_c \cos\phi_a + \delta\phi_s \sin\phi_a \quad (3)$$

where $\delta\phi_0$ is the average value of the roll phase offset, and $\delta\phi_c$ and $\delta\phi_s$ are two additional terms for the variation in the roll phase offset at the annual rate. These two additional parameters may be determined from the data analysis.

B. Covariance Analyses

1. Noise, Duty Cycle, and Duration

The statistical error in the gyroscope drift rate depends on the measurement noise, the fraction of the orbit that the guide star is in view of the telescope, and the duration of the science data collection. The values of these parameters are discussed below.

a.) Statistical Measurement Noise

The overall readout noise is the root square sum of the telescope readout noise and the SQUID readout noise. The SQUID readout noise is specified by the second part of T002, Requirement 5.

T002, Requirement 5, Science Gyroscope Readout The gyroscope readout system single-sided noise spectral density shall be less than 190 marcsec/sqrt(Hz) x sqrt(roll period/180 sec) x (130 Hz/spin speed)..

The gyroscope readout noise at a gyroscope spin speed of 130 Hz has been documented in reference [4] and is listed in the second column of Table 1 below. The readout noise is inversely proportional to the gyroscope spin speed, which is specified by T002, Requirement 6, and verified in reference [5].

T002, Requirement 6, Spin Up and Alignment The Science Gyroscope rotors shall be spun up to a speed in the range 80 to 180 Hz…

The expected gyroscope spin speed is given in the third column of Table 1. The gyroscope readout noise at the gyroscope spin speed is given in the fourth column. Note that the gyroscope readout noise assumes a satellite roll period of 3 minutes. The noise will be reduced by approximately a factor of $\sqrt{3}$ if the roll period is as short as 1 minute.

The telescope readout noise is specified by T003, Requirement 7.5.2.

7.5.2, Telescope Pointing Noise, Science As input to the Science data analysis, the standard deviation of the telescope pointing noise for each of the four telescope photodetector pairs shall be <= 75 marcsec for a measurement time of 90 ms using at most 101 samples along 90 ms of the 100 ms charge ramp for each of the 8 photodetectors. This requirement corresponds to a single-sided power spectral density of 34 marcsec/sqrt(Hz).

The telescope readout noise has been documented in reference [6]. In this requirement, the noise is specified as the standard deviation, σ, of the measured angle in a 100 msec interval. For signals sampled at intervals Δt and filtered at the Nyquist frequency, the relation between this standard deviation and the single-sided autospectral density, G, is given by [2]

$$G_r = 2\sigma^2 \Delta t\,.$$

Since the normal to the pickup loop for gyroscopes 1 and 2 is parallel to the quartz block x-axis, the y-axis telescope noise will be the dominant contribution to the combined gyroscope and telescope noise for these gyroscopes. (Note: The telescope y-axis refers to the photodiodes which measure a rotation about the y-axis.) Similarly, the x-axis telescope noise will be the dominant contribution to the overall readout noise for gyroscopes 3 and 4. Based on these references the overall readout noise for each of the four gyroscopes is summarized in the Table 1 below.

Table 1. Square Root of the Single-Sided Autospectral Density for the Combined Gyroscope and Telescope Signals

Gyro/ SQUID	Gyro Readout Noise @ 130 Hz (mas/√Hz)	Gyro Spin Speed (Hz)	Gyro Readout Noise at Spin Speed (mas/√Hz)	Telescope Readout Noise (mas/√Hz)	Overall Readout Noise (mas/√Hz)
1	117	127	120	11.2 A-side Y-axis 16.2 B-side Y-axis	121
2	145	151	125	11.2 A-side Y-axis 16.2 B-side Y-axis	126
3	133	128	135	20.5 A-side X-axis 16.5 B-side X-axis	137
4	121	147	107	20.5 A-side X-axis 16.5 B-side X-axis	109

b.) Guide Star Valid

As shown in reference [2], the overall statistical error in the gyroscope drift rate depends on the fraction of the orbit over which the data is collected. This fraction is a function of the orbit altitude, eccentricity, and inclination, which are specified by T003 Requirements 24.2 through 24.4:

24.2 Semi-Major Axis
The space vehicle shall be inserted into a near polar orbit with a nominal mean semi-major axis of 7018 km +/- 3 km (1 sigma).

24.3 Eccentricity
The satellite shall be injected into an orbit such that the mean eccentricity does not exceed 0.002.

24.4 Target Orbit
The initial orbit is a near polar orbit with the descending node close to the right ascension of the guide star. The final target orbit is a function of launch date. This final target orbit shall be chosen such that the predicted angle between the line-of-sight to guide star and the orbit plane, averaged over intervals longer than 10 months and less than 14 months starting 2 months after launch, is less than 0.025 degrees for 85% probable orbit injection errors.

In the absence of refraction effects from the earth's atmosphere, at the Gravity Probe B altitude of 640 km (T003, Req. 24.2), the guide star would be visible for 63.8% of the orbit. An analysis of the effects of the earth's atmosphere [7] demonstrates that the atmospheric deflection is less than 0.1 mas when the nearest approach altitude of a light ray is greater than 120 km. Excluding those portions of the orbit which have an atmospheric deflection greater than 0.1 mas reduces the usable fraction of the orbit to 62.4%. In spite of this analysis, a conservative value of 50% is used in this verification for the fraction of the orbit over which the science data will be valid. This estimate should be more than adequate to account for any loss of data due to particle radiation while the spacecraft is within the South Atlantic Anomaly.

c.) Experiment Duration

The duration of the science data collection is determined by the dewar lifetime after allowances have been made for the initialization and calibration phases. The dewar lifetime is specified by T003 Requirement 13.2

13.2 Lifetime
The helium dewar shall maintain the ST/Quartz Block assembly at its required temperature (Section 12) for an on orbit lifetime of at least 16.5 months. This lifetime includes operations such as spinup, flux flushing, and low temperature bakeout. The Dewar keeps the lead bag superconducting until the Science Mission is completed.

The thermal model of the dewar predicts a lifetime of 18.8 months [8] or 17.1 months with a 10% contingency. Allowing for a two-month initialization phase and a two-month calibration phase, the duration of the science data collection will be greater than 13 months allowing for the 10% contingency in the dewar lifetime. This value for the science data collection is used for this verification.

2. Statistical Error in Gyroscope Drift Rate

An analytical solution for the covariance matrix for the model given by equation (1) has been calculated and documented in reference [2]. Because of the cross-correlation between the annual aberration signal and the linear drift in orientation of the gyroscope spin axis, the standard error in the gyroscope drift rate depends on the start data of the science data collection period. Table 2 below shows the standard error in the gyroscope drift rate for each gyroscope for four different start dates. These calculated standard errors are based on the results of reference [2] using the noise estimates given in Table 1.

Table 2. Standard Error in Gyroscope Drift Rate For Different Start Dates, Basic Model

Gyroscope	1	2	3	4
Jan. 1	0.11 mas/yr	0.12 mas/yr	0.13 mas/yr	0.10 mas/yr
April 1	0.10 mas/yr	0.11 mas/yr	0.12 mas/yr	0.09 mas/yr
July 1	0.11 mas/yr	0.11 mas/yr	0.12 mas/yr	0.10 mas/yr
Oct. 1	0.10 mas/yr	0.11 mas/yr	0.12 mas/yr	0.09 mas/yr

Note that for a 13 month duration, the variation in the standard error in the gyroscope drift rate is less than 13 % for different start dates and that the differences from one gyroscope to the next are directly proportional to the overall readout noise given in Table 1. Increases in the satellite roll frequency, the gyroscope spin speeds, or the fraction of the orbit over which the guide star is valid would decrease these errors.

Including the additional terms, given by equations (2) and (3) above, for the linear drift in the scale factor will increase the standard error in the gyroscope drift rate. For these cases, the covariance matrix has been calculated numerically [9]. The increase in the standard error in the measured drift rate due to each of these additional terms is given in Tables 3 and 4 below. With these additional terms the error depends on the direction of the measured drift rate. The error in the plane of the orbit is referred to as the error in the North-South direction, and the error perpendicular to the plane of the orbit is referred to as the error in the East-West direction.

Table 3. Standard Error in Gyroscope Drift Rate in North-South Direction with Additional Parameters in Data Analysis Model

	mas/yr			
Source of Error	Gyro 1	Gyro 2	Gyro 3	Gyro 4
Statistical Error	0.11	0.12	0.13	0.10
Additional Statistical Error due to Drift In Scale Factor	0.06	0.07	0.07	0.06
Additional Statistical Error due to Annual Variation in Roll Phase Offset	0.00	0.00	0.00	0.00
Total Statistical Error	0.17	0.19	0.20	0.18

Table 4. Standard Error in Gyroscope Drift Rate in East-West Direction with Additional Parameters in Data Analysis Model

	mas/yr			
Source of Error	Gyro 1	Gyro 2	Gyro 3	Gyro 4
Statistical Error	0.11	0.12	0.13	0.10
Additional Statistical Error due to Drift In Scale Factor	0.00	0.00	0.00	0.00
Additional Statistical Error due to Annual Variation in Roll Phase Offset	0.02	0.02	0.03	0.02
Total Statistical Error	0.13	0.14	0.16	0.12

3. Statistical Error in Scale Factor and Roll Phase Offset

The calibration of the combined gyroscope and telescope readout systems may be done using any signal with a known magnitude and time signature. The optical aberration of starlight is a well known effect that may be calculated from the velocity of the spacecraft. The optical aberration signal due to the motion of the satellite about the Earth has a magnitude of 5 arc seconds, and the optical aberration signal due to the motion of the earth about the Sun has a magnitude of 20 arc seconds. Either or both of these signals may be used to determine the scale factor of the gyroscope readout and the roll phase offset, which is the orientation of the normal to the pickup loop relative to the roll phase as measured by the star trackers. The magnitudes of the additional signals due to the parallax of the guide star and the gravitational deflection of light are too small to provide a calibration with a useful accuracy.

The relative statistical error in the scale factor and the statistical error in the roll phase offset have been calculated using the analytical solution to the covariance matrix [2] assuming that both the orbital aberration signal and the annual aberration signal are used for calibration. The relative error in the scale factor and the roll phase offset are identical. The values of these statistical errors, calculated using the same assumptions as those used to calculate the statistical error in the gyroscope drift rate, are listed in Table 5. Note that there is very little variation with either start date or gyroscope.

Table 5. Statistical Error in Relative Scale Factor and Roll Phase Offset

Gyroscope	1	2	3	4
Jan. 1	2.2×10^{-6}	2.3×10^{-6}	2.5×10^{-6}	2.0×10^{-6}
April 1	2.1×10^{-6}	2.1×10^{-6}	2.3×10^{-6}	1.9×10^{-6}
July 1	2.3×10^{-6}	2.3×10^{-6}	2.5×10^{-6}	2.1×10^{-6}
Oct. 1	2.1×10^{-6}	2.1×10^{-6}	2.3×10^{-6}	1.9×10^{-6}

To determine the relativistic drift rate to an accuracy of 0.1 mas/yr, corresponding to a fractional accuracy of 1.5×10^{-5}, the relative scale factor and roll phase offset must be known to an accuracy better than 1.5×10^{-5}. Comparing the values of the statistical errors in the gyroscope drift rate in Table 2 and the statistical errors in the relative scale factor or roll phase offset in Table 3, it can be seen that for a 13 month interval the calibration of the readout system is not the limiting error.

III. Systematic Measurement Errors

Conservative limits on potential systematic errors may be made by calculating the error in the gyroscope drift rate assuming that only the basic measurement model is used in the data analysis. Any errors that arise because the true time signature of the signal is not included in the data analysis model are called unmodeled errors. If these errors can be demonstrated to be sufficiently small, then there is no need to augment the basic data analysis model to include these unmodeled errors. However, if the unmodeled errors make a significant contribution to the experiment error, <u>and</u> they have a time signature that is different than that used to determine the other data analysis parameters, the basic data analysis model may be augmented to include these additional effects. In this case there may be an increase in the statistical error associated with some of the other parameters, but the overall experiment error, including the systematic and statistical error, will be reduced. Section II above discussed the increase in statistical error due to the additional terms in the data analysis for a linear drift in the scale factor and an annual variation in the roll phase offset.

For those systematic measurement errors that are not included in the data analysis model, the unmodeled error in the critical parameters may be calculated either numerically or analytically. An accompanying document, "Analytical Solution for the Unmodeled Errors in the Gravity Probe B Experiment" [10] gives the unmodeled errors due to various temporal variations in the bias, scale factor, and phase shift of the combined gyroscope and telescope signals assuming that the basic GP-B measurement model is used in the data analysis. The following sections of this document list the requirements relevant to each of these sources of unmodeled error and discuss the contribution to these unmodeled errors to the overall experiment error. Since nonlinearity may contribute to all of these sources of systematic error, it is treated separately. The final section summarizes the contributions to the systematic experimental errors.

A. Bias Variations

1. Sources of Bias Variations

Limits on the contributions of bias variations to the systematic error in the gyroscope drift rate are specified by T002, Requirement 10:

Requirement 10, Bias Rejection
The joint effects of all bias drifts, including electronic, magnetic, optical, thermal, and mechanical, on the SG and ST readouts and on the science-data instrumentation and data reduction systems shall be reduced to less than 0.15 marcsec/year when referenced to inertial space.

References [11] and [12] verify this requirement and summarize the System Design and Performance (T003) requirements that contribute to this T002 requirement.

As discussed and documented in reference [11], the bias variations include contributions from the gyroscope readout system, the telescope readout system, and the pointing control system. An additional contribution to the roll frequency bias variation due to variations in the spacecraft power bus voltage is discussed and documented in reference [12]. Limits on variations in the output of the SQUID Readout Electronics due to variations in the power bus voltage depend on the gyroscope and the time signature of the variations in the power bus voltage. Aside from these variations in the SRE output due to the power bus voltage, the dominant source of the roll frequency bias variation is expected to be the temperature sensitivity of the SQUID readout electronics [13], which is found to have an amplitude of less than 0.021 mas.

For the telescope readout system, three sources of the roll frequency bias variations are thermal effects on the telescope optics, thermal gradients in windows #3 and #4, and temperature variations in the DMA (Detector Module Assembly) and TRE (Telescope Readout Electronics) [14]. The worst-on-worst analysis of the magnitude of these roll frequency bias variations shows that these bias variations have an amplitude of less than 0.029 mas.

In addition to roll frequency bias variations in the gyroscope and telescope readout systems, a third potential source for roll frequency bias variations in the combined gyroscope and telescope signals is due to the pointing control system. A pointing error at the satellite roll frequency combined with an error in the measurement of the relative scale factor of the gyroscope and telescope readout systems will produce a bias variation in the combined signals. This source of error is discussed in [15] and shown to not not make a significant contribution to the roll frequency bias variation in the combined measurement signal.

2. Systematic Error in Gyroscope Drift Rate due to Bias Variations

A constant roll frequency bias variation will not contribute to an error in the measured gyroscope drift rate. However, a linear variation in the amplitude or phase of the roll frequency bias variation will contribute directly to an error in the measured gyroscope drift rate. In addition, an annual variation in the roll frequency bias may cause an error in the calibration of the gyroscope readout, which will indirectly contribute to an error in the measured gyroscope drift rate. Variations in the roll frequency bias at the orbital frequency are not expected to make a significant contribution to the overall error because the temperature variations at the orbital frequency are smaller than those at the annual frequency. Also, over long intervals the annual aberration signal is a more accurate method of calibrating the gyroscope readout than the orbital aberration. Aside from the bias variations correlated with the power bus voltage, the overall contribution of the roll frequency bias variations is estimated to be less than 0.077 mas/yr for each of the supported gyroscopes and less than 0.082 mas/yr for the unsupported gyroscope [11]. Table 6 below summarized these contributions and the contributions to the error in the gyroscope drift rate due to the variations in the power bus voltage.

Table 6. Contributions to Errors in Drift Rate due to Bias Variations

Source of Error	Gyro 1	Gyro 2 (unsupported)	Gyro 3	Gyros 4
Bias Variations due to Power Bus Voltage	0.13 ± 0.12 mas/yr or < 0.25 mas/yr	0.023 mas/yr	0.023 mas/yr	0.023 mas/yr
Other Sources of Bias Variations	0.077 mas/yr	0.082 mas/yr	0.077 mas/yr	0.077 mas/yr
Root Square Sum	0.13 ± 0.14 mas/yr or < 0.27 mas/yr	0.085 mas/yr	0.080 mas/yr	0.080 mas/yr

B. Scale Factor Variations

1. Sources of Scale Factor Variation

Accurate knowledge of the gyroscope drift rate relative to the Guide Star requires that the scale factor of the combined gyroscope and telescope signals be known to the same relative accuracy. To meet the System Design and Performance (T002) Requirement 2, Detection and Calibration, the 6.6 arc second geodetic drift rate of the gyroscope must be known to an accuracy of 0.34 mas, or a fractional accuracy of 5×10^{-5}. Since this requirement includes contributions from other sources, a reasonable goal is a fractional accuracy of 1×10^{-5}.

The scale factor may be determined to this accuracy using the stellar aberration signal. Over short intervals from one orbit to several months, the scale factor is most accurately determined from the orbital aberration signal [2], which has an amplitude of 5 arc seconds but is only observed for slightly more than half of each orbit. Over longer intervals, the scale factor may be determined 10 times more accurately from the annual aberration signal because its magnitude is larger (20 arc seconds) and because it may be observed over a complete annual cycle [2]. As discussed in part II of this report, with 13 months of data, the fractional statistical error in the scale factor is 3×10^{-6} (See Table 3).

There are two types of systematic errors that may contribute to the error in the scale factor. First, the velocity of the spacecraft is used to calculate the magnitude and direction of the stellar aberration signal, which, in turn, is used to determine the scale factor. Any errors in the spacecraft velocity will produce a corresponding error in the scale factor. Secondly, the temporal variations in the scale factor, if they are not included in the data analysis model, may contribute to the systematic error in the gyroscope drift rate. Since the GP-B attitude control system is designed to drive the telescope signal to a null, the requirements on the scale factor of the gyroscope readout system are much tighter than the requirements on the scale factor of the telescope readout system. The requirements flowdown for these two types of errors are discussed in the following two subsections.

a. Calibration of Scale Factor Using the Stellar Aberration Signal

Errors in the measured velocity of the spacecraft may contribute to the error in the scale factor since the magnitude and direction of the optical aberration signal are directly proportional to the spacecraft's velocity. The requirements that specify the accuracy of the spacecraft's velocity are summarized below:

(1) Orbital Aberration

The Gravity Probe B satellite will orbit the earth in a nearly circular (eccentricity < 0.002) polar orbit at an altitude of 640 km. For this orbit the aberration signal will vary sinusoidally with a maximum value of 5 arc seconds occuring when the satellite lies along the line between the center of the earth and the guide star. This aberration signal will be known to a fractional accuracy of 10^{-5} if the velocity is known to an accuracy better than 7.5 cm/sec. For this reason the requirement on the determination of the velocity is given in T003 as

T003, 23.2.1, Post-Processed Position Accuracy
After ground processing of GPS measurements, the spacecraft position in the Earth-Centered-Earth-Fixed (ECEF) frame shall be known to better than 25m (1-sigma per axis) using 12 hours of data.

T003, 23.2.2 Post-Processed Velocity Accuracy
After ground processing of GPS measurements, the spacecraft velocity in the ECEF frame shall be known to better than 7.5 cm/s (1-sigma per axis) using 12 hours of data.

The velocity of the spacecraft with respect to the center-of-mass of the Earth will be measured by the on-board GPS receiver. T003 Requirements 23.2.1 and 23.2.2 on the accuracy of the spacecraft position and velocity as determined by the post-processed GPS data have been verified in references [16] and [17]. The worst case position error was found to be 15.7 meters and the worst case velocity error was found to be 1.3 mm/sec using 12 hours of data. Since the along-track velocity of the spacecraft is 7.5 km/sec, and the orbit has a semi-major axis of 7018 km, this worst case velocity error corresponds to a fractional accuracy of 1.7×10^{-7}. This fractional velocity error will produce the same fractional error in the scale factor determined from the orbital aberration signal.

(2) Annual Aberration, Deflection of Starlight, Parallax

The Earth's ephemeris is determined by the Julian date, which in turn may be determined by the GPS time. The telemetry monitors on the satellite are time tagged with the vehicle time. The GP-B timing system is designed to allow the correlation of the vehicle time with the GPS time. To determine the annual aberration to fractional accuracy of 10^{-6}, the GPS time must be known relative to the vehicle time to an accuracy of 30 seconds. In fact the time of all the science signals is known to an accuracy much better than 30 seconds. The top level timing requirement is

> T002, Requirement 11
> . . . The effective sample time of SG, ST, SV roll, and SV orbital tracking science signals shall be known relative to the GPS time, after ground processing of the science timing signals, to an accuracy and precision of 0.1 msec. . .

Appendix A is a summary of the accuracy of the JPL ephemerides for Earth velocity computation by Myles Standish, the author of the ephemerides. He states that the Earth's velocity is known relative the solar system barycenter (SSB) to an accuracy of 0.2 mm/sec. Since the average velocity of the Earth in its orbit around the sun is 3×10^4 m/sec, this accuracy corresponds to a fractional accuracy of 7×10^{-9}. If the critical science measurements are known relative to GPS time to an accuracy of 0.1 msec (T002, Req. 11), and the Julian date (which is used as the reference time for the JPL ephemerides) is known relative to GPS time to this same accuracy, then the fractional error in the Earth's annual velocity due to timing errors will be

$$\frac{\Delta v}{v} = \frac{10^{-4}\,\sec}{1\,year} = 3 \times 10^{-12}$$

Both the relative error in the Earth's velocity and any potential timing error are far smaller than the relative accuracy with which the scale factor from the annual aberration signal may be determined (see Table 3).

b. Stability of Gyroscope Readout System Scale Factor

The scale factor of the gyroscope readout system depends on the London Moment (and hence the gyroscope spin speed) and trapped magnetic flux in each gyroscope, the coupling of the pickup loop to the gyroscope, the coupling of the SQUID Readout Electronics (SRE) feedback current to the pickup loop, and the scale factor of the SRE analog and digital electronics used to measure the feedback current. Since there was some concern about the stability of the SRE analog and digital electronics, provisions were made to inject a calibration signal into the SRE feedback, with the thought that it would be easier to produce a stable calibration signal than to guarantee the stability of the SRE analog and digital electronics. Additional concern about the stability of the voltage reference used to produce the calibration signal over long time periods led to provisions to periodically check the stability of the calibration signal by comparison with an integral number of flux quanta in the SQUID. For a detailed discussion of these two methods of checking the stability of the scale factor, see "Monitoring the Stability of and Compensating for Changes in the Gyroscope-Readout-System Scale Factor" [18].

(1) Scale Factor Not Measured by Calibration Signal

The overall gyroscope readout system scale factor is the product of the pickup loop scale factor and the SQUID scale factor. The pickup scale factor is the change in the magnetic flux through the pickup loop for a given change in the angle between the gyroscope spin axis and the plane of the pickup loop. The SQUID scale factor is the change in the output of the SQUID Readout Electronics (SRE) for a given change in the magnetic flux through the pickup loop. Changes in the pickup loop scale factor are not measurable either with the calibration signal or flux slipping. For a more detailed discussion of the contributions to the readout loop scale factor, see references [13], [19], and [18]. The stability of the pickup loop scale factor is specified by the following requirement:

3.5.4.3 Stability
For those parts of the overall readout system, where the variation of the scale factor can not be measured by the calibration signal, the stability of the scale factor shall be equal to or better than the above requirements (Section 3.5.3) on the relative stability of the amplitude of the calibration signal.

Contributions to the stability of the pickup loop scale factor are discussed in reference [19]. In this reference, the long-term stability of the pickup loop scale factor due to changes in the trapped magnetic flux and due to changes in the polhode path were shown to be negligible. However, as the gyroscope spin speed changes due to the residual gas pressure, the magnitude of the London moment signal will change in proportion to the spin speed, but the magnitude of the trapped magnetic flux will remain constant. If the contribution of the trapped magnetic flux to the readout signal is unknown, then, based on the measured spin down rate, it is only possible to place an upper limit on the change in the gyroscope scale factor [19]. In this reference, a worst-case upper limit on the uncertainty in the drift of the scale factor over a 90 day period was shown to be 8×10^{-6}. The probable value for this number is significantly smaller

(1) since the measured trapped flux is smaller in three out of four of the gyroscopes,
(2) since the component of the trapped magnetic field along the spin axis will be a fraction of the trapped magnetic flux, and
(3) since the residual pressure is expected to be several orders of magnitude smaller than the measured pressure.

The average gyroscope spin speed will be measured on orbit to an accuracy better than 1 microHertz over a 10 hour interval. This accuracy is significantly better than the spin down rate due to the required value of the residual gas pressure. As long as the fractional spindown rate is less than 4×10^{-5}/year, the contribution to the fractional change in the pickup loop scale factor will be less than 3×10^{-6}/year.

Since the measured value of the trapped flux is different in each of the four gyroscopes, the worst-case limit on the change in the scale factor will be different for each of the four gyroscopes. Table 7 gives the trapped magnetic flux measured in each of the four gyroscopes [20], and the limit on the uncertainty in the gyroscope scale factor corresponding to the required value of the pressure [21].

Table 7. Uncertainty in Fractional Change in Pickup Loop Scale Factor Due to Trapped Magnetic Flux and Required Value of Residual Gas Pressure

	Gyro 1	Gyro 2	Gyro 3	Gyro 4
Trapped Magnetic Flux	8.3 μgauss	1.75 μgauss	2.5 μgauss	0.9 μgauss
Uncertainty in Fractional Change in Scale Factor	3.2×10^{-5}/yr	0.7×10^{-5}/yr	1×10^{-5}/yr	0.36×10^{-5}/yr

(2) Short Term Stability

The primary check on the stability of the scale factor of the gyroscope readout system is the calibration signal. For that reason, the requirements on the SQUID readout scale factor only cover the short-term stability of the scale factor and those parts of the gyroscope readout system that can not be checked by the calibration signal:

3.5.4 Scale Factor Variation
The scale factor of the sampled low frequency SQUID readout signal at the roll frequency and the calibration signal frequency (including effects from the SQUID, the SQUID electronics, and the A/D sampling) meets the following requirements:

3.5.4.1 Orbital Variation
Over any 15 day period, the variation in the SQUID scale factor at orbital frequency shall be less than 2E-5.

3.5.4.2 Linear Drift
Over any 15 day period, the linear drift in scale factor shall be less than 2E-5.

The principal reason for these requirements is that it takes 4.6 days to determine the amplitude of the calibration signal to a relative accuracy of 10^{-5} using the injected calibration signal. Over these short intervals variations in the scale factor at this level can not be measured by the calibration signal. Over longer intervals, the scale factor may be monitored by the calibration signal, and the flux slipping procedure may be used at regular intervals to determine the stability of the scale factor. Therefore, if a change in the scale factor is known to occur at some point, the flux slipping procedure may be used to determine the scale factor before and after that point. Otherwise, if the scale factor is stable over other periods to the required accuracy, it will have a negligible impact on the overall accuracy of the experiment. References [22] and [23] verify the requirements on the short-term stability of the gyroscope readout scale factor and show that they are within the specified values.

(3) Calibration Signal

The stability of the scale factor of the SQUID readout system relies on the stability of the injected calibration signal for time scale up to 90 days. The following requirements specify the characteristics of the calibration signal:

3.5.3 Low Frequency Calibration Signal

3.5.3.1 Description
Provision shall be made for a periodic calibration signal to be injected into the SQUID input circuit having an amplitude up to a London moment equivalent of 30 ± 10 arcsec.

3.5.3.2 Period
The period of the body-fixed calibration signal shall be between 45 minutes and 20 seconds.

3.5.3.3 Stability of Low Frequency Calibration Signal at Orbital Period
The stability of the amplitude and phase at the orbital period have the following requirements:

3.5.3.3.1 Stability of Low Frequency Calibration Amplitude at Orbital Period
The stability of the amplitude at the orbital period shall be less than 1 part in 100,000.

3.5.3.3.4 Drift of Low Frequency Calibration Signal
The linear drift of the amplitude and phase of the calibration signal have the following requirements

3.5.3.3.4.1 Drift of Amplitude of Low Frequency Calibration Signal
The linear drift of the amplitude of the calibration signal shall be less than 1 part in 100,000 over any 90 day period.

Over intervals between 15 and 90 days, the linear drift and the orbital variation of the gyroscope readout scale factor may be monitored using the injected calibration signal. In this case, any variations in the scale factor may be monitored and compensated for [18]. The stability of the calibration signal amplitude at the orbital period is shown to be 2.2 ppm in reference [24], and the linear drift in the calibration signal amplitude is shown in reference [25] to be less than 8.6 ppm over any 90 day period. Note that additional supporting test data for Requirement 3.5.3.4.1 is documented in the VLOA for this requirement.

The SQUID scale factor at the frequency interval from roll-orbital to roll+orbital is checked by measuring its response at the calibration signal frequency. The difference between this signal frequency interval and the calibration frequency places the following requirements on the gyroscope readout transfer function

3.5.7 Readout Transfer Function

3.5.7.1 Stability
The frequency dependence of the magnitude (and phase shift) of the SQUID transfer function over the interval between (roll - orbital) frequency and (roll + orbital) shall be stable to 1/100,000 (and 1/100,000 radians).

3.5.7.3 Calibration Frequency
In addition, the ratio of the scale factor at the calibration signal frequency to the scale factor at the roll frequency shall be stable to 1E-5, and the ratio of the phase shift at the calibration signal frequency to the phase shift at the roll frequency shall be stable to 1E-5 radians.

(4) Flux Slipping

Over periods longer than 90 days the stability of the scale factor of the gyroscope readout may be checked using a flux slipping technique. The following requirement specifies the capability to make this measurement:

3.7 Flux Quantum Calibration
The Readout System shall have the capability of measuring changes in the scale factor to an accuracy of better than 1e-5 using the flux quantum in the SQUID.

The process of calibrating the gyroscope scale factor using the flux quantum in the SQUID is documented in reference [18]. By continuously monitoring the output of the SQUID Readout Electronics at the calibration signal frequency and performing the flux slipping calibration at intervals of once per month, the long term stability of the SQUID scale factor may be monitored and corrected to an accuracy of 4.8×10^{-6}.

(5) Summary

Combining the uncertainty in the long term drift of the pickup loop scale factor (estimated above) with the uncertainty in the long term drift of the SQUID scale factor (from [18]), the overall uncertainty in the long term drift of the gyroscope readout system scale factor is shown in Table 8 below. The contributions from the pickup loop scale factor are taken from Table 7.

Table 8. Uncertainty in Long Term Drift of Gyroscope Readout System Scale Factor

Contribution	Gyro 1	Gyro 2	Gyro 3	Gyro 4
Long Term Drift				
Pickup Loop Scale Factor	32×10^{-6}/yr	7×10^{-6}/yr	10×10^{-6}/yr	3.6×10^{-5}/yr
SQUID Scale Factor	4.8×10^{-6}/yr	4.8×10^{-6}/yr	4.8×10^{-6}/yr	4.8×10^{-6}/yr
Root Sum Square	32×10^{-6}/yr	8.5×10^{-6}/yr	11×10^{-6}/yr	6.0×10^{-6}/yr
Annual Variation				
Pickup Loop Scale Factor	$< 1 \times 10^{-6}$/yr	$< 1 \times 10^{-6}$/yr	$< 1 \times 10^{-6}$/yr	$< 1 \times 10^{-6}$/yr
SQUID Scale Factor	4.3×10^{-6}/yr	4.3×10^{-6}/yr	4.3×10^{-6}/yr	4.3×10^{-6}/yr
Root Sum Square	4.4×10^{-6}/yr	4.4×10^{-6}/yr	4.4×10^{-6}/yr	4.4×10^{-6}/yr

2. Systematic Errors in Gyroscope Drift Rate

An error in the scale factor will produce a corresponding error in the gyroscope drift rate. As discussed above, errors in the measured orbital velocity of the spacecraft by the GPS receiver may produce fractional errors in the scale factor as large as 1.7×10^{-7}, which in turn would produce a fractional error in the measured drift rate of the same magnitude. If the geodetic drift rate is 6.6 arc seconds/year, the corresponding error in the drift rate is 0.001 mas/yr. Clearly, this error is negligible. Over longer intervals, the annual aberration signal dominates the determination of the scale factor. The JPL ephemeris and the timing system will produce a fractional error in the measured velocity of the spacecraft of less than 7×10^{-9}. The 6.6 arc sec/yr drift rate and this fractional error in the scale factor will produce an error in the gyroscope drift rate of less than 1 μas. Neither of these two potential error sources will make a significant contribution to the overall error.

The effects of unmodeled variations in the scale factor of the gyroscope readout system have been calculated in reference [10]. The magnitude of the unmodeled error in the gyroscope drift rate is directly proportional to the amplitude of the linear variation of the scale factor or the annual variation in the scale factor. The long-term drift in the gyroscope readout system scale factor will be predominantly linear since the largest uncertainty is due to the linear decrease in the gyroscope spin speed. As shown in reference [10], an unmodeled linear drift in the gyroscope readout system scale factor will produce an error in the measured gyroscope drift rate that depends on the start date of the data acquisition and on the direction of the gyroscope drift rate. For all possible start dates, the unmodeled error in the gyroscope drift rate was shown to be less than 0.13 mas/yr perpendicular to the plane of the orbit (East-West direction) for a linear variation in the scale factor of 1×10^{-5}/yr. The corresponding error is less than 0.10 mas/yr in the plane of the orbit (North-South direction).

Because these worst case unmodeled errors will make a significant contribution to the overall error budget, the possibility of including an additional term for the linear variation in the gyroscope readout system scale factor was investigated. The computer program for this covariance analysis are included in reference [9]. The increase in the standard errors due to the addition of a linear variation in the scale factor is discussed in the section on the statistical errors and the results of this numerical covariance analysis are given in Tables 3 and 4. The increase in the standard error with the addition of the linear drift in the scale factor to the data analysis model increases the error primarily for the drift rate in the plane of the orbit (North –South direction).

There will also be an unmodeled error due to the annual variation in the scale factor. The magnitude and direction of this error will vary according to the start date of the science data collection and the phase of the annual variation. Using the magnitude of the fractional annual variation from Table 8 of 4.4×10^{-6}, the worst case error in the gyroscope drift rate is 0.079 mas/yr based on the analysis in [10].

C. Phase Shift Variations

1. Sources of Phase Shift Errors

a. Calibration of Roll Phase Offset Using the Stellar Aberration Signal

The roll phase offset is also calibrated using the direction of orbital and annual aberration signals, and the same lower level requirements apply as those listed in the calibration of the scale factor. The requirements on the orbital velocity and the timing, which determine the accuracy of these calibrating signals are discussed in section III.B.1.a above.

Errors in the measured velocity or position of the spacecraft will produce corresponding errors in the roll phase offset. Although the error in the roll phase offset depends on different components of the velocity than those that will contribute to errors in the scale factor, the magnitude of the errors is the same. For this reason the estimated fractional error in the roll phase offset is exactly the same as the estimated fractional error in the scale factor. As noted in the section on the requirements verification for the calibration of the gyroscope readout scale factor, the orbital aberration signal will be known to an fractional accuracy of 1.7×10^{-7}, and the annual aberration signal will be known to a fractional accuracy of 7×10^{-9}. Then, in the absence of other errors, the roll phase offset could be determined to 1.7×10^{-7} radians from the orbital aberration signal and 7×10^{-9} radians from the annual aberration signal. Both of these errors are significantly smaller than the statistical error in the roll phase offset (see Table 3).

b. Stability of Roll Phase Offset

(1) Phase Shift in Gyroscope Readout System

An error in the phase shift in the gyroscope readout system at the satellite roll frequency will produce a corresponding error in the roll phase offset. As long as the error in the phase shift is constant, this phase shift will be in the roll phase offset and will have no net effect on the determination of the gyroscope drift rates. However, variations in this phase shift will have a direct impact on the relativistic drift rate. The following requirements are placed on the phase shift of the gyroscope readout system.

3.5.5 Phase Shift Variations
The body-fixed phase shift of the sampled SQUID readout signal at the roll frequency and the calibration signal frequency (including effects from the SQUID, the SQUID electronics, and the A/D sampling) meets the following requirements:

3.5.5.1 Orbital Variation
Over any 15 day period, the variation in the phase shift at orbital frequency shall be less than 2E-5 radians.
3.5.5.2 Linear Drift

Over any 15 day period, the linear drift in phase shift shall be less than 2E-5 radians.

3.5.5.3 Stability
For those parts of the overall readout system, where the variation of the body-fixed phase shift at the roll frequency and the calibration signal frequency can not be measured by the calibration signal, the stability of the phase shift shall be equal to or better than the above requirements (Section 3.5.3) on the relative stability of the phase of the calibration signal.

The phase shift of the gyroscope readout system may be divided into a contribution from the phase shift in the pickup loop circuit and a phase shift in the SQUID readout electronics. The phase shift of the pickup loop circuit is expected to be negligible because the pickup loop is superconducting, and T003 Requirement 3.5.5.3 is verified in document [19].

The analog signal processing electronics and digital filtering of the SQUID readout electronics were designed to minimize the phase shift and any variations in the phase shift. At the satellite roll frequency, the phase shift of the gyroscope readout system is dominated by the 4 Hz analog low pass filter in the low frequency signal processing chain. At a three minute roll period, this analog low pass filter produces a phase shift, which at low frequencies increases linearly with frequency and is equivalent to a time delay of 44.3 msec or 2.5×10^{-4} radians at a 3 minute satellite roll period [26]. Since the Flux Locked Loop (FLL) board in the SQUID Readout Electronics (SRE) is temperature controlled, temperature variation of the components which produce this phase shift are expected to be negligible. T003 requirements 3.5.5.1 and 3.5.5.2 specify the linear and orbital phase shift over 15 day periods. Verification of these requirements is documented in [27].This document gives upper limits on both of these phase shifts of 0.4 microradians compared to a required value of 20 microradians.

(2) Calibration Signal

Note that there are no requirements on the long-term stability of the phase shift listed above. It is assumed that long-term drifts in the phase of the gyroscope readout system may be measured by the injected calibration signal. The requirements on the phase stability of the injected calibration signal are listed below:

3.5.3.3.2 Stability of Calibration Signal Phase at the Orbital Period
The stability of the phase of the calibration signal at the orbital period shall be less than 1e-5 radians.

3.5.3.4.2 Drift of Phase of Low Frequency Calibration Signal
The linear drift of the phase of the calibration signal shall be less than 1 part in 100,000 over any 90 day period.

Since the digital-to-analog (D/A) Converter which generates the calibration signal is driven by the same clock which runs the analog-to-digital (A/D) converter, there will be no phase drift in the calibration signal. T003 Requirements 3.5.3.3.2, on the stability of the calibration signal phase at the orbital period, and 3.5.3.4.2, on the linear drift of the phase of the low frequency calibration signal are verified by reference [24].

(3) Errors in Roll Phase Reference

The roll phase offset is the difference between the roll phase as measured by either of the two star trackers and the roll phase of the normal to the pick-up loop for any one of the four gyroscopes. This roll phase offset may be determined from the science data because the direction of the aberration signal is known from the orbital and annual velocity of the spacecraft. Temporal variations in this roll phase offset may be caused by the thermally induced mechanical variations in the orientation of the post which supports the attitude reference platform or thermal variations in the attitude reference platform itself.

The accuracy of the measurement of the roll phase is specified by the first part of T002, Requirement 4:

T002, Requirement 4, Roll Measurement and Control
The Space Vehicle (SV) shall roll about the line of sight to the guide star. This roll rate and phase shall be measured and controlled with sufficient accuracy to permit resolution of the measured inertial drifts into the two coordinate axes with the accuracy specified in reqmt. 2 . . .

The specific requirements on the roll attitude reference are contained in T003, Requirement 20:

20. Roll Attitude Reference

20.1 Description
The roll angle of the space vehicle relative to a suitable field of stars offset from the line of sight to the guide star is measured by a roll attitude reference mounted on the payload.

20.2 QB Roll Reference
The roll reference system bias is calibrated by the data reduction system using the known orbital aberration of starlight.

20.2.1 Uncertainty
The uncertainty (electrical and mechanical) of the roll angle reference system with respect to a quartz block reference shall not exceed 1 deg at the beginning of Science Data measurement.

20.2.2 Variation
Total random changes in this number shall not exceed 10 arcsec/year.

The requirement in the error in the roll attitude reference (T003 Requirement 20.2.2), as measured by the star trackers, is verified in reference [28], and the contribution of these errors to errors in the measured gyroscope drift rate (T002 Requirement 4) are discussed in [29].

(5) Timing

Errors in the time tagging of the signals from the roll phase measurement relative to the measurement of the SQUID readout system will produce an error in the roll phase offset determined by the data analysis. The satellite roll rate is specified by the following requirement:

21.2 Science Roll Rate
The space vehicle shall provide a science roll rate of at least 0.1 rpm and, if sufficient thruster authority is available for the on-orbit space vehicle mass property variations, up to 1 rpm. The final value of the science roll rate will be chosen on orbit.

At the fastest roll rate of 1 rpm (~0.1 radians/sec), an error of 0.1 msec will produce a shift in the roll phase offset of 10^{-5} radians. For this reason the overall timing requirement is set at 0.1 msec.

T002, Requirement 11
. . . The effective sample time of SG, ST, SV roll, and SV orbital tracking science signals shall be known relative to the GPS time, after ground processing of the science timing signals, to an accuracy and precision of 0.1 msec. . .

Note that this requirement specifies that the effective sample time of each of these signals be known relative to GPS time. This requirement is tighter than necessary because it is only necessary to know the relative time of these signals. Also, a constant offset in the effective sample time of the gyroscope (SG) relative to the space vehicle (SV) roll would produce a constant error in the roll phase offset as long as the satellite roll rate were constant. This top level timing requirement is implemented with the timing requirements from section 16 of the System Design and Performance Requirements.

An error in the timing of the measured star tracker signal relative to the measured output of the SQUID readout electronics will produce an roll phase offset. Variations in the relative timing of these two signals will produce a variation in the roll phase offset, which, in turn, may lead to a error in the measured gyroscope drift rate. Both of these signals are measured relative to the 16 fo clock in the aft SRE. The star sensor data is time tagged relative to the Interface Unit 10 Hz pulse, which is synchronized with the SRE 10 Hz pulse within 2 microseconds (VLOA, T003 Requirement 16.6.5). The star sensor data is time tagged within 52 microseconds of the IU 10 Hz pulse (VLOA, T003 Requirement 16.6.5). The sampling of the SRE output is driven directly by the 16 fo clock and will have a negligible timing error. Time delay due to variations in the phase shift due of the analog low pass filter are discussed above. These variations are expected to be negligible but may also be measured by the calibration signal [30]. The overall error and the long term drift in the timing of the star tracker signals relative to the gyroscope readout signals is expected to be less than 54 microseconds. At a 3 minute roll rate, this time delay corresponds to a variation in the roll phase offset of 3×10^{-7} radians.

2. Systematic Error in Gyroscope Drift Rate

The error in the calibration of the roll phase offset, due to errors in the velocity or position of the spacecraft, will produce an error in the components of the gyroscope drift rate along the north-south and east-west directions. The estimated error in the roll phase offset of 1.7×10^{-7} radians due to an error in the orbital aberration will produce an error in the components of the expected 6.6 arc sec drift rate of 0.001 mas/yr. This error is not a significant contribution to the overall drift rate. The error in the drift rate due to incorrect values of the roll phase offset determined from the annual aberration is more than an order of magnitude smaller.

To estimate the errors in the gyroscope drift rate due to variations in the roll phase offset of the star trackers, the data analysis model was augmented to include additional terms for the variation in the roll phase offset at an annual period [29]. Since these errors are included in the data analysis model, they will not contribute to the systematic error but will increase the statistical error in the measured gyroscope drift rate. These additional terms increased this error in the gyroscope drift rate by 0.04 mas/yr.

The other potential contribution to the variation in the roll phase offset is an error in the timing of the star tracker signals relative to the gyroscope readout signals. As shown above, this variation is expected to be less than 3×10^{-7} radians. From reference [10], a linear variation of this magnitude will contribute an error of less than 0.0015 mas/yr to the gyroscope drift rate, and an annual variation of this magnitude will contribute an error of less than 0.003 mas/yr.

D. Linearity

Nonlinearity in the gyroscope and telescope readout systems are treated separately from other sources of systematic errors because the nonlinearities may introduce scale factor, phase shift, or bias variations. Because the telescope is operated close to its null point, nonlinearities in this readout system are less of a concern than those for the gyroscope readout system. The following section lists the lower level requirements related to the nonlinearity in the gyroscope and telescope readout systems, the next section discusses the verification of these requirements, and the final section places limits on the overall readout error due to nonlinearity.

1. Gyroscope Readout System

The overall impact of the nonlinearity in the gyroscope readout system is specified by the first part of T002 Requirement 5:

T002, Req. 5, Science Gyroscope Readout
The SG readout system shall have a linearity consistent with an error of less than 0.3 marcsec/year.

The magnitude of the nonlinearity is specified in the System Design and Performance Requirements by the requirements in paragraph 3.2.1:

2.2.1 Linearity
The readout system shall meet the following linearity requirements in the presence of trapped flux levels as specified in requirement 1.5:

2.2.1.1 High Frequency
In the frequency range 100-1000 Hz the harmonic distortion of each of harmonics 2-5 of sinusoids with amplitude corresponding to the trapped flux levels of section 1.5 shall be less than 1.5e-4.

2.2.1.2 Low Frequency
For frequencies less than 1 Hz, the harmonic distortion of each harmonic 2-5 of sinusoids with amplitude up to 80 arcsec London moment equivalent shall be less than 1.5e-4.

Reference [31] verifies T002, Requirement 5 and discusses the contributions of the nonlinearity in the gyrsocope readout system to the overall experiment error. The dominant effect of nonlinearity in the gyroscope readout system is to introduce changes in the scale factor of the gyroscope readout system. These changes in the scale factor may be classified according to whether they depend on the bias or the amplitude of the signals. With the value of the low frequency harmonic distortion specified by T003 Requirement 3.2.1.2, the overall contribution to the error in the gyroscope drift rate is less than 0.10 mas/yr. The high frequency harmonic distortion, specified by T003 Requirement 3.2.1.1 produces low frequency signals. However these signals are at frequencies which are incommensurate with the satellite roll frequency and will make no significant contribution to the overall experiment error.

2. Telescope Readout System

The impact of the nonlinearity of the telescope readout on the overall experiment error is specified by the first part of T002, Requirement 7:

T002, Req. 7, Science Telescope
Within the central range of +/- 100 marcsecs, the ST readout system, after appropriate filtering and calibration, shall have a linearity consistent with an error of less than 0.1 mas/yr.

The System Design and Performance Requirements, T003, specifies that the linearity shall be consistent with the T002 requirement given above

7.2.3 Linear Range The fused quartz ST shall meet the linear range and error requirements of T002, Reqmt 7.

An additional requirement on the stability of the nonlinearity has been deleted because analysis showed that the stability of the nonlinearity made no significant contribution to the overall experiment error [32].

The effects of nonlinearity in the telescope readout system on the overall experiment error are discussed in reference [32]. The nonlinearity in the telescope readout system may distort the roll frequency pointing error when combined with the telescope noise or the dither in the pointing control system. Using likely values for these parameters and allowing for a worst-case drift of the distortion over the course of a year, the worst-on-worst error due to the telescope nonlinearity is 0.011 mas/yr and therefore not a significant contribution to the overall error.

IV. Summary

The contributions to the gyroscope readout error for each of the gyroscopes are summarized in Table 10 and 11 below. The statistical error is taken from Tables 3 and 4, and the error in the drift rate due to bias variations is taken from Table 6. The other contributions are as described in the text. The root square sum of these errors is listed in the last line of this table. All four gyroscopes meet the requirement of 0.34 mas/yr.

Table 9. Dominant Contributions to Error in Measurement of Gyroscope Drift Rate in NS Direction.

	mas/yr			
Source of Error	Gyro 1	Gyro 2	Gyro 3	Gyro 4
Statistical Error (from Table 3)	0.17	0.19	0.20	0.18
Bias Variations	0.14	0.085	0.080	0.080
Residual Annual Variation in Scale Factor	0.08	0.08	0.08	0.08
Nonlinearity	0.10	0.10	0.10	0.10
Root Sum Square	0.25	0.24	0.25	0.23

Table 10. Contributions to Error in Measurement of Gyroscope Drift Rate in EW Direction

	mas/yr			
Source of Error	Gyro 1	Gyro 2	Gyro 3	Gyro 4
Statistical Error (from Table 4)	0.13	0.14	0.16	0.12
Bias Variations	0.14	0.085	0.080	0.080
Residual Annual Variation in Scale Factor	0.08	0.08	0.08	0.08
Nonlinearity	0.10	0.10	0.10	0.10
Root Sum Square	0.23	0.21	0.22	0.19

Appendix A – Accuracy of Earth's Velocity in JPL Ephemerides

Date: Thu, 25 Sep 2003 09:58:16 -0700
From: e m standish <ems@smyles.jpl.nasa.gov>
To: ilya@caltech.edu
Subject: Accuracy of JPL ephemerides for Earth velocity computation
Ilya Mandel
September 25, 2003

The velocity of the earth w.r.t. the sun, given in the ICRF (International Celestial Reference Frame), is accurate to about 0.2 mm/sec. Most of this comes from the orientation into the reference frame, which is good to about 0.001-0.002 arcseconds, implying a 1 km rotation at the distance of the earth from the sun. A sinusoid of that amplitude with a one-year period will give a velocity of about 0.2 mm/sec.

The location of the solar system barycenter (i.e., the sun with respect to the SSB), which depends upon the uncertainties in the masses of the planets, is much less well-known as far as the position is concerned. Most of the positional uncertainty comes from Saturn and Pluto, and, fortunately, these are slow-moving, so their contributions to the SSB velocity uncertainty are small. Thus, while the SSB's position is uncertain to a number of kilometers, the velocity is good to 0.02 mm/sec or better.

The bottom line: the earth's velocity in the SSB is known to 0.2 mm/sec.

I hope this helps,

Myles Standish
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